# Two-Dimensional Rotational Dynamics
## Challenge Problem Solutions

**Problem 1: Estimation**

Your car has a flat and you try to loosen the lugs on the wheel with a tire iron shown in the figure on the left below. You can't budge the lugs but then you try the 4-way wrench shown in the figure on the right and the lugs loosen. Based on the figures below, estimate how much torque you needed to apply to loosen the lugs.

**Problem 1 Possible Answer:**

There are many considerations in this problem, and if you've had some experience with changing flat tires, you're aware of what they are. Your experience may suggest that the 4-way is overall easier to use.

Start by keeping in mind that the car should be up on a jack, and many jacks don't supply good overall stability. Applying too much net force, up or down, on the wheel could cause the car to fall off the jack. As an example, for a 1983 VW Rabbit (since departed), the car could be lifted off the jack. Typically, a person can exert an upward force of 2-3 times the person's weight. So, using the simple wrench on the left and pulling up with both hands warrants caution. Pushing down, the maximum downward force exerted can't exceed the person's weight. As an estimate of the net torque, use a force with magnitude equal to your weight and a moment arm of about half a meter. For me, that's about $500\,\mathrm{N \cdot m}$. Of course, the angle between the applied force and the wrench should be $90°$; the wrench arm should be horizontal.

For the 4-way wrench, assume the same magnitude of force applied on each of the two arms used to turn the wrench, one force directed up and the other down. Ideally, this would mean that you apply no net force the car, and falling off the jack is less likely. Also, you would be able to position yourself more or less symmetric about the wheel, making the process a bit more comfortable. (But the forces on your feet wouldn't be the same – see Problem 3.) The forces you exert then form a "couple," and the torque would be the product of the applied force and the moment arm, times 2 for the two arms of the wrench. From the figure, it looks like the moment arms are about the same as for the simple wrench, so the net torque is roughly your weight times the moment arm of about half a meter times 2, or $1000\,\mathrm{N \cdot m}$; much easier to undo the lug nuts.

**Problem 2: Moment of Inertia and Rotational Kinematics:** *Turntable*

A turntable is a uniform disc of mass $1.2\ \text{kg}$ and radius $1.3\times10^{-1}\ \text{m}$. The turntable is spinning at a constant rate of $f_0 = 0.5\ \text{Hz}$. The motor is turned off and the turntable slows to a stop in 8.0 s with constant angular deceleration.

a) What is the moment of inertia of the turntable?

b) What is the initial rotational kinetic energy?

c) What is the angular deceleration of the turntable while it is slowing down?

d) What is the total angle in radians that the turntable spins while slowing down?

e) What is the magnitude of the frictional torque?

f) Find the work done by the frictional torque two different ways: use the work-kinetic energy theorem, and calculate the work done by the frictional torque through the total angle that the turntable moves while slowing down.

g) What is the average power of the frictional torque in stopping the turntable?

**Problem 2 Solutions:**

a) The moment of inertia of the turntable about an axis passing perpendicular to the disc and through the center of mass is

$$I_{\text{cm}} = \frac{1}{2}MR^2 = \frac{1}{2}(1.2\ \text{kg})(1.3\times10^{-1}\text{m})^2 = 1.01\times10^{-2}\ \text{kg}\cdot\text{m}^2. \qquad \textbf{(2.1)}$$

b) Initially, the disc is spinning with a frequency $f_0 = 0.5\ \text{Hz}$, so the initial angular velocity is

$$\omega_0 = 2\pi f_0 = \left(2\pi\frac{\text{radian}}{\text{cycle}}\right)\left(0.5\frac{\text{cycles}}{\text{s}}\right) = \pi\ \text{rad}\cdot\text{s}^{-1}. \qquad \textbf{(2.2)}$$

The initial rotational kinetic energy is

$$K_{\text{rot}} = \frac{1}{2}I_{\text{cm}}\omega_0^2 = \frac{1}{2}(1.01\times10^{-2}\ \text{kg}\cdot\text{m}^2)(\pi\ \text{rad}\cdot\text{s}^{-1})^2 = 5.0\times10^{-2}\ \text{J}. \qquad \textbf{(2.3)}$$

c) The final angular velocity is zero, so the angular acceleration is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_0}{t_f - t_0} = \frac{-\pi \text{ rad}\cdot\text{s}^{-1}}{8.0 \text{ s}} = -3.9\times10^{-1} \text{ rad}\cdot\text{s}^{-2}\,. \qquad \textbf{(2.4)}$$

Because the angular acceleration is negative, the disc is slowing down.

d) The disc turns through angle

$$\begin{aligned}\Delta\theta &= \omega_0\,\Delta t + \frac{1}{2}\alpha\,\Delta t^2 = (\pi \text{ rad}\cdot\text{s}^{-1})(8.0\text{ s}) + \frac{1}{2}(-3.9\times10^{-1}\text{ rad}\cdot\text{s}^{-2})(8.0\text{ s})^2 \\ &= 12.7 \text{ rad}\end{aligned} \qquad \textbf{(2.5)}$$

e) The magnitude of the frictional torque is

$$\left|\tau^{\text{total}}_{\text{friction}}\right| = I_S\left|\alpha\right| = (1.01\times10^{-2}\text{ kg}\cdot\text{m}^2)(3.9\times10^{-1}\text{ rad}\cdot\text{s}^{-2}) = 4.0\times10^{-3}\text{ N}\cdot\text{m}\,. \qquad \textbf{(2.6)}$$

f) In our example, the component of the torque is negative because the angular acceleration is negative, so the rotational work is given by

$$W_{\text{friction}} = \int_{\theta=\theta_0}^{\theta=\theta_f} \tau_S\,d\theta = \tau_S\,\Delta\theta = (-4.0\times10^{-3}\text{ N}\cdot\text{m})(12.6\text{ rad}) = -5.0\times10^{-2}\text{ J}\,. \qquad \textbf{(2.7)}$$

The work-rotational energy theorem is that

$$W_{\text{rot}} = \frac{1}{2}I_{\text{cm}}\omega^2_{\text{cm},f} - \frac{1}{2}I_{\text{cm}}\omega^2_{\text{cm},0} = \Delta K_{\text{rot}}\,. \qquad \textbf{(2.8)}$$

In our example the final angular velocity is zero, so using Equation (2.3)

$$W_{\text{rot}} = -\frac{1}{2}I_{\text{cm}}\omega^2_{\text{cm},0} = -5.0\times10^{-2}\text{ J}\,, \qquad \textbf{(2.9)}$$

in agreement with Equation (2.7)

g) The average power is the total work done divided by the time interval, so

$$P_{\text{ave}} = \frac{W_{\text{rot}}}{\Delta t} = \frac{-5.0\times10^{-2}\text{ J}}{8.0\text{ s}} = -6.2\times10^{-3}\text{ W}\,. \qquad \textbf{(2.10)}$$

The instantaneous rotational power is defined as the rate of doing rotational work,

$$P_{\text{rot}} \equiv \frac{dW_{\text{rot}}}{dt} = \tau_S \frac{d\theta}{dt} = I_S \, \alpha \, \omega \, . \tag{2.11}$$

The angular velocity as a function of time for constant angular acceleration is given by

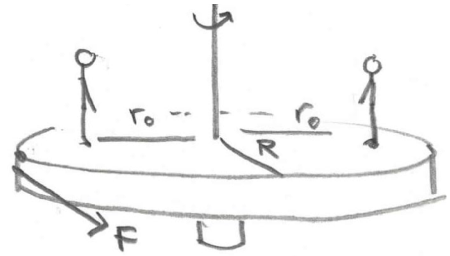
$$\omega(t) = \omega_0 + \alpha t \, , \tag{2.12}$$

so the rotational power is given by

$$P_{\text{rot}} = I_S \alpha (\omega_0 + \alpha t) \, . \tag{2.13}$$

**Problem 3:**

A playground merry-go-round has a radius of $R = 4.0m$ and has a moment of inertia $I_{cm} = 7.0\times10^3 kg\cdot m^2$ about an axis passing through the center of mass. There is negligible friction about its vertical axis. Two children each of mass $m = 25kg$ are standing on opposite sides a distance $r_0 = 3.0m$ from the central axis. The merry-go-round is initially at rest. A person on the ground applies a constant tangential force of $F = 2.5\times10^2 N$ at the rim of the merry-go-round for a time $\Delta t = 1.0\times10^1 s$.




a) What is the magnitude of angular acceleration of the merry-go-round?

b) What is the angular speed of the merry-go-round when the person stopped applying the force?

c) What average power does the person put out while pushing the merry-go-round?

d) What is the rotational kinetic energy of the merry-go-round when the person stopped applying the force?

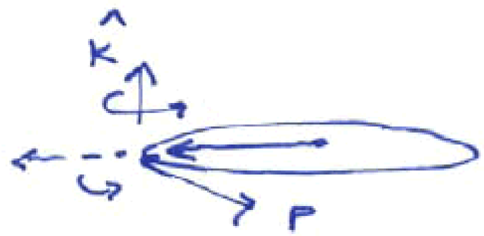
The two children then walk inward and stop a distance of $r_1 = 1.0m$ from the central axis of the merry-go-round.

e) What is the angular velocity of the merry-go-round when the children reach their final position?

f) What is the change in rotational kinetic energy of the merry-go-round when the children reached their final position?

**Problem 3 Solutions:**

The torque diagram is shown below.




The torque about the center of the merry-go-round is given by

$$\vec{\tau}_O = \vec{r}_O \times \vec{F} = RF\hat{k} \,. \tag{3.1}$$

The moment of inertia about the axis passing perpendicularly through the center of the merry-go-round is

$$I_z = I_{cm} + 2mr_0^{\,2} \,. \tag{3.2}$$

Therefore the torque equation $\tau_z = I_z\alpha_z$ becomes

$$RF = (I_{cm} + 2mr_0^{\,2})\alpha_z \,. \tag{3.3}$$

So the z-component of angular acceleration is

$$\alpha_z = \frac{RF}{I_{cm} + 2mr_0^{\,2}} = \frac{(4.0m)(2.5\times10^2 N)}{7.0\times10^3 kg\cdot m^2 + 2(25kg)(3.0m)^2} = 1.3\times10^{-1} rad\cdot s^{-2} \,. \tag{3.4}$$

We shall use two different approaches to find the angular speed of the merry-go-round when the person stopped applying the force. The magnitude of the angular impulse equals the change in angular momentum

$$\tau_z \Delta t = \Delta L_z = I_{cm}\omega_1 \,. \tag{3.5}$$

Therefore the final angular speed is

$$\omega_1 = \frac{\tau_z \Delta t}{I_{cm} + 2mr_0^{\,2}} = \alpha_z \Delta t = (1.3\times10^{-1}\ rad\cdot s^{-2})(1.0\times10^1 s) = 1.3\, rad\cdot s^{-1} \,. \tag{3.6}$$

The average power exerted by the person while pushing the merry-go-round is

$$P_{ave} = \tau_z \omega_{ave} = \frac{1}{2}\tau_z \omega_1 = \frac{1}{2}(4.0m)(2.5\times 10^2 N)(1.3\, rad\cdot s^{-1}) = 670\, W\,. \qquad \textbf{(3.7)}$$

The rotational kinetic energy of the merry-go-round when the person stopped applying the force is

$$\begin{aligned} K_{rot,1} &= \frac{1}{2}(I_{cm} + 2mr_0^{\,2})\omega_1^{\,2} = \frac{1}{2}(7.0\times 10^3 kg\cdot m^2 + 2(25kg)(3.0m)^2)(1.3\, rad\cdot s^{-1})^2 \\ &= 6.7\times 10^3\ J \end{aligned} \qquad \textbf{.(3.8)}$$

Note that $K_{rot} = P_{ave}\Delta t$ .

We assume that there are no external torques about the center-of the merry-round as the two children walk inward and stop a distance of $r_1 = 1.0m$ from the central axis. Therefore the angular momentum about the center of the merry-go-round is constant so

$$(I_{cm} + 2mr_0^{\,2})\omega_1 = (I_{cm} + 2mr_1^{\,2})\omega_f \quad . \qquad \textbf{(3.9)}$$

We can solve for the angular speed after they are finished walking inward

$$\begin{aligned} \omega_f &= \frac{(I_{cm} + 2mr_0^{\,2})}{(I_{cm} + 2mr_1^{\,2})}\omega_1 = \frac{(7.0\times 10^3 kg\cdot m^2 + 2(25kg)(3.0m)^2)}{(7.0\times 10^3 kg\cdot m^2 + 2(25kg)(1.0m)^2)}(1.3\, rad\cdot s^{-1}) \\ &= 1.4\, rad\cdot s^{-1} \end{aligned} \quad . \qquad \textbf{(3.10)}$$

The final rotational kinetic energy is

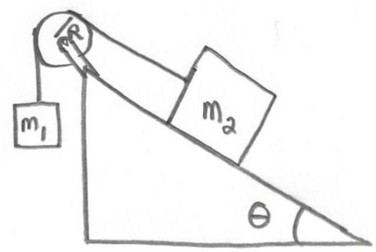
$$\begin{aligned} K_{rot,f} &= \frac{1}{2}(I_{cm} + 2mr_1^{\,2})\omega_f^{\,2} = \frac{1}{2}(7.0\times 10^3 kg\cdot m^2 + 2(25kg)(1.0m)^2)(1.4\, rad\cdot s^{-1})^2 \\ &= 7.1\times 10^3\ J \end{aligned} \qquad \textbf{.(3.11)}$$

So the change in rotational kinetic energy of the merry-go-round when the children reached their final position is

$$\Delta K = K_{rot,f} - K_{rot,1} = 3.8\times 10^2\ J\,. \qquad \textbf{(3.12)}$$

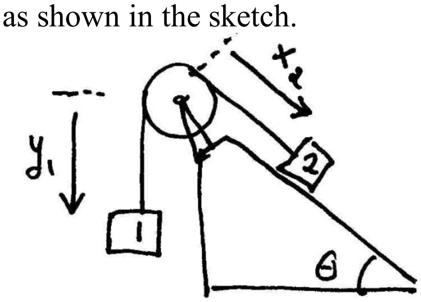
**Problem 4:**

A wheel in the shape of a uniform disk of radius $R$ and mass $m_{\mathrm{p}}$ is mounted on a frictionless horizontal axis. The wheel has moment of inertia about the center of mass $I_{\mathrm{cm}} = (1/2)m_{\mathrm{p}}R^2$. A massless cord is wrapped around the wheel and one end of the cord is attached to an object of mass $m_2$ that can slide up or down a frictionless inclined plane. The other end of the cord is attached to a second object of mass $m_1$ that hangs over the edge of the inclined plane. The plane is inclined from the horizontal by an angle $\theta$. Once the objects are released from rest, the cord moves without slipping around the disk. Calculate the speed of block 2 as a function of distance that it moves down the inclined plane using energy techniques. Assume there are no energy losses due to friction and that the rope does slip around the pulley

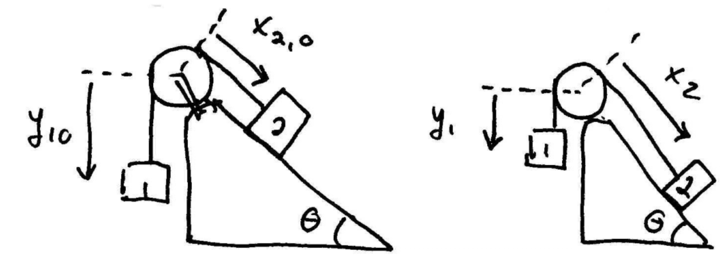



**Problem 4 Solution:**

Define a coordinate system as shown in the sketch.




Choose the zero for the gravitational potential energy at a height equal to the center of the pulley.



$$U_0 = -m_1 g y_{10} - m_2 g x_{2,0} \sin\theta \qquad U = -m_1 g y_1 - m_2 g x_2 \sin\theta$$

$$K_0 = 0 \qquad K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I_P \omega^2$$

Then the initial mechanical energy is

$$E_0 = U_0 = -m_1 g y_{1,0} - m_2 g x_{2,0} \sin\theta . \tag{4.1}$$

The mechanical energy, when block 2 has moved a distance

$$d = x_2 - x_{2,0} \tag{4.2}$$

is given by

$$E = U + K = -m_1 g y_1 - m_2 g x_2 \sin\theta + \frac{1}{2} m_1 v_1^{\,2} + \frac{1}{2} m_2 v_2^{\,2} + \frac{1}{2} I_P \omega^2 . \tag{4.3}$$

Since the rope connects the two blocks, they move at the same speed so

$$v \equiv v_1 = v_2 . \tag{4.4}$$

The rope does slip on the pulley so the as the rope moves around the pulley the tangential speed of the rope is equal to the speed of the blocks

$$v_{\tan} = R\omega = v . \tag{4.5}$$

Thus Eq. (4.3) can be simplified

$$E = U + K = -m_1 g y_1 - m_2 g x_2 \sin\theta + \frac{1}{2}\left( m_1 + m_2 + \frac{I_P}{R^2} \right) v^2 . \tag{4.6}$$

Because we have assumed that there is no loss of mechanical energy, we can set $E_0 = E$ and find that

$$-m_1 g y_{1,0} - m_2 g x_{2,0} \sin\theta = -m_1 g y_1 - m_2 g x_2 \sin\theta + \frac{1}{2}\left( m_1 + m_2 + \frac{I_P}{R^2} \right) v^2 \tag{4.7}$$

which simplifies to

$$-m_1 g\left(y_{1,0}-y_1\right)-m_2 g\left(x_2-x_{2,0}\right)\sin\theta=\frac{1}{2}\left(m_1+m_2+\frac{I_P}{R^2}\right)v^2 \tag{4.8}$$

We finally note that the movement of block 1 and block 2 are constrained by the relationship

$$d=x_2-x_{2,0}=y_{1,0}-y_1\,. \tag{4.9}$$

Then Eq. (4.8) becomes

$$gd(m_1-m_2\sin\theta)=\frac{1}{2}\left(m_1+m_2+\frac{I_P}{R^2}\right)v^2\,. \tag{4.10}$$

We can now solve for the speed as a function of distance $d=x_2-x_{2,0}$ that block 2 has traveled down the incline plane

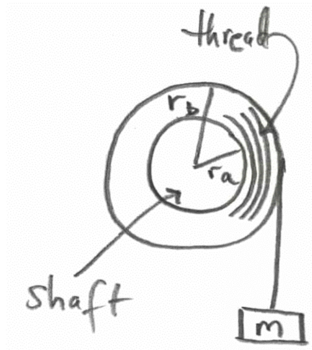
$$v=\sqrt{\frac{2gd(m_1-m_2\sin\theta)}{\left(m_1+m_2+(I_P/R^2)\right)}}\,. \tag{4.11}$$

If we assume that the moment of inertial of the pulley is $I_{\text{cm}}=(1/2)m_{\text{p}}R^2$, then the speed becomes

$$v=\sqrt{\frac{2gd(m_1-m_2\sin\theta)}{\left(m_1+m_2+(1/2)m_P\right)}}\,. \tag{4.12}$$

**Problem 5:** ***Stall Torque of Motor***

The following simple experiment can measure the stall torque of a motor. (See sketch.) A mass $m$ is attached to one end of a thread. The other end of the thread is attached to the motor shaft so that when the motor turns, the thread will wind around the motor shaft. The motor shaft without thread has radius $r_0 = 1.2 \times 10^{-3} m$. Assume the thread winds evenly effectively increasing the radius of the shaft. Eventually the motor will stall.




a) Suppose a mass $m = 5.0 \times 10^{-2} kg$ stalls the motor when the wound thread has an outer radius of $r_f = 2.4 \times 10^{-3} m$. Calculate the stall torque.

b) Suppose the motor has an unloaded full throttle angular frequency of $\omega_0 = 2\pi f_0 = 2\pi\left(6.0 \times 10^1 Hz\right)$ (unloaded means that the motor is not applying any torque). Suppose the relation between angular frequency $\omega$ and the applied torque $\tau$ of the motor is given by the relation

$$\omega = \omega_0 - b\tau$$

where $b$ is a constant with units $s/kg - m^2$. Using your result from part a), calculate the constant $b$. Make a graph of $\omega$ vs. $\tau$.

c) Graph the power output of the motor vs. angular frequency $\omega$. At what angular frequency is the power maximum? What is the power output at that maximum? Briefly explain the shape of your graph. In particular, explain the power output at the extremes $\tau = 0$ and $\tau = \tau_{stall}$.

d) What torque does the motor put out at its maximum power output?

**Problem 5 Solutions:**

**a)** The torque is $\tau_{st} - r_f mg \approx 1.1\times10^{-3}\text{N}\times m$.

**b)** When the motor is stalled, $\omega = 0$. Thus $\omega_0 = b\tau_{st}$ and

$$b = \frac{\omega_0}{\tau_{st}} = \frac{\omega_0}{r_f mg} \approx 3.2\times10^5 \frac{s}{kg\times m^2}$$

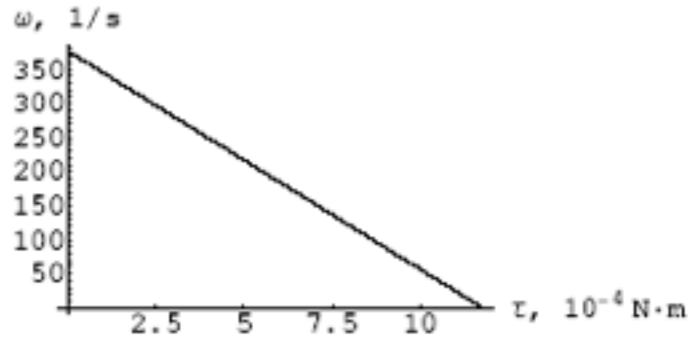



Figure 1: Angular velocity vs. Torque

**c)** The power output is given by

$$P = \tau\omega = \frac{\omega_0 - \omega}{b}\omega .$$

The plot is a parabola (Figure 2).

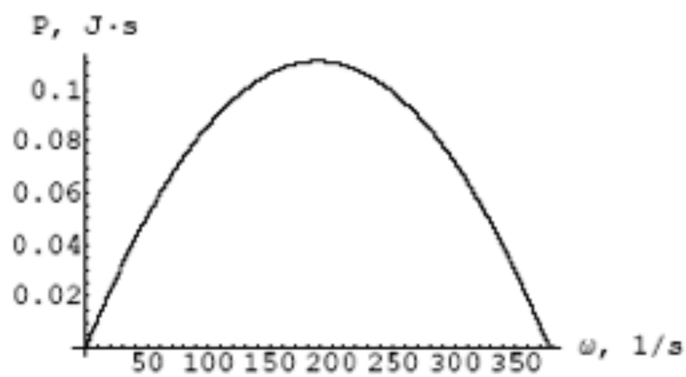
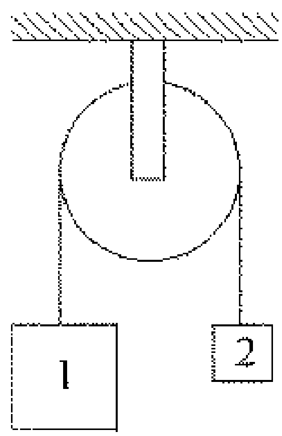



Figure 2: Power output vs. angular velocity

**d)** The maximum is reached at $\omega = \omega_0 / 2$. The corresponding output power is $\omega_0^2 / (4b) \approx 0.1W$.

**Problem 6: Torque and Angular Acceleration:** ***Atwood Machine Solutions***

A pulley of mass $m_p$, radius $R$, and moment of inertia about the center of mass $I_{cm}$, is suspended from a ceiling. An inextensible string of negligible mass is wrapped around the pulley and attached on one end to an object of mass $m_1$ and on the other end to an object of mass with $m_1 > m_2$. At time $t = 0$, the objects are released from rest. How long does it take the objects to move a distance d? If $I_{\text{cm}} = \frac{1}{2} m_p R^2$, what does your answer reduce to?




We will use Newton's Second Law, and the Rotational Law to find the acceleration of all the objects in the system. We can then use kinematics to calculate the time it takes for each object to move a certain distance. As a strategy we will answer the following questions.

a) What are the free body force diagrams on the pulley and the two objects. Is the tension in the string constant?

b) Write down Newton's Second Law for the pulley and the two objects.

c) Write down the rotational equation of motion for the pulley.

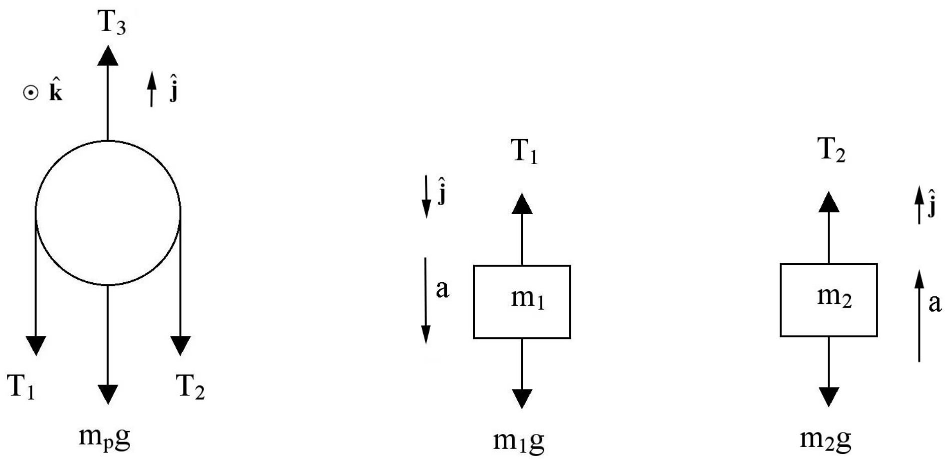
d) What is the constraint condition between the magnitude of the angular acceleration of the pulley and the magnitude of the acceleration of either object.

e) Find the direction and magnitude of the acceleration of the two objects.

f) How long does it take the objects to move a distance d? If $I_{\text{cm}} = \frac{1}{2} m_p R^2$, what does your answer reduce to?

**Problem 6 Solutions:**

a) Free body diagrams:




The two tensions in the string each produce a torque on the pulley in opposite directions. The pulley has rotational inertial: the moment of inertia is not zero. Hence if the pulley undergoes angular acceleration there must be a net non-zero torque. (Note the pulley is not massless. Earlier in the semester we always assumed that the pulley is massless to avoid the complication we are considering here.) The two tensions in the string each produce a torque on the pulley in opposite directions. The net torque is proportional to the difference in tensions in the string on the two sides of the pulley. Therefore the tensions cannot be equal when the pulley undergoes angular acceleration.

b) Equation of linear motion for pulley (Pulley is at rest):

$$T_3 - T_1 - T_2 - m_p g = 0 . \qquad (6.1)$$

Equation of linear motion for $m_1$:

$$m_1 g - T_1 = m_1 a . \qquad (6.2)$$

Equation of linear motion for $m_2$:

$$T_2 - m_2 g = m_2 a . \qquad (6.3)$$

c) Equation of rotational motion for pulley:

$$R\left(T_1 - T_2\right) = I_{\text{cm}}\alpha\,. \tag{6.4}$$

d) For pure rotation,

$$a = R\alpha\,. \tag{6.5}$$

e) From (2) and (3):

$$T_1 - T_2 = \left(m_1 - m_2\right)g - (m_1 + m_2)a\,. \tag{6.6}$$

From (4) and Eq (5) along with the moment of inertia:

$$T_1 - T_2 = \frac{I_{\text{cm}}\alpha}{R} = \frac{I_{\text{cm}}a}{R^2}\,. \tag{6.7}$$

Equating the expressions for $T_2 - T_1$ in (6) and (7) we obtain:

$$\frac{I_{\text{cm}}a}{R^2} = \left(m_1 - m_2\right)g - (m_1 + m_2)a\,, \tag{6.8}$$

which we can solve for the acceleration.

$$a = \frac{\left(m_1 - m_2\right)g}{\left(\dfrac{I_{\text{cm}}}{R^2} + (m_1 + m_2)\right)}\,. \tag{6.9}$$

f) $d = \dfrac{1}{2}at^2$ and the results from part e) =>

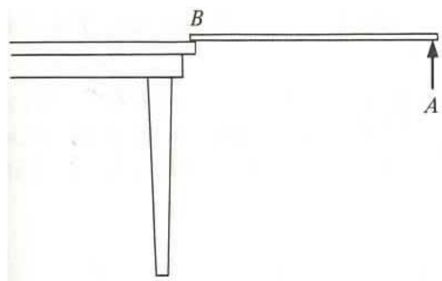
$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2d\left(\dfrac{I_{\text{cm}}}{R^2} + (m_1 + m_2)\right)}{\left(m_1 - m_2\right)g}}\,. \tag{6.10}$$

If $I_{\text{cm}} = \dfrac{1}{2}m_p R^2$, then

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{d\left(m_p + 2(m_1 + m_2)\right)}{\left(m_1 - m_2\right)g}}\,. \tag{6.11}$$

**Problem 7:**

A uniform stick of mass $m$ and length $l$ is suspended horizontally with end $B$ at the edge of a table and the other end $A$ is held by hand. Point $A$ is suddenly released. At the instant after release:




a) What is the torque about the end $B$ on the table?

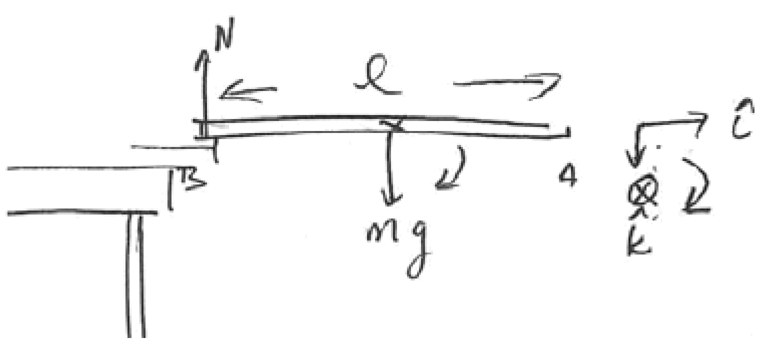
b) What is the angular acceleration about the end $B$ on the table?

c) What is the vertical acceleration of the center of mass?

d) What is the vertical component of the hinge force at $B$? Does the hinge force have a horizontal component at the instant after release?

**Problem 7 Solution:**

The torque diagram is shown in the figure below.




The toque about the point $B$ is

$$\vec{\tau}_B = \vec{r}_{B,mg} \times m\vec{g} = \frac{l}{2} mg\hat{k}. \tag{7.1}$$

The moment of inertia about the axis passing perpendicularly through the end of the stick is

$$I_z = I_{cm} + m(l/2)^2 = (1/12)ml^2 + m(l/2)^2 = (1/3)ml^2 . \qquad \textbf{(7.2)}$$

Therefore the torque equation $\vec{\tau}_B = I_B \vec{\alpha}$ becomes

$$(l/2)mg = (1/3)ml^2\alpha . \qquad \textbf{(7.3)}$$

So the angular acceleration is

$$\alpha = 3g/2l . \qquad \textbf{(7.4)}$$

Newton's Second Law in the vertical direction is

$$mg - N = ma_z . \qquad \textbf{(7.5)}$$

The component of the angular acceleration about the end and the linear acceleration of the center of mass are related by

$$a_z = (l/2)\alpha = (l/2)(3g/2l) = 3g/4 . \qquad \textbf{(7.6)}$$

Therefore the vertical component of the hinge force at $B$ (the normal force)

$$N = mg - ma_z = mg/4 . \qquad \textbf{(7.7)}$$