Adjoint-based techniques for designing next-generation aerospace vehicles

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The overarching research goal

Automated design of next generation aerospace vehicles under uncertainty.

- Unsteady flows with moving boundary.
- Multi-disciplinary coupling of unsteady fluid-structural dynamics.
- Simulation and design in an uncertain environment.
A common theme: adjoint based approaches

- Back-propagating sensitivities through the underlying physics.
  - Forward simulation computes \( f(x_1, \ldots, x_n) \)
  - Adjoint simultaneously computes \( \partial f / \partial x_i, i = 1, \ldots, n \)

- **Computational cost is independent of** \( n \).

- Engineering design optimization with many design variables

- Uncertainty quantification with many random parameters.
Adjoint based methods in my research

- Checkpointing schemes for unsteady adjoint solvers,
- Windowing for optimizing periodic dynamical systems,
- Variance reduction for computational risk assessment,
- Robust mesh deformation for unsteady flows simulation with moving boundary,
- Adjoint based optimization of fluid and structural dynamics,
- Reduced order modeling and experiment design,
- Aviation environment impact analysis,
- Uncertainty analysis and tuning of turbulence models.
Checkpointing scheme for unsteady adjoint equations

- Naïve solution of unsteady adjoint equation requires storing the entire time history.
- Major obstacle for using unsteady adjoint.
- My dynamic checkpointing scheme solves this problem with minimal increase in computation time:

<table>
<thead>
<tr>
<th></th>
<th>1000 time steps</th>
<th>10,000 time steps</th>
<th>100k time steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 checkpoints</td>
<td>180%</td>
<td>270%</td>
<td>370%</td>
</tr>
<tr>
<td>50 checkpoints</td>
<td>100%</td>
<td>190%</td>
<td>280%</td>
</tr>
<tr>
<td>100 checkpoints</td>
<td>90%</td>
<td>150%</td>
<td>200%</td>
</tr>
</tbody>
</table>

- Do not require a priori knowledge of the number of time steps.
Windowing for optimizing periodic dynamical systems

• Objective function is often a infinite time averaged quantity.

• Finite time average approximation produces the wrong gradient, when the period is sensitive.

• Solution: windowed averaging over finite time

  • Perfect window: calculates the exact gradient when there is a single, known harmonic frequency.

  • Long window: approximates the gradient for multiple, non-rational frequencies. Fast convergence as window length increases.
Adjoint based variance reduction in computational risk assessment

- Idea: build an adjoint based surrogate function as a basis of adapting the sampling strategy.

**Control variates, importance sampling, stratified sampling**

- **Unbiased estimate of failure probability,**
- **Significantly reduce computation cost for very nonlinear problems,** including in *unsteady vortex shedding* and in *hypersonic propulsion.*

<table>
<thead>
<tr>
<th></th>
<th>Flow solutions</th>
<th>Failure probability</th>
<th>Acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic Monte Carlo</td>
<td>3000</td>
<td>between 7.2% and 9.2%</td>
<td></td>
</tr>
<tr>
<td>Stratified Sampling</td>
<td>3000</td>
<td>between 7.9% and 8.7%</td>
<td>525%</td>
</tr>
</tbody>
</table>
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Mesh deformation for moving boundary flows

- CFD methods with **body-conforming** mesh is preferred for flows with **thin boundary layers**.
- Under large boundary movements, to **deform the mesh** and maintain good mesh quality is challenging.

Current methods: **structural analogy; interpolation**.

- Handles moderate boundary deformations;
- Requires human intervention when they fail.
Variable stiffness mesh deformation

Introduce a scalar field $a$ in PDE based structural analogy methods, for example:

$$\nabla \cdot \nabla \nabla \cdot \nabla X = 0 \quad \Rightarrow \quad \nabla \cdot a \nabla \nabla \cdot a \nabla X = 0$$

$$a \equiv 1$$
Variable stiffness mesh deformation

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$a_{\text{inner}} > a_{\text{outer}}$
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Variable stiffness mesh deformation

Introduce a scalar field $a$ in PDE based structural analogy methods, for example:

$$\nabla \cdot \nabla \nabla \cdot \nabla X = 0 \quad \rightarrow \quad \nabla \cdot a \nabla \nabla \cdot a \nabla X = 0$$

What's the optimal $a$?

$a \equiv 1$

$a_{\text{inner}} > a_{\text{outer}}$

$a_{\text{inner}} \gg a_{\text{outer}}$
Optimal stiffness mesh deformation

Use adjoint based optimization to compute the best stiffness field for a set of deformation scenarios.

- Much more robust than existing methods in handling large boundary deformations.
- The optimization can be efficiently performed on-line if the current stiffness field fails to produce a good mesh.
Design and optimal control of structural dynamics: motivation

- Structural dynamics of helicopter blades, wind turbines and flapping flight;
- Bullwhip cracking – supersonic
  - Possible for space propulsion?
Geometrically exact beam model

- Use Chebyshev spectral element method to discretize the differential-integral equation.
  - High accuracy, low computation cost

- 8th order polynomial, 9 modes
- 16th order polynomial, 17 modes
Geometrically exact beam model

• Use **Chebyshev spectral element method** to discretize the differential-integral equation.

  • High accuracy, low computation cost

  8\textsuperscript{th} order polynomial, 9 modes

  16\textsuperscript{th} order polynomial, 17 modes
Adjoint based optimal control

- The unsteady adjoint equation for the nonlinear differential-integral equation is solved.
- The torque on the root is optimized to maximize tip velocity at $t=4$ (open loop optimal control)

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1 seconds</td>
<td><img src="image1" alt="Diagram" /></td>
</tr>
<tr>
<td>1-2 seconds</td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td>2-3 seconds</td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td>3-4 seconds</td>
<td><img src="image4" alt="Diagram" /></td>
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</tbody>
</table>
Adjoint based optimal control: result

Future work:

- Simultaneous optimization of structural properties and control.
- Couple with flow solvers to perform multi-disciplinary adjoint based optimization of aerodynamics, structure and control.
Multi-disciplinary adjoint based design optimization

Simultaneous computation of sensitivity gradient to all design variables through multi-disciplinary coupling of adjoint solvers.
Gradient based reduced order modeling and experiment design

- Given a nonlinear input-output relation, find the linear combinations of parameters that affect the output most.

- Example application in experiment design for quantifying the effect of manufacturing error:

  - Potential applications in complex fuel combustion.
Adjoint based analysis of aviation environmental impact

Airplane emission estimated to cause over 10000 deaths / year.

- Complex chemical reactions in the atmosphere,
- Species transported by global atmosphere circulation.

An adjoint solution computes the sensitivity of the total health impact to arbitrary perturbation in airplane emission.

- Provides valuable information to aviation policy makers
Model uncertainty in Reynolds Averaged Navier-Stokes simulations

- **RANS**: most popular CFD technique for turbulent flows.

- A few quantities are used to determine the effect of the high dimensional turbulent motions in the flow. The resulting non-uniqueness leads to model uncertainty.

- Adjoint based inversion computes the error between the model estimated effect and the true effect of turbulence in various flow fields, from which a model of the error is built.
Conclusion

- Adjoint based methods efficiently computes high-dimensional sensitivity gradient,
  - Solvers high dimensional optimization in many engineering problems, that otherwise would be too expensive, or impracticable to solve.
  - Inverse problems and inference,
  - Surrogate modeling and uncertainty quantification.
- An invaluable tool in designing next generation aerospace vehicles in an uncertain environment