# "Practice in Theory" Solution 

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## 1 Puzzle solution

The answer to each problem is a large integer which is actually a hep-th arXiv preprint identifier, as clued by the flavor text. For some problems, you are given seven boxes, which means it is an old style (pre April 2008) arXiv identifier (i.e. one which is formatted hep-th/YYMMxxx), and for others, you are given eight boxes, which means it is a new one (i.e. one which is formatted YYMM.xxxx). Some 8-box problems will have 7-digit answers, and some 7-box problems have 6 -digit answers. For these, you must prepend a zero (i.e. it was published in 200x). There are no answers which require two or three prepended zeros (i.e. there are none published in the year 2000). Each paper has a unique, distinguished letter in the title that you extract. Sorting by date gives you "GULAG ARCHIPELAGO AUTHOR", which clues SOLZHENITSYN. In addition, there is a check step: each word problem has a character with some name, and that name is the first name of the first author on each paper.

The full list of answers is given below, and the solutions to each problem follow afterwards:

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G - MATTHIAS (second one) - 9407042
U - TIM - 9505003
L - JIANYONG - 0412227
A - YUJI - 0507057
G - HERBERT - 0602108
A - PAUL - 08064592
R - NICK - 08073679
C - MATTHIAS (first one) - 08113892
H - SERGEI - 10035333
I - SEUNG-YEOP - 10050369
P - SOPHIA - 10082963
E - ANDREAS - 10102218
L - MARIO - 11035468
A - YU - 11102586
G - JOHANN - 12051835
O-ALI - 12073717
A - PHILLIP - 12093025
U - BIN - 13040568
T - EUGENIU - 13040772
H - ROBERTO - 13070705
O-ANN - 14013504
R - TSUGUHIKO - 14082649
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## 2 Solutions to each problem

Here are detailed solutions to each problem, pedagogically written in case it's been a while since you last thought about physics.

## Problem 1 (Phillip)

If $g$ is the gravitational field strength, the equation for balancing the torques is:

$$
\begin{equation*}
(48372100 \mu \mathrm{~g}) g \cdot 117000 \mu \mathrm{~m}=(x g) \cdot 468000 \mu \mathrm{~m} \Longrightarrow x=12093025 \mu \mathrm{~g} \tag{2.1}
\end{equation*}
$$

arXiv identifier 1209.3025 points to "Comments on a-maximization from gauged supergravity" by Phillip Szpietowski, which gives you the letter A for extraction.

## Problem 2 (Sophia)

If the fish is thrown with initial horizontal velocity $v$ from a height $h$ and travels a distance $d$ in a time $t$ before hitting the ground, the kinematical equations in the horizontal and vertical directions, respectively, are:

$$
\begin{align*}
d & =v t  \tag{2.2}\\
h & =\frac{1}{2} g t^{2} \tag{2.3}
\end{align*}
$$

where $g$ is the gravitational acceleration. Simultaneously solving the equations with $h=1524000 \mu \mathrm{~m}$, $d=5623188 \mu \mathrm{~m}$, and $g=9800000 \mu \mathrm{~m} / \mathrm{s}^{2}$ gives a velocity $v=10082963 \mu \mathrm{~m} / \mathrm{s}$.
arXiv identifier 1008.2963 points to "Setting the scale of the p p and p bar p total cross sections using AdS/QCD" by Sophia K. Domokos, et al., which gives you the letter $\mathbf{P}$ for extraction.

## Problem 3 (Johann)

Let $H$ be the height of the barrel, $h$ be the height at which the hole is located, $v$ be the fluid velocity at the hole, $\rho$ be the fluid density, $P_{\text {atm }}$ be the atmospheric pressure, and $g$ be the gravitational field strength. Since the fluid velocity at the top of the barrel is basically zero, Bernoulli's principle yields

$$
\begin{align*}
P_{\mathrm{atm}} & +\frac{1}{2} \rho v^{2}+\rho g h=P_{\mathrm{atm}}+\rho g H  \tag{2.4}\\
\Rightarrow v & =\sqrt{2 g(H-h)} \tag{2.5}
\end{align*}
$$

Now we have to solve the kinematics problem as in the previous problem. Johann and friends will land at a distance $d$ away from the hole after a time $t$ determined by the equations

$$
\begin{align*}
d & =v t  \tag{2.6}\\
h & =\frac{1}{2} g t^{2} \tag{2.7}
\end{align*}
$$

This yields the quadratric equation

$$
\begin{equation*}
d=2 \sqrt{(H-h) h} \tag{2.8}
\end{equation*}
$$

That is, the solution does not depend on $g$ at all. Plugging in the given values of $H$ and $d$, solving the quadratic, and taking the smaller value of $h$ gives $h=12051835 \mu \mathrm{~m}$.
arXiv identifier 1205.1835 points to "A Cusp in QED at $\mathrm{g}=2$ " by Johann Rafelski and Lance Labun, which gives you the letter $\mathbf{G}$ for extraction.

## Problem 4 (Tim)

The moment of inertia of a cylinder of radius $R$ and mass $M$ with respect to its axis is $I=\frac{1}{2} M R^{2}$. Let $h$ be the height of the hill and $g$ be the gravitational field strength. Let $v$ be the translational velocity of the cylinder and $\omega$ be the angular velocity of the cylinder with respect to its axis. Conservation of energy yields:

$$
\begin{equation*}
M g h=\frac{1}{2} M v^{2}+\frac{1}{2} I \omega^{2} \tag{2.9}
\end{equation*}
$$

Substituting our expression for $I$ and the rolling without slipping condition $v=r \omega$, we obtain $\omega=\frac{1}{R} \sqrt{\frac{4 g h}{3}}$. Plugging in the given numbers, we find $\omega=9505003 \mathrm{Ms}^{-1}$.
arXiv identifier hep-th/9505003 points to "Two Phases for Compact $\mathrm{U}(1)$ Pure Gauge Theory in Three Dimensions" by Tim Morris, which gives you the letter $\mathbf{U}$ for extraction.

## Problem 5 (Bin)

Let $H$ be the initial height, and let $M$ and $m$ be the masses of Bin and the pearl, respectively. Conservation of energy tells us that the initial velocities of the two spheres at the point of collision with the floor are the same. For the pearl, e.g.:

$$
\begin{equation*}
m g h=\frac{1}{2} m v_{0}^{2} \Longrightarrow v_{0}=\sqrt{2 g H} \tag{2.10}
\end{equation*}
$$

When Bin collides with the floor, his velocity changes sign, but not magnitude, since the collision is elastic. Immediately afterwards, Bin collides with the pearl. Let $V$ and $v$ be Bin's velocity and the pearl's velocity after this second collision. Conservation of momentum and conservation of energy, respectively, yield:

$$
\begin{align*}
M v_{0}-m v_{0} & =m v+M V  \tag{2.11}\\
\frac{1}{2}(M+m) v_{0}^{2} & =\frac{1}{2} M V^{2}+\frac{1}{2} m v^{2} \tag{2.12}
\end{align*}
$$

This is a system of two equations for two unknowns ( $v$ and $V$ ) and therefore can be solved explicitly by standard algebraic techniques. The solution for $v$ corresponding to upward motion of the pearl is

$$
\begin{equation*}
v=\frac{3 M-m}{M+m} \sqrt{2 g H} \tag{2.13}
\end{equation*}
$$

Let $h$ be the maximum height of the pearl after this first bounce. Conservation of energy applied to the pearl after the bounce yields:

$$
\begin{equation*}
m g h=\frac{1}{2} m v^{2} \tag{2.14}
\end{equation*}
$$

What we really want is the height relative to its initial position - i.e. $h-H$. Substituting all quantities and simplifying, we obtain

$$
\begin{equation*}
h-H=\frac{8 H(1-m / M)}{(1+m / M)^{2}} \tag{2.15}
\end{equation*}
$$

Substituting the given numbers, we obtain $h-H=13040568 \mu \mathrm{~m}$. arXiv identifier 1304.0568 points to "Smooth U(1) Gauge Potentials on the de Sitter Spacetime" by Bin Zhou, et. al., which yields $\mathbf{U}$ for extraction.

## Problem 6 (Roberto)

The resonant frequency of a series RLC circuit occurs at the frequency where the inductive and capacitive impedances cancel each other out, thereby minimizing the total impedance:

$$
\begin{equation*}
i \omega_{0} L+\frac{1}{i \omega_{0} C}=0 \Longrightarrow \omega_{0}=\frac{1}{\sqrt{L C}} \tag{2.16}
\end{equation*}
$$

The quality factor is defined to be $Q=\omega_{0} / \Delta \omega$, where $\Delta \omega$ is the width of the average power curve at half-maximum. The average power is

$$
\begin{equation*}
P=\bar{I}^{2} R=\bar{V}^{2} R / Z^{2} \tag{2.17}
\end{equation*}
$$

where $\bar{I}$ and $\bar{V}$ are the RMS current and voltage, respectively. Subsituting in the impedance $Z=R+i \omega L+1 / i \omega C$, a short computation gives that the quality factor is

$$
\begin{equation*}
Q=\frac{\omega_{0} L}{R} \tag{2.18}
\end{equation*}
$$

Since you are given $Q, \omega_{0}$, and $R$ in the problem statement, this equation can be used to solve for $L$, and then equation 2.16 can be used to solve for $C$. This turns out to be 13070705 fF . arXiv identifier 1307.0705 points to "QCD corrections to $\mathrm{H} \rightarrow \mathrm{gg}$ in FDR" by Roberto Pittau, which gives you the letter $\mathbf{H}$ for extraction.

## Problem 7 (Tsuguhiko)

The standard way to solve this would be by using the principle of superposition. You would compute the electric field at the given point due to a solid sphere of charge density $\rho$ with no hole and the electric field at the given point due to a solid sphere of charge density $-\rho$ located where the hole is located, and add the two results together. However, since Tsuguhiko is located at the center of the hole - i.e. the center of this second sphere, this second contribution is just zero, so it suffices to computer the electric field at that point assuming the hole does not even exist.

The force Tsuguhiko feels is $F=q E$, where $q$ is his charge and $E$ is the electric field at his location. $E$ can be computed from the integral form of Gauss's law:

$$
\begin{equation*}
\oint_{\Sigma} E \cdot \mathrm{~d} A=Q / \epsilon_{0} \tag{2.19}
\end{equation*}
$$

where $\Sigma$ is some closed surface, $E$ is the electric field $\mathrm{d} A$ is the surface element, $Q$ is the enclosed charge and $\epsilon_{0}$ is the permittivity of free space. Here, we can choose $\Sigma$ to be a sphere centered at the center of the large sphere with radius $R / 2$. Since we may ignore the hole for the special point we have chosen, spherical symmetry implies that the left hand side is equal to $4 \pi(R / 2)^{2} E$. Then, $Q=V \rho$, where $V=\frac{4}{3} \pi(R / 2)^{3}$ is the volume of the sphere enclosed by $\Sigma$. Solving for $E$ and plugging in all the numbers, one finds that the force Tsuguhiko feels is $F=14082649 \mu \mathrm{~N}$.
arXiv identifier 1408.2649 points to "Poisson-generalized geometry and R-flux" by Tsuguhiko Asakawa, et. al, which gives you the letter $\mathbf{R}$ for extraction.

## Problem 8 (Matthias I)

Let $x=0$ be the top of the spring with the $x$ axis oriented so that $x$ increases as you go down the spring. Consider some infinitesimal segment of the spring from $x$ to $x+d x$. There is some tension $T(x)$ at the top of the spring and $T(x+d x)$ at the bottom of the spring, pulling the infinitesimal
segment in opposite directions. There is also the force due to gravity, which pulls downward with magnitude $\lambda d x g$. Newton's second law therefore yields that

$$
\begin{align*}
T(x+d x)+\lambda d x g-T(x) & =0 \\
\Rightarrow \frac{d T}{d x} & =-\lambda g \tag{2.20}
\end{align*}
$$

It is well-known that if you cut a spring of constant $k$ into $n$ pieces, you get $n$ springs each with constant $k n$ (i.e. "springs are like capacitors") ${ }^{1}$. So the spring constant of this tiny piece is $k=K L / d x$.

Now, let $a(x)$ be the displacement of the spring at position $x$ relative to its equilibrium position (i.e. the position it would have if $g=0$ ). The tension $T(x)$ at the location of the infinitesimal piece is given by $T(x)=k[a(x+d x)-a(x)]=K L d a / d x .^{2}$ Substituting this expression into 2.20, we obtain

$$
\begin{equation*}
\frac{d T}{d x}=K L \frac{d^{2} a}{d x^{2}}=-\lambda g \tag{2.21}
\end{equation*}
$$

The solution to this equation is

$$
\begin{equation*}
a(x)=-\frac{\lambda g x^{2}}{2 K L}+C x+D \tag{2.22}
\end{equation*}
$$

where $C$ and $D$ are integration constants. These are determined by the boundary conditions $T(L)=0$ and $a(0)=0$. imposing them, we find

$$
\begin{equation*}
a(x)=\frac{-\lambda g x^{2}}{2 K L}+\frac{\lambda g x}{K} \tag{2.23}
\end{equation*}
$$

We want to compute $a(1 / 2)$. Substituting the given values of $K, L, \lambda$, and $g$, we obtain $a(1 / 2)=$ 8113892 nm . This answer appears to have too few digits since there are 8 boxes given and the answer is only 7 digits long. We prefix with a 0 , since it is the same number even with a prefixed zero. A more motivated way to know that one should do this comes from understanding the extraction mechanism. We want valid arXiv identifiers, so we need to prepend a digit, and the only one that doesn't change the numerical value of the answer is 0 .
arXiv identifier 0811.3892 refers to "Zhu's algebra, the $C_{2}$ algebra, and twisted modules" by Matthias Gaberdiel and Terry Gannon, which yields the letter $\mathbf{C}$ for extraction.

## Problem 9 (Matthias II)

This problem may be solved by the method of images. Recall that the classic example of a point charge sitting above a plane is solved by putting a image charge of opposite charge at the point given by reflecting the charge's position across the plane. The same trick works here, except the given geometry suggests we need to reflect "all the way around" - i.e. we should put 7 image charges at the vertices of a regular hexagon with the real charge sitting at the 8th vertex. The charges alternate $+q$ and $-q$ around the hexagon, where $q$ is the given charge that Matthias carries. It is a tedious algebraic exercise to verify that the required boundary conditions for the electric field at the conducting surfaces are indeed satisfied (i.e. the electric field at those surfaces is perpendicular to the surface), and we will not reproduce the argument here.

[^0]For a hexagon of radius 1 , simple trigonometry gives that the edge length $a=\sqrt{2-\sqrt{2}}$ and the three diagonals have length $d_{1}=\sqrt{2}, d_{2}=a(1+\sqrt{2}), d_{3}=2$. It is clear from the geometry of the situation that the total force will be parallel to the line connecting Matthias to the center of the hexagon. Hence, the total force is:

$$
\begin{equation*}
F=\frac{q^{2}}{4 \pi \epsilon_{0}}\left(2 \frac{\sin (\pi / 8)}{a^{2}}-2 \frac{\sin (\pi / 4)}{d_{1}^{2}}+2 \frac{\sin (3 \pi / 8)}{d_{2}^{2}}-\frac{\sin (\pi / 2)}{d_{3}^{2}}\right)=9407042 \mu \mathrm{~N} \tag{2.24}
\end{equation*}
$$

arXiv identifier hep-th/9407042 points to "Equivariant Kaehler Geometry and Localization in the G/G Model" by Matthias Blau and George Thompson, which yields the letter G for extraction.

## Problem 10 (Herbert)

For a point on the wall at angle $\theta$, the difference in the distance the light between adjacent slits must travel is $\Delta L=d \sin \theta$. Hence, light from adjacent slits will be out of phase when they arrive at the wall. The difference in phase $\delta$ is given by the fraction of a period of oscillation which elapsed over this extra distance: $\delta=2 \pi \frac{\Delta L}{\lambda}$. Therefore, at the screen, the amplitude of the three waves will be given by

$$
\begin{align*}
& A_{0}=A \sin (\omega t-\delta)  \tag{2.25}\\
& A_{1}=A \sin (\omega t)  \tag{2.26}\\
& A_{2}=A \sin (\omega t+\delta) \tag{2.27}
\end{align*}
$$

where here $\omega$ is the angular frequency of the light and we have ignored the possibility of a constant phase offsetting all three amplitudes by choosing the origin of our time coordinate appropriately. The sum of these waves can be simplified by using the trigonometric identities for sums and differences of angles

$$
\begin{align*}
A & =A_{0}+A_{1}+A_{2}  \tag{2.28}\\
& =\ldots  \tag{2.29}\\
& =A(1+2 \cos \delta) \sin (\omega t) \tag{2.30}
\end{align*}
$$

Thus, a dark spot will be observed when $\cos \delta=-1 / 2$. The smallest such $\theta$ will be generated when $\delta=2 \pi / 3$, i.e. $\theta=\sin ^{-1}(\lambda / 3 d)$. Substituting in the numbers, we obtain that $\theta=607228 \mu \mathrm{rad}$. Again, we appear to be one digit short of the required 7 digits, so we prepend a 0 . arXiv identifier hep-th/0607228 points to "Renormalization Group Running of Newton's G: The Static Isotropic Case" by Herbert Hamber and Ruth Williams, which yields the letter G for extraction.

## Problem 11 (Sergei)

We need to compute the efficiency of the given "box cycle" for a photon gas. Using the thermodynamic identity

$$
\begin{equation*}
d U=T d S-P d V \tag{2.31}
\end{equation*}
$$

on a path that does not change the volume $(d V=0)$, we find that

$$
\begin{equation*}
4 \sigma V T^{3}=T d S \Rightarrow S=\frac{4}{3} \sigma V T^{3} \tag{2.32}
\end{equation*}
$$

where we have set the boundary condition for the differential equation by using the third law of thermodynamics ( $S=0$ at $T=0$ ). Plugging this back in to the thermodynamic identity, but now considering a general deformation, we obtain:

$$
\begin{equation*}
4 \sigma V T^{3} d T+\sigma T^{4} d V=T\left(4 \sigma V T^{2} d T+\frac{4}{3} \sigma T^{3} d V\right)-P d V \Rightarrow P=\frac{1}{3} \sigma T^{4} \tag{2.33}
\end{equation*}
$$

Or to put it another way

$$
\begin{equation*}
3 P V=U \tag{2.34}
\end{equation*}
$$

Let's consider the heat taken in at each point in the cycle. In the isochoric expansion phase, no work is done, so the entire change in internal energy during this phase is heat intake. Thus, $\Delta U=\Delta Q=3 V \Delta P=3 \times 77 \times P V=231 P V$. In the isobaric expansion phase, the work done is $78 P \times 56 \mathrm{~V}$, and the change in internal energy is $3 P \Delta V=3 \times 78 P \times 56 V$. Thus, $\Delta Q=\Delta U+\Delta W=$ $4 \times 78 P \times 56 V=17472 P V$. There is no heat intake in the contraction phases. So the total heat taken is $17703 P V$. The total work done is $77 P \times 56 V=4312 P V$. So the efficiency is $4312 / 17703$ and so the total work done is $41200255.125 \mathrm{~J} \times 4312 / 17703=10035333 \mathrm{~J}$.
arXiv identifier 1003.5333 points to "Quantum Sine(h)-Gordon Model and Classical Integrable Equations" by Sergei Lukyanov and Alexander Zamolodchikov, which yields $\mathbf{H}$ for extraction.

## Problem 12 (Mario)

The fastest way to solve this problem is to notice that the given "interaction" is not really an interaction at all in the sense that a judicious change of basis factorizes the Hamiltonian into two decoupled sectors. In fact, the good basis to use is just the center of mass basis. Define the following quantities:

$$
\begin{align*}
x & =x_{1}-x_{2}  \tag{2.35}\\
X & =\frac{x_{1}+x_{2}}{2} \tag{2.36}
\end{align*}
$$

The appropriate conjugate momenta that satisfy the canonical commutation relations are:

$$
\begin{align*}
& p=\frac{p_{1}-p_{2}}{2}  \tag{2.37}\\
& P=p_{1}+p_{2} \tag{2.38}
\end{align*}
$$

Now define $\mu=m / 2$ and $M=2 m$. Then, we can rewrite the Hamiltonian as:

$$
\begin{align*}
H & =H_{\mu}+H_{M}  \tag{2.39}\\
H_{\mu} & =\frac{p^{2}}{2 \mu}+\frac{\mu x^{2}}{2}\left(\omega^{2}-\frac{g}{m}\right)  \tag{2.40}\\
H_{\mu} & =\frac{P^{2}}{2 M}+\frac{M X^{2}}{2}\left(\omega^{2}+\frac{g}{m}\right) \tag{2.41}
\end{align*}
$$

where we have $\left[H_{\mu}, H_{M}\right]=0$. Hence, we have two independent harmonic oscillators of frequencies $\omega_{ \pm}^{2}=\omega^{2} \pm g / m$. We know from elementary quantum mechanics that the spectrum is therefore indexed by a pair of non-negative integers $\left(n_{\mu}, n_{M}\right)$ corresponding to a state of energy

$$
\begin{equation*}
E_{n_{\mu}, n_{M}}=\hbar \omega_{+}\left(n_{M}+\frac{1}{2}\right)+\hbar \omega_{-}\left(n_{\mu}+\frac{1}{2}\right) \tag{2.42}
\end{equation*}
$$

The ground state is $n_{\mu}=n_{M}=0$. The first excited state is given by $n_{\mu}=1, n_{M}=0$. Running the numerical values, one obtains $E_{1,0}=11035468 \mathrm{fJ}$.
arXiv identifier 1103.5468 points to "Generalised massive gravity one-loop partition function and AdS/(L)CFT" by Mario Bertin, et. al., which yields $\mathbf{L}$ for extraction.

## Problem 13 (Andreas)

Observe the following two facts. First, the joint wavefunction, which is the tensor product of the spin part and the spatial part, must be antisymmetric with respect to exchange of the fermions. This means that either the spatial or spin part must be odd under exchange and the other must be even. Second, since Andreas measures either 0 or $2 \hbar$ for $\left(S_{1}+S_{2}\right)^{2}$, there must be a ground-state degeneracy.

The single-particle spatial eigenstates are simply given by the free result. For a circle of circumference 1 , this is:

$$
\begin{equation*}
\psi_{n}(x)=e^{2 \pi i n x} \tag{2.43}
\end{equation*}
$$

where $n=0,1,2, \ldots$ and the corresponding energy $E_{n}=\hbar^{2}(2 \pi n)^{2} /(2 m)$. The lowest-energy twoparticle spatial wavefunction is $\psi_{0}\left(x_{1}\right) \psi_{0}\left(x_{2}\right)$ - i.e. both particles are in the ground state, which is symmetric under exchange. The next-lowest energy level of the two-particle spatial Hamiltonian is infinitely degenerate, but one representative which is an eigenstate of the exchange operator is

$$
\begin{equation*}
\frac{1}{\sqrt{2}}\left(\psi_{0}\left(x_{1}\right) \psi_{1}\left(x_{2}\right)-\psi_{1}\left(x_{0}\right) \psi_{0}\left(x_{1}\right)\right) \tag{2.44}
\end{equation*}
$$

i.e. the antisymmetrized combination of one particle in the ground state, the other in the excited state, which is antisymmetric under exchange.

Consider now the spin piece of the wavefunction. We work in a basis of eigenstates of $\left(S_{1}+S_{2}\right)^{2}$. Here we have the triplet of symmetric two-particle states, $|\uparrow \uparrow\rangle, \frac{1}{\sqrt{2}}(|\uparrow \downarrow\rangle+|\downarrow \uparrow\rangle),|\downarrow \downarrow\rangle$ and the antisymmetric singlet state $\frac{1}{\sqrt{2}}(|\uparrow \downarrow\rangle-|\downarrow \uparrow\rangle)$. Direct computation yields that the triplet states have $\left(S_{1}+S_{2}\right)^{2}$ eigenvalue equal to $2 \hbar^{2}$ and the singlet has eigenvalue 0 . Moreover, $S_{1}^{2}=S_{2}^{2}=\frac{3}{4} \hbar^{2}$ for all of these states. Therefore, the triplet states have $S_{1} \cdot S_{2}$ eigenvalue $\hbar^{2} / 4$ and the singlet state has $S_{1} \cdot S_{2}$ eigenvalue $-\frac{3}{4} \hbar^{2}$.

Combining spatial and spin parts, the symmetric spatial ground state with energy 0 must be paired with the singlet antisymmetric spin state, giving a total energy of $\frac{3 g}{4} \hbar^{2}$. The antisymmetric first-excited spatial wavefunction with energy $E_{1}=\hbar^{2}(2 \pi)^{2} /(2 m)$ must be paired with a symmetric spin state (a triplet state), for a total energy $\hbar^{2}(2 \pi)^{2} /(2 m)-g \hbar^{2} / 4$.

That Andreas measures two values for $\left(S_{1}+S_{2}\right)^{2}$ indicates a degeneracy. Since we are given the value of $g$, we must set these two energies equal in order to determine the mass:

$$
\begin{align*}
\frac{\hbar^{2}(2 \pi)^{2}}{2 m}-\frac{g \hbar^{2}}{4} & =\frac{3 g}{4} \hbar^{2}  \tag{2.45}\\
\Longrightarrow m=\frac{2 \pi^{2}}{g} & =10102218 \mathrm{eV} \tag{2.46}
\end{align*}
$$

arXiv identifier 1010.2218 points to "PT invariant complex E(8) root spaces" by Andreas Fring and Monique Smith, which yields $\mathbf{E}$ for extraction.

## Problem 14 (Eugeniu)

We solve the Laplace equation by separation of variables. We begin by finding a potential $V(r, \theta, \phi)$ that satisfies the appropriate boundary conditions. As is customary in the separation-of-variables approach, we take as an ansatz a $V$ of the form $V=R(x) \Theta(\theta) \Phi(\phi)$ - or, more specifically, a linear combination of terms of that form. The Laplace equation in spherical coordinates in a system with azimuthal symmetry specifies that

$$
\begin{equation*}
\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \frac{\partial V}{\partial r}\right)+\frac{1}{r^{2} \sin (\theta)} \frac{\partial}{\partial \theta}\left(\sin (\theta) \frac{\partial V}{\partial \theta}\right)=0 \tag{2.47}
\end{equation*}
$$

The Laplace equation along with the assumption of separation of variables implies that $V$ is of the form

$$
\begin{equation*}
V=\sum_{n=0}\left(A_{n} r^{n}+B_{n} r^{-n-1}\right) P_{n}(\cos (\theta)) \tag{2.48}
\end{equation*}
$$

where the $P_{n}$ are Legendre polynomials.
Our boundary conditions specify that $\left.V\right|_{(r=R)}=0$, where $R$ is the radius of the sphere, since the sphere is a metal sphere, and $V(r \rightarrow \infty)=-E_{0} r \cos (\theta)$, since we began with a uniform electric field $E_{0} \hat{z}$ filling all of space. Applying these to the expression above, we find that

$$
\begin{equation*}
V(r, \theta, \phi)=-E_{0} R \cos (\theta)\left(\frac{r}{R}-\frac{R^{2}}{r^{2}}\right) . \tag{2.49}
\end{equation*}
$$

The surface charge is then given by

$$
\begin{equation*}
\sigma=-\left.\epsilon_{0} \vec{n} \cdot \nabla V\right|_{r=R}=3 \epsilon_{0} E_{0} \cos (\theta) \tag{2.50}
\end{equation*}
$$

Plugging in the numbers given, we note that the value of $\phi$ does not matter and $\sigma(\theta=0.42681131 \pi)=$ $13040772 \mathrm{nC} / \mathrm{cm}^{2}$.
arXiv identifier 1304.0772 points to "Short-Range Entangled Bosonic States with Chiral Edge Modes and T-duality of Heterotic Strings" by Eugeniu Plamadeala, et. al., which yields T for extraction.

## Problem 15 (Nick)

We begin by writing the single-jellyfish partition function: $Z=\sum_{i} e^{-\beta E_{i}}$, where $i$ runs over the $g+1$ states. This is just $Z=1+e^{a \lambda} e^{-\beta h \lambda}$. The total partition function is then $Z^{N} . f$ is therefore given by

$$
\begin{equation*}
f=-\frac{1}{h \lambda Z} \partial_{\beta} Z=\frac{e^{a \lambda-\beta h \lambda}}{1+e^{a \lambda-\beta h \lambda}} . \tag{2.51}
\end{equation*}
$$

Taking a limit as $\lambda \rightarrow \infty$, we find that $f$ is either 1 or 0 depending on whether $a-\beta h$ is positive or negative, respectively. The critical $\beta$ therefore occurs at $\beta=\frac{a}{h}$; that is, $T=\frac{h}{a k_{B}}=8073679$ millikelvins. As before, this answer is one digit short, which we rectify with a leading 0 . arXiv identifier 0807.3679 points to "R-Charge Chemical Potential in a $2+1$ Dimensional System", by Nick Evans and Ed Threlfall, giving $\mathbf{R}$ for extraction.

## Problem 16 (Ann)

The Lagrangian in cylindrical coordinates $(r, \theta, z)$ is

$$
\begin{equation*}
\mathcal{L}=\frac{1}{2} m\left(\dot{r}^{2}+r^{2} \dot{\theta}^{2}+\dot{z}^{2}\right)-m g z \tag{2.52}
\end{equation*}
$$

Since it is rotating with constant angular velocity $\omega, \dot{\theta}=\omega$. From the shape of the wire given, we have $\dot{z}=\frac{d z}{d r} \dot{r}=3 k r^{2} \dot{r}$. Thus, the Lagrangian is actually a function of just the variables $r$ and $\dot{r}$ :

$$
\begin{equation*}
\mathcal{L}=\frac{1}{2} m\left(\dot{r}^{2}+r^{2} \omega^{2}+9 k^{2} r^{4} \dot{r}^{2}\right)-m g k r^{3} \tag{2.53}
\end{equation*}
$$

We are looking for equilibrium points, so we compute the equation of motion from $r$ arising from this Lagrangian and set all the time derivatives equal to zero. This yields the following simple equation:

$$
\begin{equation*}
m r \omega^{2}-3 g k r^{2}=0 \tag{2.54}
\end{equation*}
$$

The nontrivial fixed point is given by

$$
\begin{equation*}
r=\frac{\omega^{2}}{3 g k} \tag{2.55}
\end{equation*}
$$

Substituting all the given numerical values gives $r=14013504 \mathrm{~nm}$. arXiv identifier 1401.3504 points to "Corner contribution to the entanglement entropy of an $\mathrm{O}(3)$ quantum critical point in $2+1$ dimensions" by Ann Kallin, et. al., which yields $\mathbf{O}$ for extraction.

## Problem 17 (Seung-Yeop)

The photon trajectories are given by the null geodesics along $x$ and $y$ at the $z=0$ plane.

$$
\begin{align*}
& 0=d s^{2}=-d t^{2}+(1+A \cos (\omega t)) d x^{2} \Longrightarrow d x \approx d t\left(1-\frac{1}{2} A \cos (\omega t)\right)  \tag{2.56}\\
& 0=d s^{2}=-d t^{2}+(1-A \cos (\omega t)) d y^{2} \Longrightarrow d y \approx d t\left(1+\frac{1}{2} A \cos (\omega t)\right) \tag{2.57}
\end{align*}
$$

Integrating both sides so that $\int d x=\int d y=2 L$, we obtain

$$
\begin{align*}
& 2 L=t-\frac{A}{2 \omega} \sin (\omega t)  \tag{2.58}\\
& 2 L=t+\frac{A}{2 \omega} \sin (\omega t) \tag{2.59}
\end{align*}
$$

So simultaneous arrival is only possible when $\sin (\omega t)=0$. The smallest $L$ is generated by the smallest nonzero value of $t$, which is $\pi / \omega$, or $L=\pi / 2 \omega$. Restoring the factor of $c$, we have $L=c \pi / 2 \omega$, and plugging in the given value of $\omega$, we obtain $L=10050369 \mathrm{~m}$.
arXiv identifier 1005.0369 points to "Viscous shocks in Hele-Shaw flow and Stokes phenomena of the Painleve I transcendent" by Seung-Yeop Lee, et. al., which yields I for extraction.

## Problem 18 (Yuji)

There's only one Feynman diagram to evaluate since there is only one term in the Standard Model Lagrangian that couples the $W$ to leptons:

$$
\begin{equation*}
\Delta \mathcal{L}=\frac{1}{\sqrt{2}} \frac{e}{\sin \theta_{w}} W_{\mu}^{+} \sum_{i} \bar{\nu}_{i \mathrm{~L}} \gamma^{\mu} e_{i \mathrm{~L}} \tag{2.60}
\end{equation*}
$$


where the sum on $i$ is over generations, and $\left(\bar{\nu}_{i}, e_{i}\right)$ are the lepton pairs, and $L$ is the left-handed spinor part of the lepton fields. Inserting the left-handed projector explicitly, the amplitude for the decay into any particular lepton pair is

$$
\begin{equation*}
i \mathcal{M}=\frac{i e}{\sqrt{2} \sin \theta_{w}} \epsilon_{\mu}(k) \bar{u}(p) \gamma^{\mu} \frac{1-\gamma^{5}}{2} v(q) \tag{2.61}
\end{equation*}
$$

Here, $\epsilon$ is the $W^{+}$polarization vector, $u, v$ are the standard spinors solving the massless Dirac equation, and $k, p, q$ are the momenta of the $W^{+}$, anti-neutrino, and corresponding lepton. In the unitary gauge (which is typically most convenient at tree level), the spin averaged amplitude is

$$
\begin{equation*}
\frac{1}{3} \sum_{s}|i \mathcal{M}|^{2}=\frac{e^{2}}{6 \sin ^{2}\left(\theta_{w}\right)}\left(-g_{\mu \nu}+\frac{k_{\mu} k_{\nu}}{m_{W}^{2}}\right) \operatorname{tr}\left(\not p \gamma^{\mu} \frac{1-\gamma^{5}}{2} \not q \nu^{\nu} \frac{1-\gamma^{5}}{2}\right) \tag{2.62}
\end{equation*}
$$

In the center of mass frame, $k=\left(m_{W}, 0,0,0\right), p=\left(m_{W} / 2,0,0, m_{W} / 2\right), q=\left(m_{W} / 2,0,0,-m_{W} / 2\right)$. A bunch of trace algebra later, one obtains the simple answer

$$
\begin{equation*}
\frac{1}{3} \sum_{s}|i \mathcal{M}|^{2}=\frac{m_{W}^{2} e^{2}}{3 \sin ^{2}\left(\theta_{w}\right)} \tag{2.63}
\end{equation*}
$$

The natural linewidth is given by applying Fermi's golden rule:

$$
\begin{equation*}
\Gamma=\int d \Gamma=\int \frac{d^{3} p d^{3} q}{(2 \pi)^{6} 2 E_{p} E_{q}}\left(\frac{m_{W}^{2} e^{2}}{3 \sin ^{2}\left(\theta_{w}\right)}\right) \frac{1}{2 m_{W}}(2 \pi)^{4} \delta^{(4)}(k-p-q) \tag{2.64}
\end{equation*}
$$

After some more algebra:

$$
\begin{equation*}
\Gamma=\frac{\alpha m_{W}}{12 \sin ^{2}\left(\theta_{w}\right)} \tag{2.65}
\end{equation*}
$$

where $\alpha=e^{2} / 4 \pi$ is the fine structure constant. Plugging in all the numbers and multiplying by the given number of generations, we obtain 507057 GeV . As we have seen in previous problems, we should prepend a 0 to this to obtain a valid arXiv hep-th identifier.
arXiv identifier hep-th:0507057 points to "Five-dimensional Supergravity Dual of a-Maximization" by Yuji Tachikawa, which yields $\mathbf{A}$ for extraction.

## Problem 19 (Paul)

Let's work in $c=1$ units until the end. A flat FRW-universe is described by the metric

$$
\begin{equation*}
d s^{2}=-d t^{2}+a^{2}(t)\left(d r^{2}+r^{2} d \Omega^{2}\right) \tag{2.66}
\end{equation*}
$$

where $a$ is the scale factor and $r$ is the comoving distance. We can just read off the size of the sphere at the location of the object from the metric. If the angular size is $\theta$, the proper size $s$ is equal to $a\left(t^{*}\right) r \theta$, where $t^{*}$ is the time at which the light we are seeing today was emitted. Of course, neither $a$ nor $r$ is observable, so we have to convert those into observable quantities.
$a$ can be handled just by knowing the definition of the redshift factor. $a$ today is 1 and so $a\left(t^{*}\right)=1 /(1+z) . r$ can be determined by integration on radially directed null geodesics from the object to our present position:

$$
\begin{equation*}
0=d s^{2}=-d t^{2}+a^{2} d r^{2} \Longrightarrow r=\int \frac{d t}{a} \tag{2.67}
\end{equation*}
$$

We may now use the definition of the Hubble constant $H=\dot{a} / a$ and the definition of the redshift $a=1 / 1+z$ to simplify:

$$
\begin{equation*}
r=\int_{t^{*}}^{t_{0}} \frac{d t}{a}=\int_{a\left(t^{*}\right)}^{1} \frac{d a}{a^{2} H(a)}=-\int_{z}^{0} \frac{d z}{H(z)} \tag{2.68}
\end{equation*}
$$

The Hubble constant may be related to the redshift using the Friedmann equation. We're told that the universe is matter dominated, so we may neglect all other energy sources so that the Friedmann equation is simply

$$
\begin{equation*}
H^{2}=\frac{8 \pi G}{3} \rho_{\mathrm{m} 0}(1+z)^{3} \tag{2.69}
\end{equation*}
$$

where $\rho_{\mathrm{m} 0}$ is the matter density today. But we know that $H=H_{0}$ at redshift zero (i.e. now), so this is actually equal to

$$
\begin{equation*}
H^{2}=H_{0}^{2}(1+z)^{3} \tag{2.70}
\end{equation*}
$$

Substituting this equation back into 2.68 , we have

$$
\begin{equation*}
r=\int_{0}^{z} \frac{d z}{H_{0}(1+z)^{3 / 2}} \tag{2.71}
\end{equation*}
$$

Performing this integral, restoring $c$, and collecting all our results, we have the final answer

$$
\begin{equation*}
s=\frac{2 c \theta\left(1-\frac{1}{\sqrt{1+z}}\right)}{H_{0}(1+z)} \tag{2.72}
\end{equation*}
$$

Plugging in all the numbers, we get $s=8064592 \mathrm{mpc}$, which is, as with some previous problems, a digit (0) short.
arXiv identifier 0806.4592 points to "Renormalizable $A_{4}$ Model for Lepton Sector" by Paul Frampton and Shinya Matsuzaki, which yields $\mathbf{A}$ for extraction.

## Problem 20 (Ali)

This is a paraphrasing of problem 14.2 from Polchinski's "String Theory, Volume 2", and the solution given below closely follows Matthew Headrick's solution manual (arxiv:0812.4408).

Ali is a point mass with respect to the dimensions parallel to the brane. Therefore, if we integrate the Dirac-Born-Infeld action (that Ali, being a D1-brane, is described by) along the brane directions, we ought to obtain the familiar point-particle action in the residual $5+1$ dimensional space:

$$
\begin{equation*}
S_{\text {point-particle }}=-m \int d \tau \sqrt{-\partial_{\tau} X^{\mu} \partial_{\tau} X_{\mu}} \tag{2.73}
\end{equation*}
$$

In particular, since the Poincare symmetry is broken along the brane directions, all the $\sigma$ dependence in $X$ should be contained in the brane directions so that $X^{m}=X^{m}(\sigma)$ and $X^{\mu}=X^{\mu}(\tau)$. Under this ansatz, the off diagonal terms of $G+B$ vanish since the given supergravity solution for $G$ and $H=d B$ contain no terms in that mix latin and greek indices (i.e. brane and parallel directions). Plugging in, a little algebra shows that

$$
\begin{align*}
S_{\mathrm{Ali}} & =-\frac{1}{2 \pi \alpha^{\prime}} \int d \tau d \sigma e^{-\Phi} \sqrt{-\operatorname{det}\left(G_{a b}+B_{a b}\right)}  \tag{2.74}\\
& =-\frac{1}{2 \pi \alpha^{\prime}} \int d \sigma g^{-1 / 2} \sqrt{-\partial_{\sigma} X^{m} \partial_{\sigma} X_{m}} \int d \tau \sqrt{-\partial_{\tau} X^{\mu} \partial_{\tau} X_{\mu}} \tag{2.75}
\end{align*}
$$

Therefore, we conclude from comparison with the point-particle action that

$$
\begin{equation*}
m=\frac{g^{-1 / 2}}{2 \pi \alpha^{\prime}} \int d \sigma\left|\partial_{\sigma} X^{m}\right| \tag{2.76}
\end{equation*}
$$

This can be integrated since this is the coordinate, not proper line element on $\sigma$ :

$$
\begin{equation*}
m=\frac{g^{-1 / 2}\left|x_{2}^{m}-x_{1}^{m}\right|}{2 \pi \alpha^{\prime}} \tag{2.77}
\end{equation*}
$$

Plugging in everything, we find that $m=12073717$. arXiv identifier 1207.3717 points to "Bounces with $\mathrm{O}(3) \times \mathrm{O}(2)$ symmetry" by Ali Masoumi and Erick Weinberg, which yields $\mathbf{O}$ for extraction.

## Problem 21 (Jianyong)

This problem is well-suited to the Ryu-Takayanagi prescription for computing entanglement entropy (EE) holographically. We must compute the minimum length of a geodesic $\gamma$ anchored on the boundary of AdS, such that the "boundary" (here, endpoints) $\partial \gamma$ matches the boundary $\partial A$ of the interval $A$. Such geodesics are semicircles extending into the bulk of AdS. The explicit computation of the desired geodesic length, " $\lambda_{*}$ " was computed in Ryu and Takayanagi's paper arXiv:hepth/0605073. It satisfies the following equation:

$$
\begin{equation*}
\cosh \left(\lambda_{*} / R\right)=1+2 \sinh ^{2}\left(\rho_{0}\right) \sin ^{2}(\pi / 2) . \tag{2.78}
\end{equation*}
$$

In the large- $\rho_{0}$ limit, we find that

$$
\begin{align*}
\lambda_{*} & =2 R \log \left(e^{\rho_{0}} \cdot 1\right)  \tag{2.79}\\
& =2 R \rho_{0} \tag{2.80}
\end{align*}
$$

and therefore

$$
\begin{equation*}
S=\frac{\lambda_{*}}{4}=\frac{R \rho_{0}}{2} \tag{2.81}
\end{equation*}
$$

Taking the logs of both sides,

$$
\begin{align*}
\log (S)=\log (R)+\log \left(\rho_{0}\right)-\log (2) & =52991.4818246993+359236.211322482-\log (2)  \tag{2.82}\\
& =412227 . \tag{2.83}
\end{align*}
$$

As we have done multiple times before, we pad this answer with a 0 . arXiv identifier hep-th/0412227 points to "Constraints on the Dark Energy from the holographic connection to the small l CMB Suppression" by Jianyong Shen, et. al., which yields $\mathbf{L}$ for extraction.

## Problem 22 (Yu)

The duality referenced in the problem was established in the paper "Branes And Supergroups" by Victor Mikhaylov and Edward Witten (arxiv identifier 1410.1175). It is a duality between $\operatorname{OSp}(2 m+1 \mid 2 n)$ and $\operatorname{OSp}(2 n+1 \mid 2 m)$. In this problem, $n=989117, m=4562175$; therefore the required sum is $2(n+m)+2=11102586$. This indicates hep-th/1110.2586, "On $\epsilon$ - conjecture in $a$ theorem" by Yu Nakayama, giving A for extraction.


[^0]:    ${ }^{1}$ Simple way to see this: if you stretch a spring of constant $k$ by some amount $l$, each of the $n$ pieces you would cut it into stretches by an amount $l / n$ - i.e. the spring constant is $n$ times bigger.
    ${ }^{2}$ Here, we're assuming $T$ is the same throughout the infinitesimal piece, which is correct up to terms subleading in $d x$.

