The Strange Case of ^{229}Pa: Equivalent Atomic and Nuclear Energy Scales

- Nuclear Enhancements of P and P+T Violation
- The ^{229}Pa Case
- Concident atomic and nuclear scales

New Opportunities for Fundamental PhysicsResearch with Radioactive Molecules
June 28-July 22, 2021

Wick Haxton

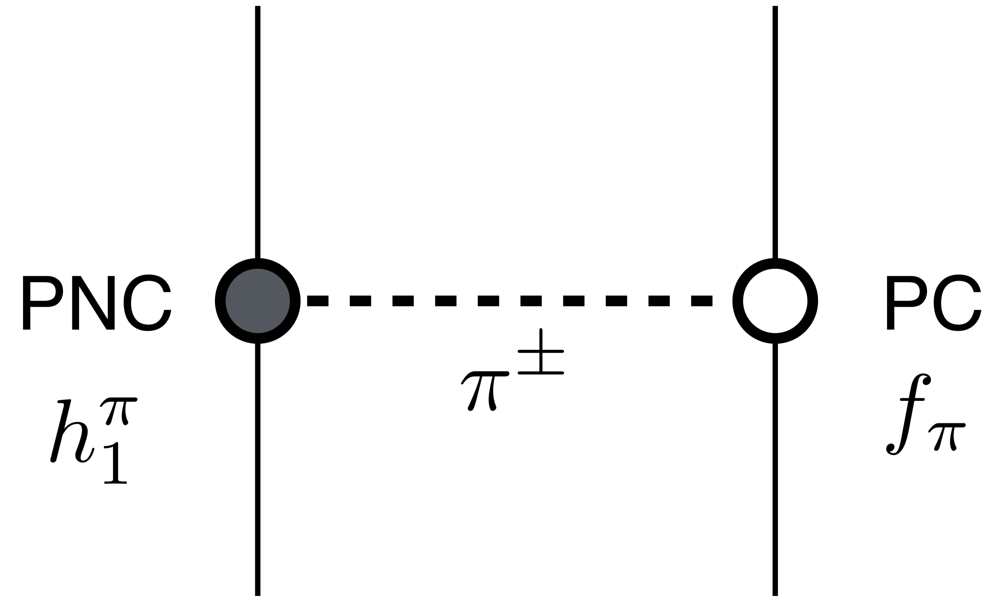
UC Berkeley and Berkeley Lab

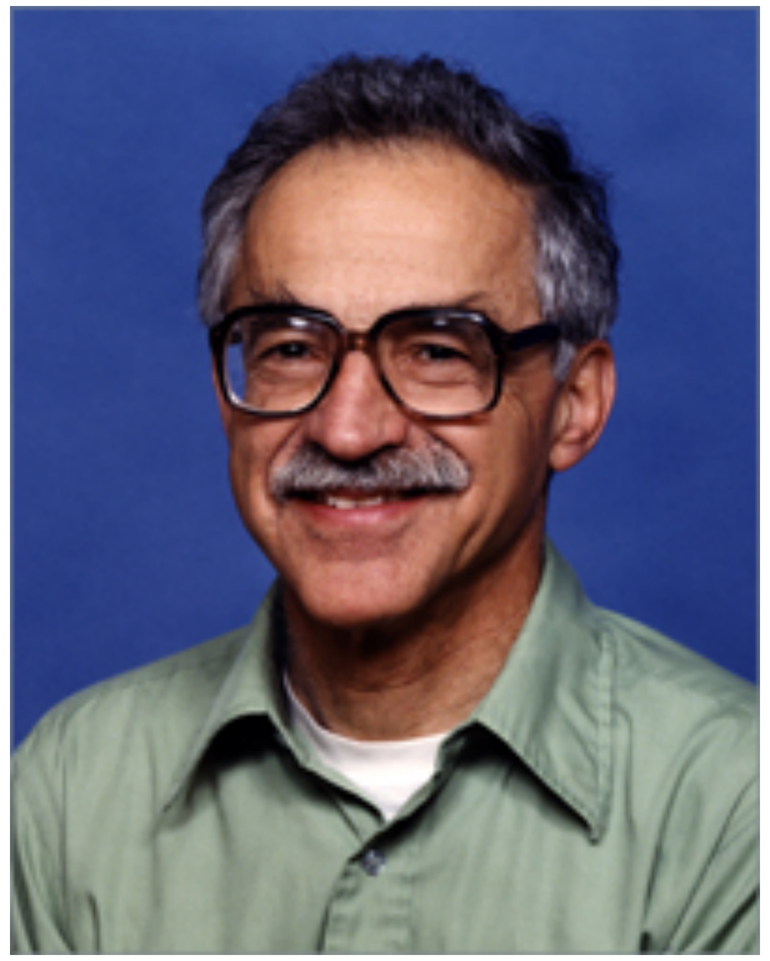
29 June 2021

I. Introduction: Enhancements of Symmetry Violation

Motivated by hadronic PNC

e.g.,





Ernest Henley
6/10/1924-3/27/2017

and the effort to characterize this interaction at low energies

Nuclei can filter interactions:

- *the quantum labels of nuclear states allow one to isolate parts of interactions of particular interest*

They can enhance the PNC signal:

- *Through nuclear degeneracies: mixing of nearby states*

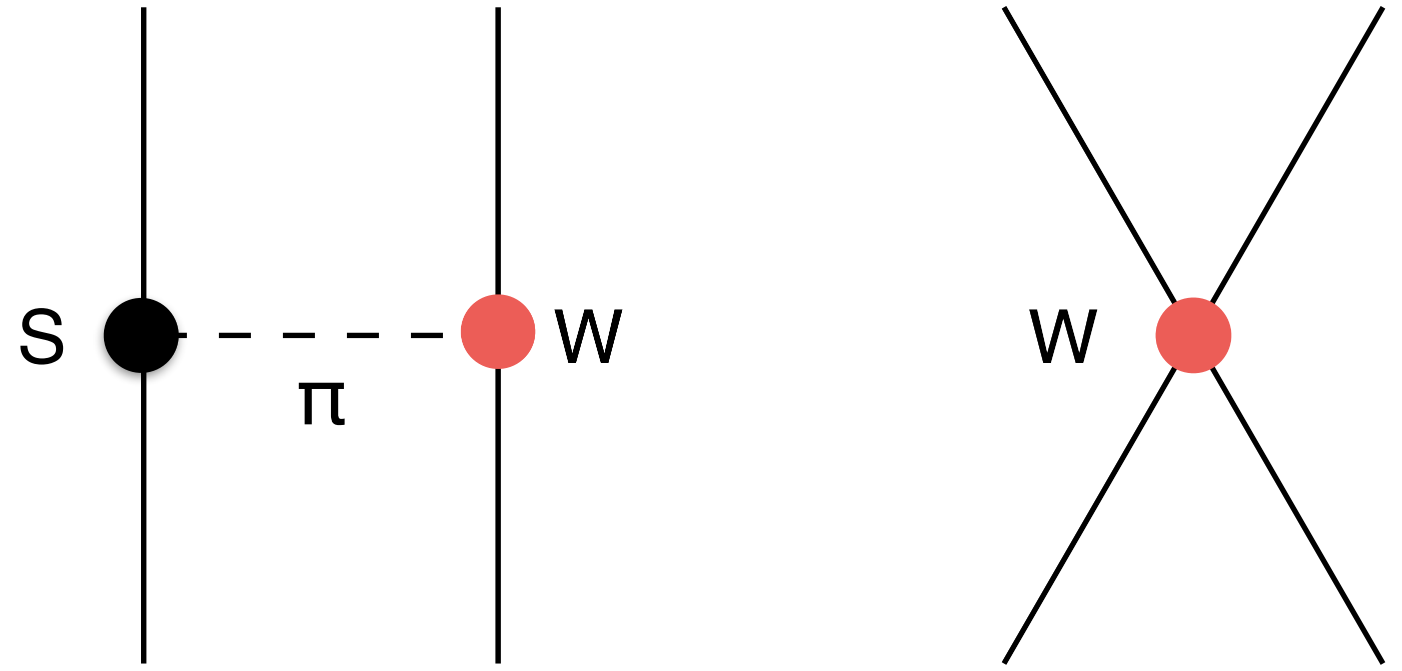
- *By competing symmetry-allowed, suppressed transitions against symmetry-forbidden strong ones*

The low-energy PNC NN interaction can be described in terms of five S-P (Danilov) amplitudes

Transition	$I \leftrightarrow I'$	ΔI	n-n	n-p	p-p	NN system exchanges
$^3S_1 \leftrightarrow {}^1P_1$	$0 \leftrightarrow 0$	0		x		ρ ,ω
$^1S_0 \leftrightarrow {}^3P_0$	$1 \leftrightarrow 1$	0	x	x	x	ρ ,ω
		1	x		x	ρ ,ω
		2	x	x	x	ρ
$^3S_1 \leftrightarrow {}^3P_1$	$0 \leftrightarrow 1$	1		x		$\pi^{\pm}$, ρ ,ω

This physics can be encoded into an effective theory, with enough degrees of freedom to describe the five amplitudes and the pion's range

e.g., $^3S_1 \leftrightarrow {}^3P_1$:



or nearly equivalently, into a potential with the same DoFs (DDH)

Remains an active, interesting topic: new ideas about the LEC hierarchy

EFTs:
- S. L. Zhu et al., Nucl. Phys. A748 (2005) 435
- L. Girlanda, Phys. Rev. C77 (2008) 067001
- D. R. Phillips, M. R. Schindler, and R. P. Springer, Nucl. Phys. A822 (2009) 1

1/N$_{c}$:
- D. Phillips, D. Samart, and C. Schat, PRL 114 (2015) 062301
- M. R. Schindler, R. P. Springer, and J. Vanasse, Phys. Rev. C93 (2016) 025502

Coeff	DDH	Girlanda	Zhu
$\Lambda_{0\ DDH}^{^1S_0-^3P_0}$	$-g_\rho h_\rho^0(2+\chi_V) - g_\omega h_\omega^0(2+\chi_S)$	$2(\mathcal{G}_1+\tilde{\mathcal{G}}_1)$	$2(\mathcal{C}_1+\tilde{\mathcal{C}}_1+\mathcal{C}_3+\tilde{\mathcal{C}}_3)$
$\Lambda_{0\ DDH}^{^3S_1-^1P_1}$	$g_\omega h_\omega^0\chi_S - 3g_\rho h_\rho^0\chi_V$	$2(\mathcal{G}_1$-$\tilde{\mathcal{G}}_1)$	$2(\mathcal{C}_1$-$\tilde{\mathcal{C}}_1$-$3\mathcal{C}_3+3\tilde{\mathcal{C}}_3)$
$\Lambda_{1\ DDH}^{^1S_0-^3P_0}$	$-g_\rho h_\rho^1(2+\chi_V) - g_\omega h_\omega^1(2+\chi_S)$	$\mathcal{G}_2$	$(\mathcal{C}_2+\tilde{\mathcal{C}}_2+\mathcal{C}_4+\tilde{\mathcal{C}}_4)$
$\Lambda_{1\ DDH}^{^3S_1-^3P_1}$	$\frac{1}{\sqrt{2}}g_{\pi NN}h_\pi^1\left(\frac{m_\rho}{m_\pi}\right)^2+g_\rho(h_\rho^1$-$h_\rho^{1\prime}) - g_\omega h_\omega^1$	$2\mathcal{G}_6$	$(2\tilde{\mathcal{C}}_6+\mathcal{C}_2$-$\mathcal{C}_4))$
$\Lambda_{2\ DDH}^{^1S_0-^3P_0}$	$-g_\rho h_\rho^2(2+\chi_V)$	$-2\sqrt{6}\mathcal{G}_5$	$2\sqrt{6}(\mathcal{C}_5+\tilde{\mathcal{C}}_5)$

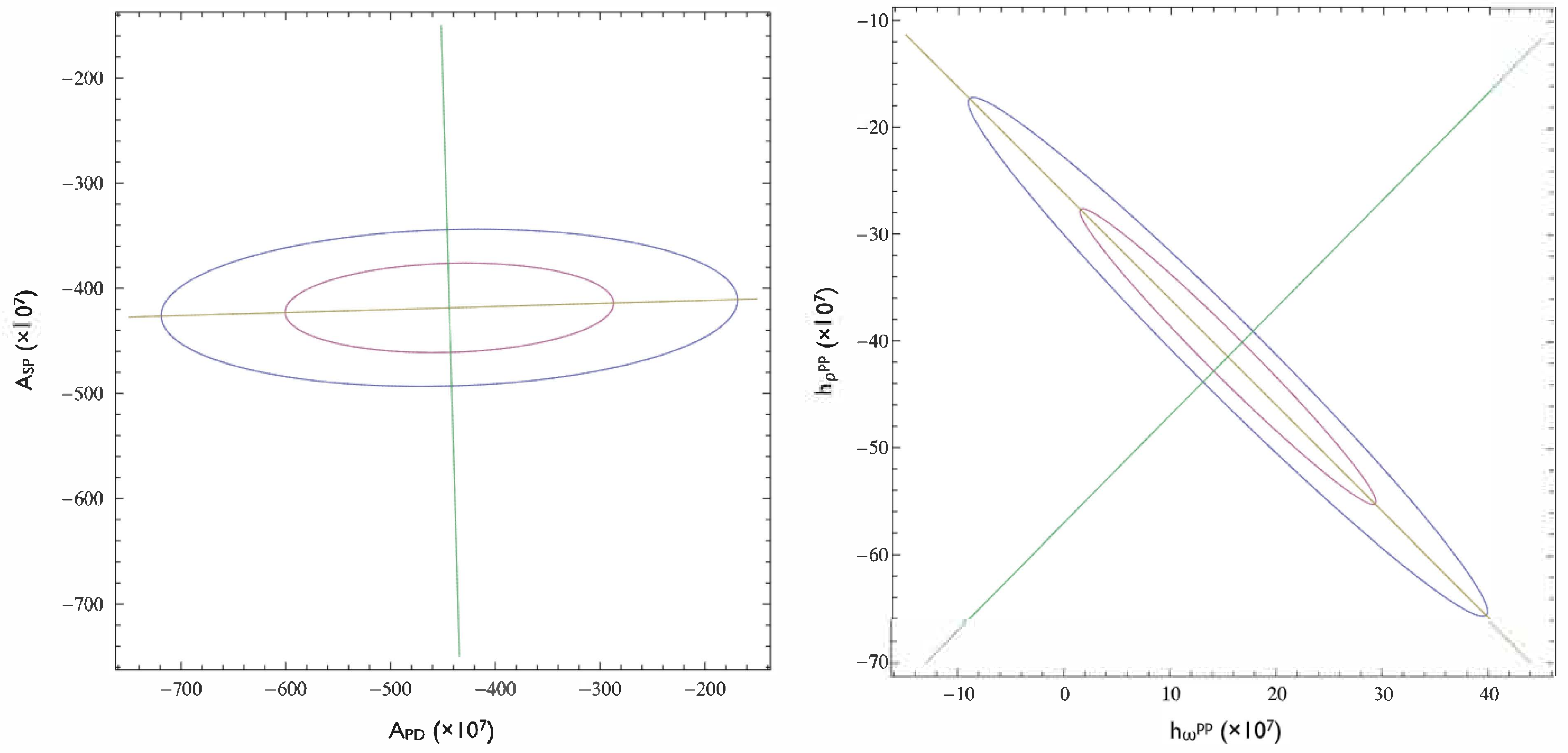
but experiments challenging due to relevant scale

$$\frac{4\pi G_F m_\pi^2}{g_{\pi NN}^2} \sim 10^{-7}$$

$\vec{p}+p$ asymmetry:

at 13.6, 45, 221 MeV

available, interpretable constraints



$$A_L^{\vec{p}+p}(45\ \mathrm{MeV}) = (-1.57 \pm 0.23) \times 10^{-7}$$

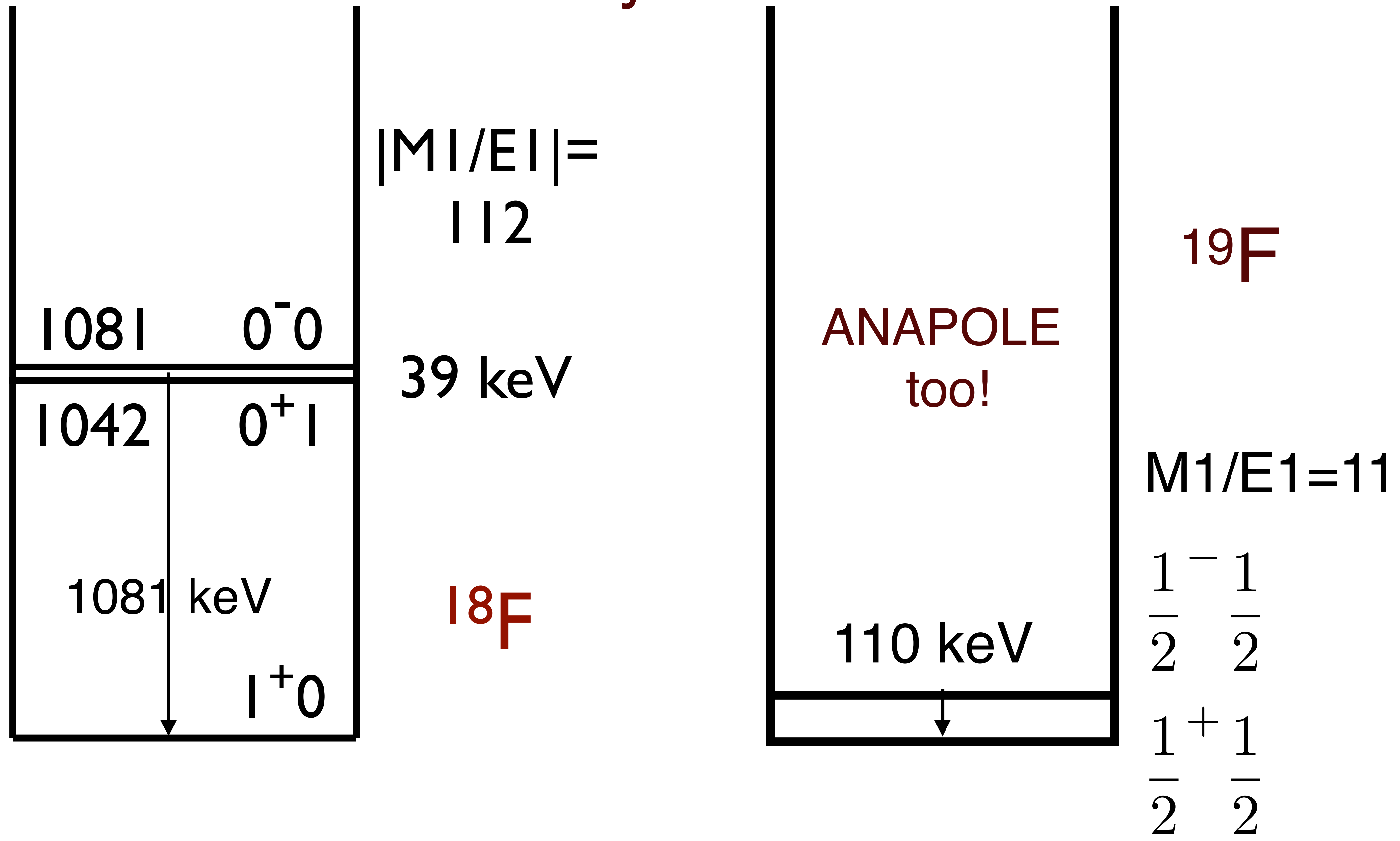
$$A_L^{\vec{p}+\alpha}(46\ \mathrm{MeV}) = (-3.34 \pm 0.93) \times 10^{-7}$$

(NC) $$P_\gamma^{^{18}\mathrm{F}}(1081\ \mathrm{keV}) = (1.2 \pm 3.8) \times 10^{-4}$$

$$A_\gamma^{^{19}\mathrm{F}}(110\ \mathrm{keV}) = (0.74 \pm 0.19) \times 10^{-4}$$

25 year wait ……

Isolated Parity Doublets



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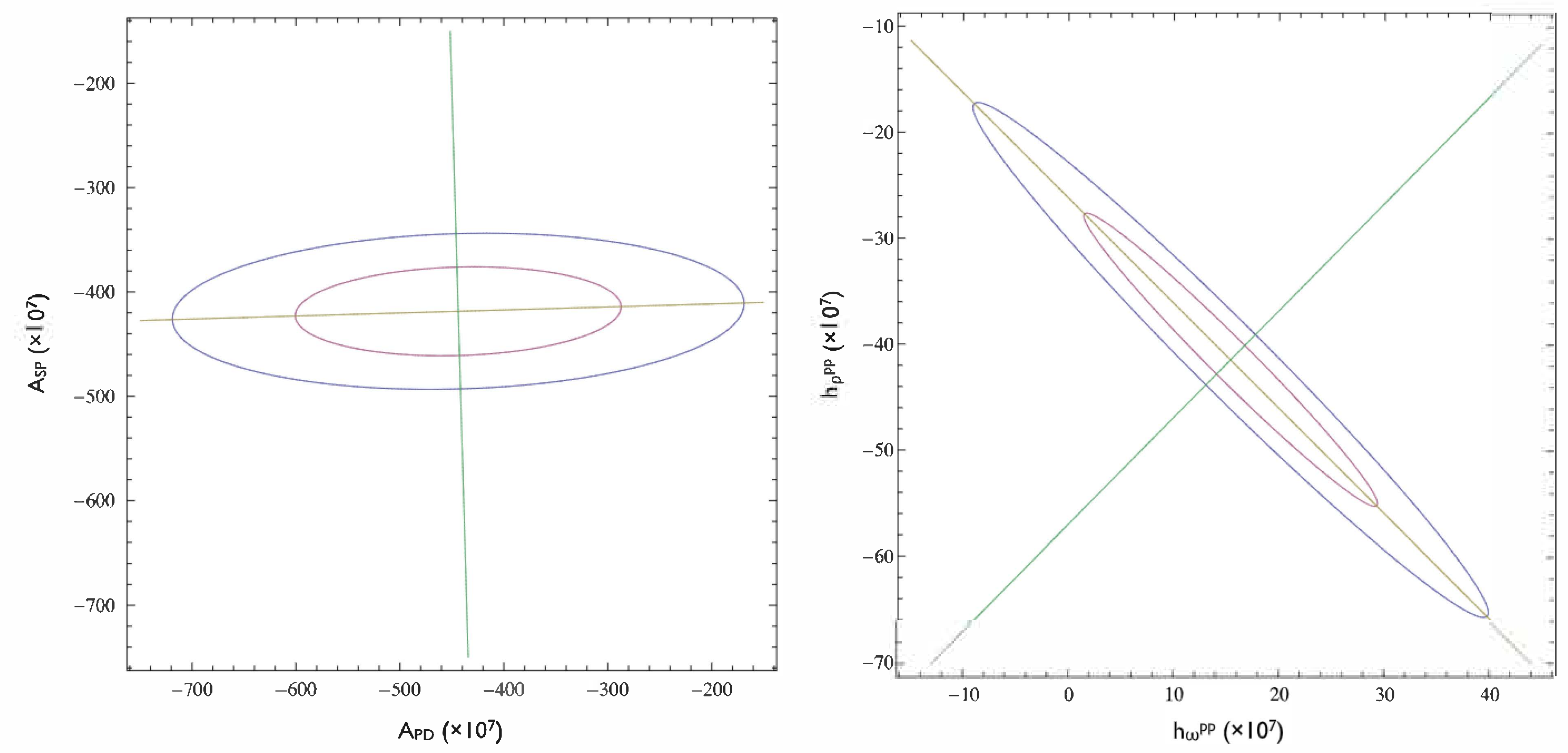

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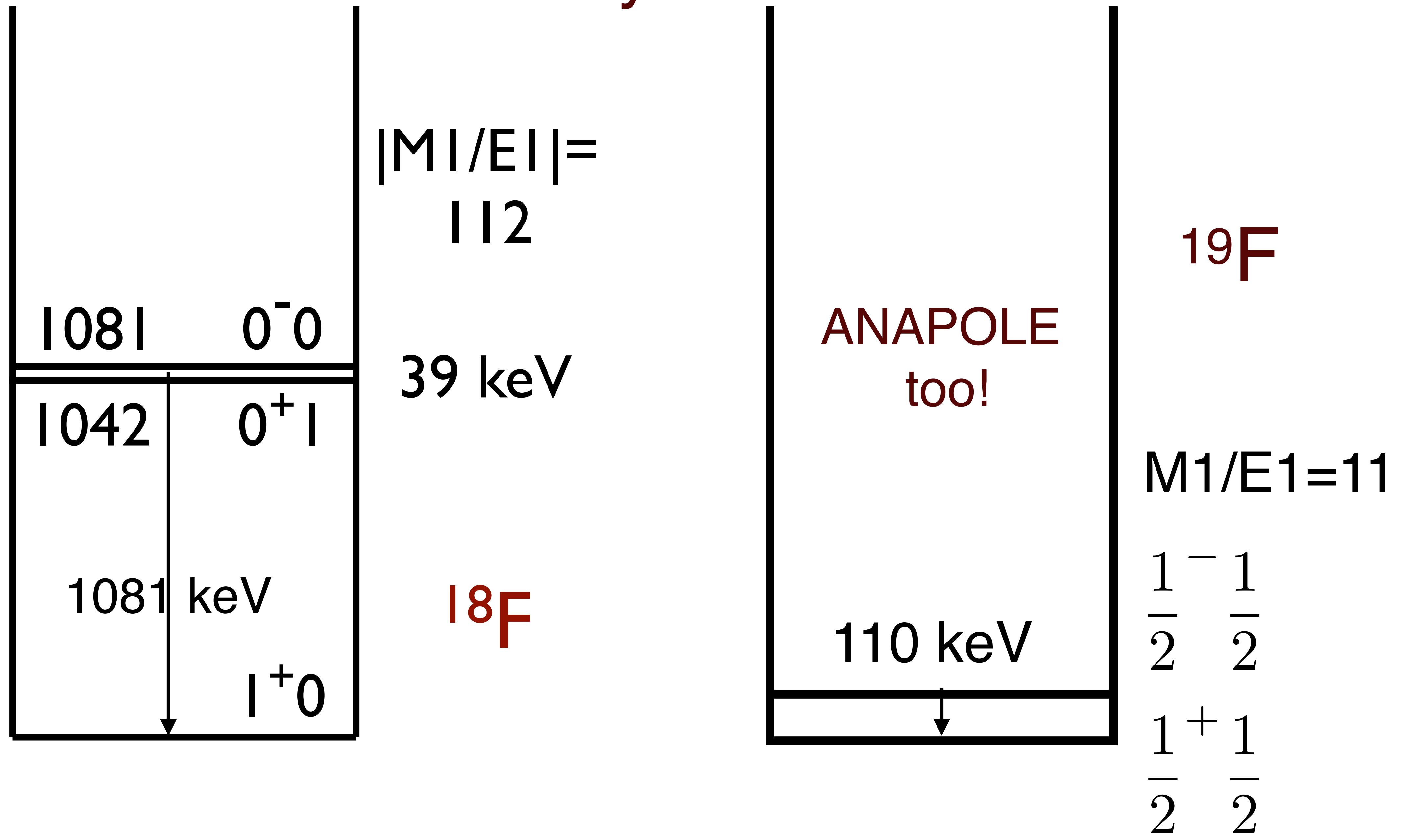
$$SNS \begin{cases} A_\gamma^{\vec{n}+p\to d+\gamma} = (-3.0 \pm 1.4 \pm 0.2) \times 10^{-8} \\ A_p^{\vec{n}+{}^3\mathrm{He}\to{}^3\mathrm{H}+p} = (1.58 \pm 0.97 \pm 0.24) \times 10^{-8} \end{cases}$$

Isolated Parity Doublets



Enhancement in ^{18}F

$$P_\gamma(1081\ \mathrm{keV}) = 2Re\left[\frac{\langle +|V_{\rm PNC}|-\rangle}{39\ \mathrm{keV}}\ \frac{\langle g.s.|M1|+\rangle}{\langle g.s.|E1|-\rangle}\right]$$

1/E:
100 times typical nuclear scale ~ few MeV

⇒ ~10^{-5} vs natural scale 10^{-7}

PC E1: isoscalar E1 in a self-conjugate nucleus: leading-order forbidden

PNC M1: unusually strong 10.3 W.u.⇒

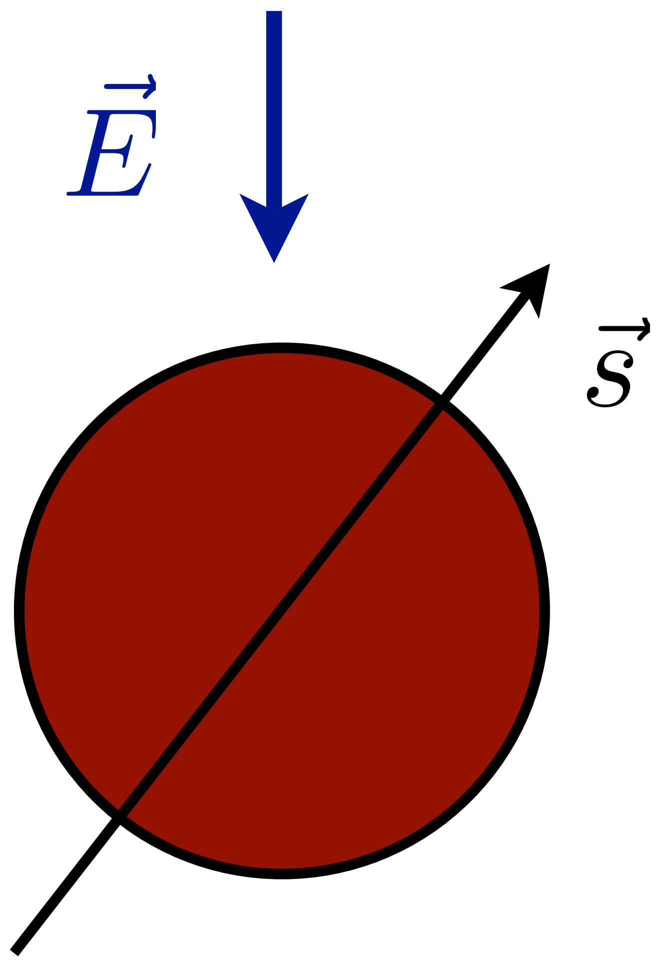
enhancement ~ 110

- so expected effect ~10^{-3}
- heroic efforts by Queens, Florence, other groups $(1.2 \pm 3.8) \times 10^{-4}$
- now recognized that this coupling might be small $\quad h^1_\pi \sim LO \longrightarrow NNLO$

THEME: rare to find both conditions satisfied in edm studies - ^{229}Pa may be an exception

II. Nuclear enhancements of T-odd nuclear moments

^{19}F: similar g.s. parity doublets would enhance sensitivity to P- and T- violating NN interactions



$$H_{edm} = d\ \vec{E} \cdot \vec{s}$$

$$\vec{E}(t \rightarrow -t) \rightarrow \vec{E}$$
$$\vec{s}(t \rightarrow -t) \rightarrow -\vec{s}$$

$$\Rightarrow H_{edm} \rightarrow -H_{edm}$$

Would like to understand in a complex nucleus within a neutral atom

— the moments that contribute to the response

— the decomposition of this response: single nucleon edms, polarization, long-range currents

General classification of electromagnetic moments:

Multipole	P-even, T-even	P-odd, T-odd	P-odd, T-even	P-even, T-odd
$\langle C_J^M \rangle$	even J≥0	odd J≥1	x	x
$\langle M_J^M \rangle$	odd J≥1	even J≥2	x	x
$\langle E_J^M \rangle$	x	x	odd J≥1	even J≥2

General current for a spin-1/2 fermion:

$$\langle p|J_\mu^{\text{em}}|p\rangle = \bar{N}(p')\left(F_1\gamma_\mu + F_2\sigma_{\mu\nu}q^\nu + \frac{a(q^2)}{M^2}(\not{q}q_\mu - q^2\gamma_\mu)\gamma_5 + d(q^2)\sigma_{\mu\nu}q^\nu\gamma_5\right)N(p)$$

Charge Magnetic Anapole Electric Dipole

General classification of electromagnetic moments:

Multipole	P-even, T-even	P-odd, T-odd	P-odd, T-even	P-even, T-odd
$\langle C_J^M \rangle$	even $J \geq 0$	odd $J \geq 1$	x	x
$\langle M_J^M \rangle$	odd $J \geq 1$	even $J \geq 2$	x	x
$\langle E_J^M \rangle$	x	x	odd $J \geq 1$	even $J \geq 2$

P- and T-odd moments for a nucleus:

$$\langle j_N | J_\mu^{em} | j_N \rangle = \begin{cases} C_1 & j_N \geq \frac{1}{2} \\ C_1,\ M_2 & j_N \geq 1 \\ C_1,\ M_2,\ C_3 & j_N \geq \frac{3}{2} \end{cases}$$

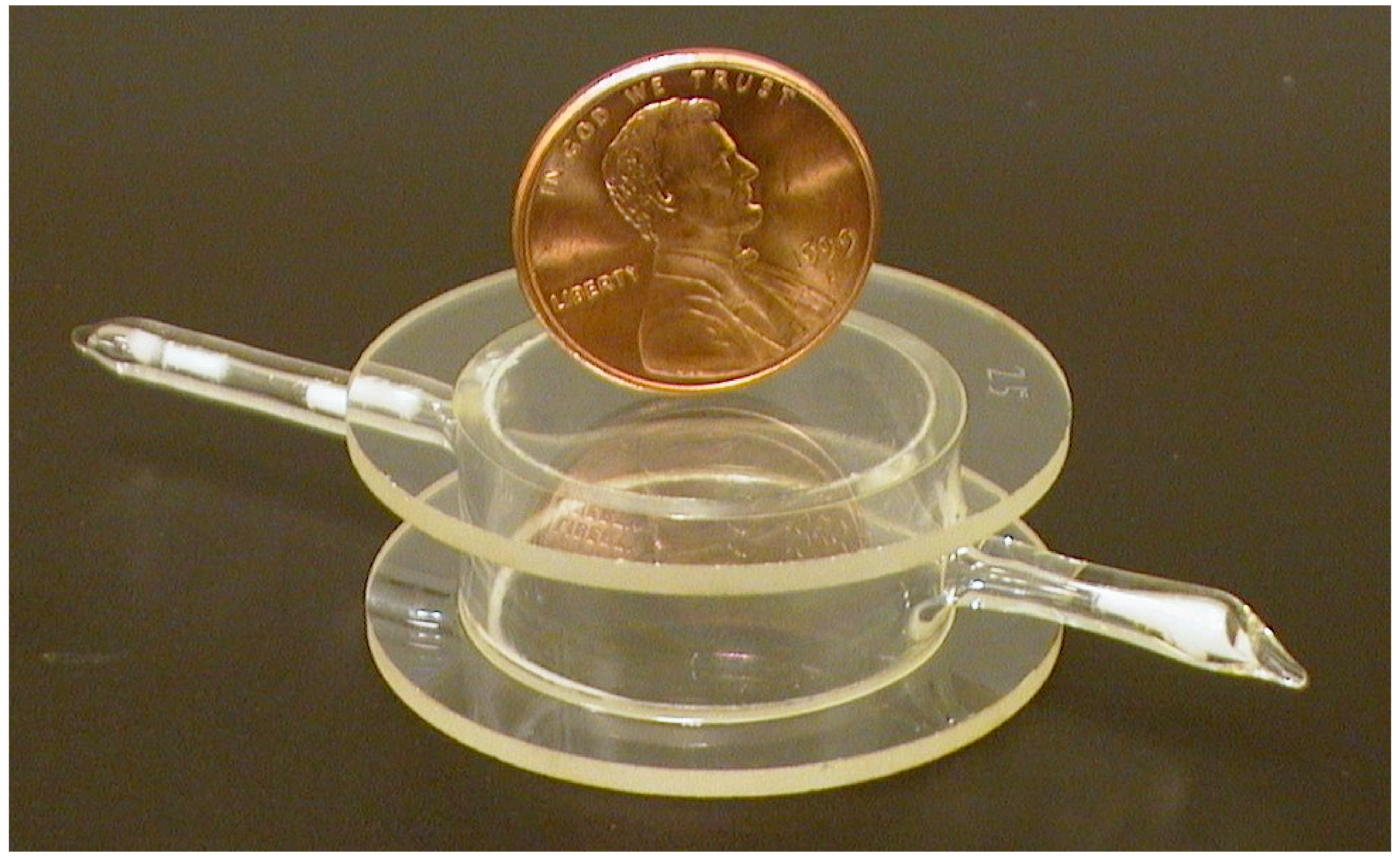
$o(R_N/R_A) \to o(R_N^3/R_A^3)$

$o(R_N^2/R_A^2)\ o(v_N/c)$

$o(R_N^3/R_A^3)$

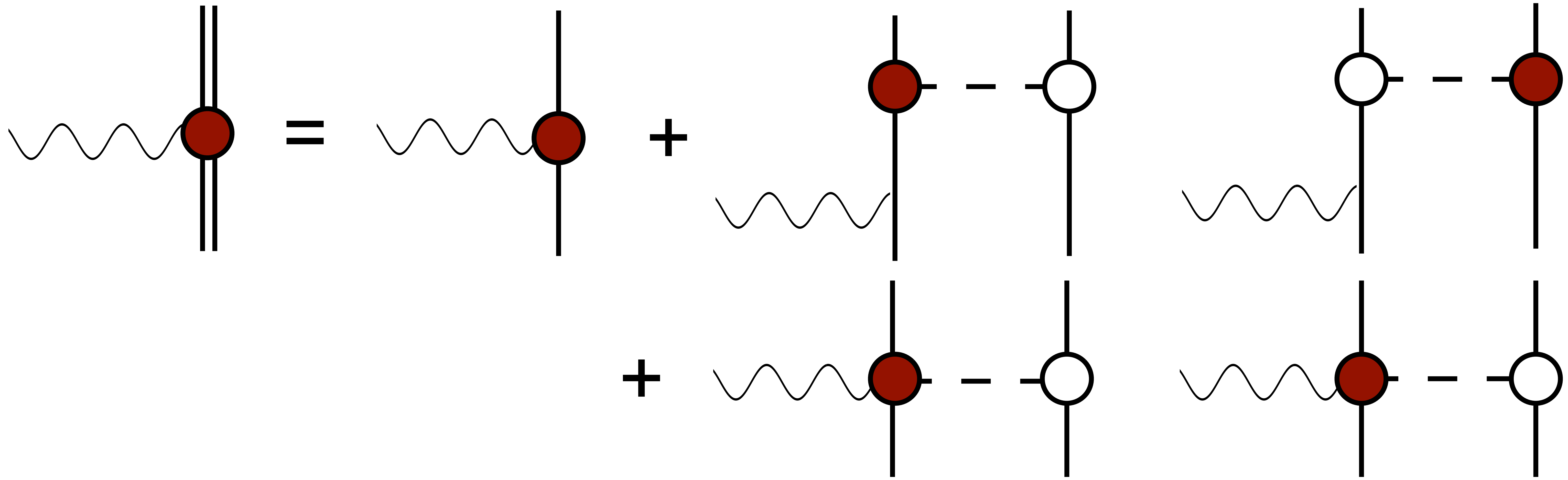
e.g., a case like ^{199}Hg

$j_N = 1/2$:   C1



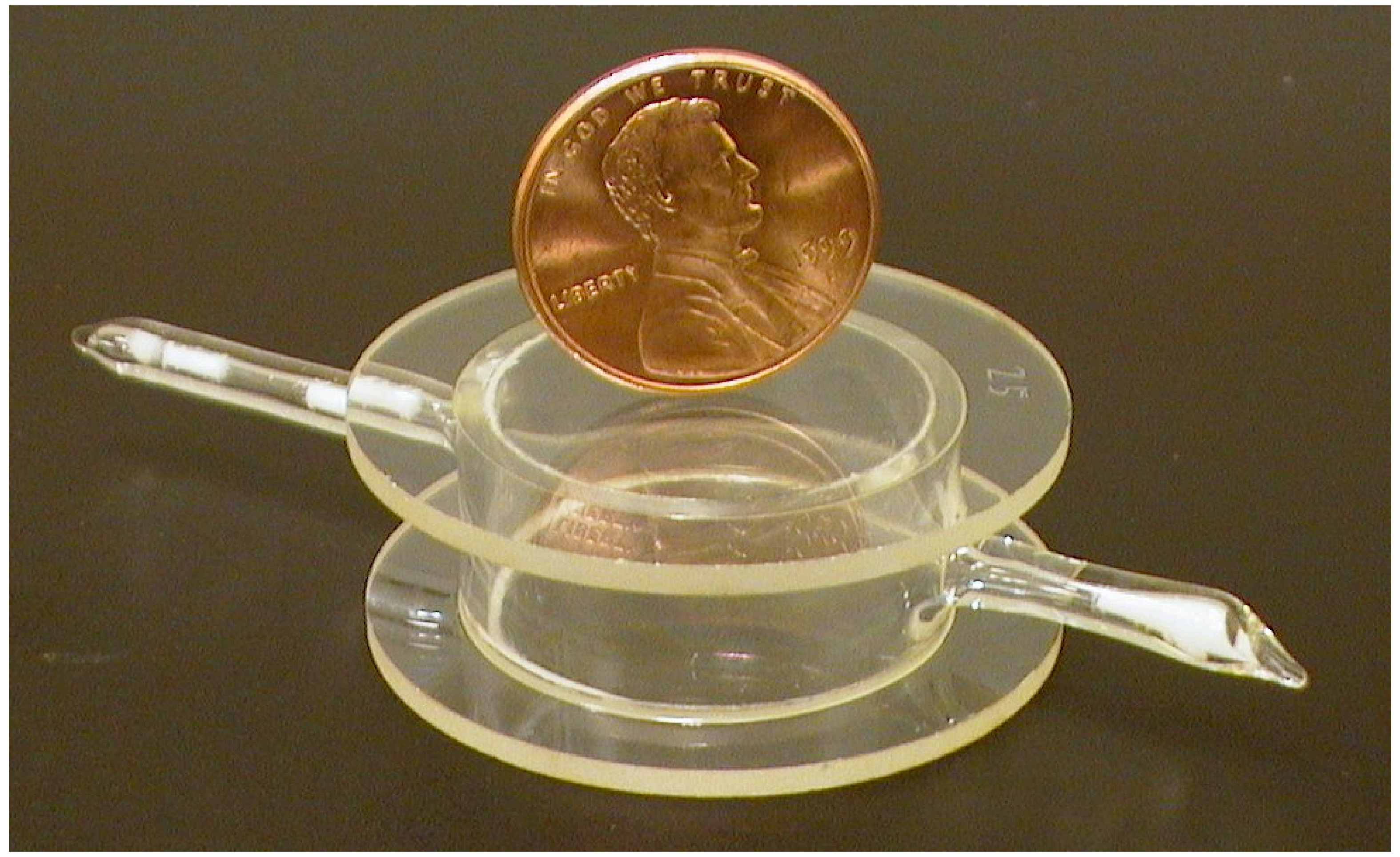
– Number of ^{199}Hg atoms: 10^{14}
– Leakage currents at 10 kV: 0.5 – 1 pA
– N_2 + CO buffer gas (500 Torr)
– Paraffin wall coating
– Spin relaxation time: 100 – 200 sec

Nuclear edm:



nuclear edm = 1-body + polarization + exchange current e.g., a case like ^{199}Hg

$j_N = 1/2$: C1



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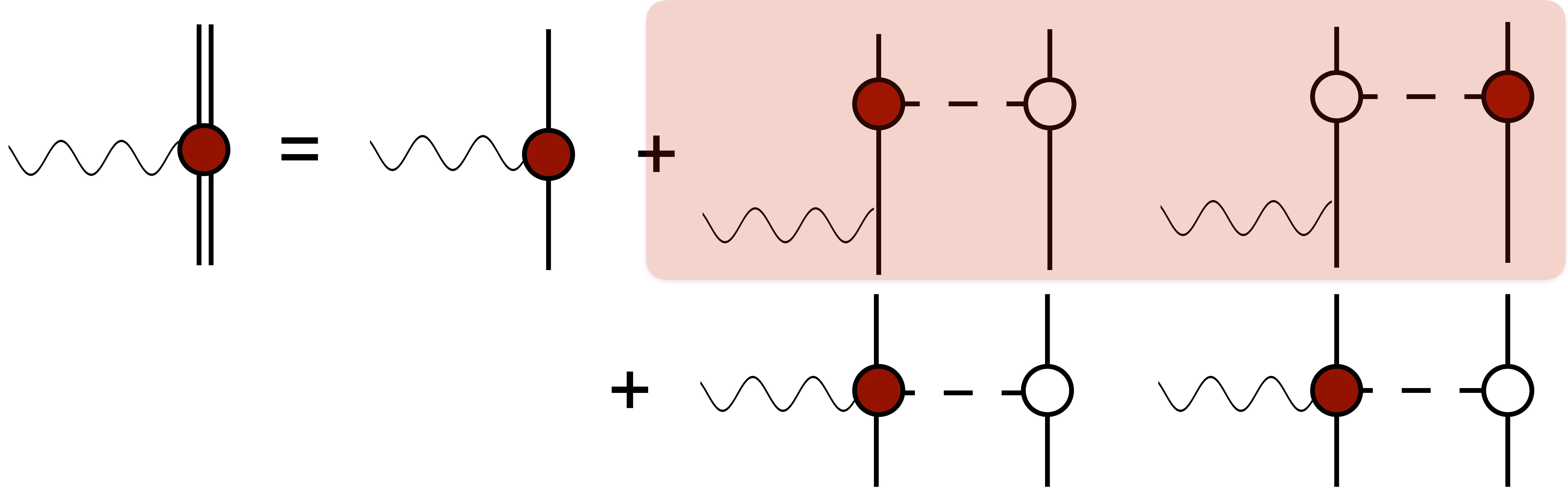
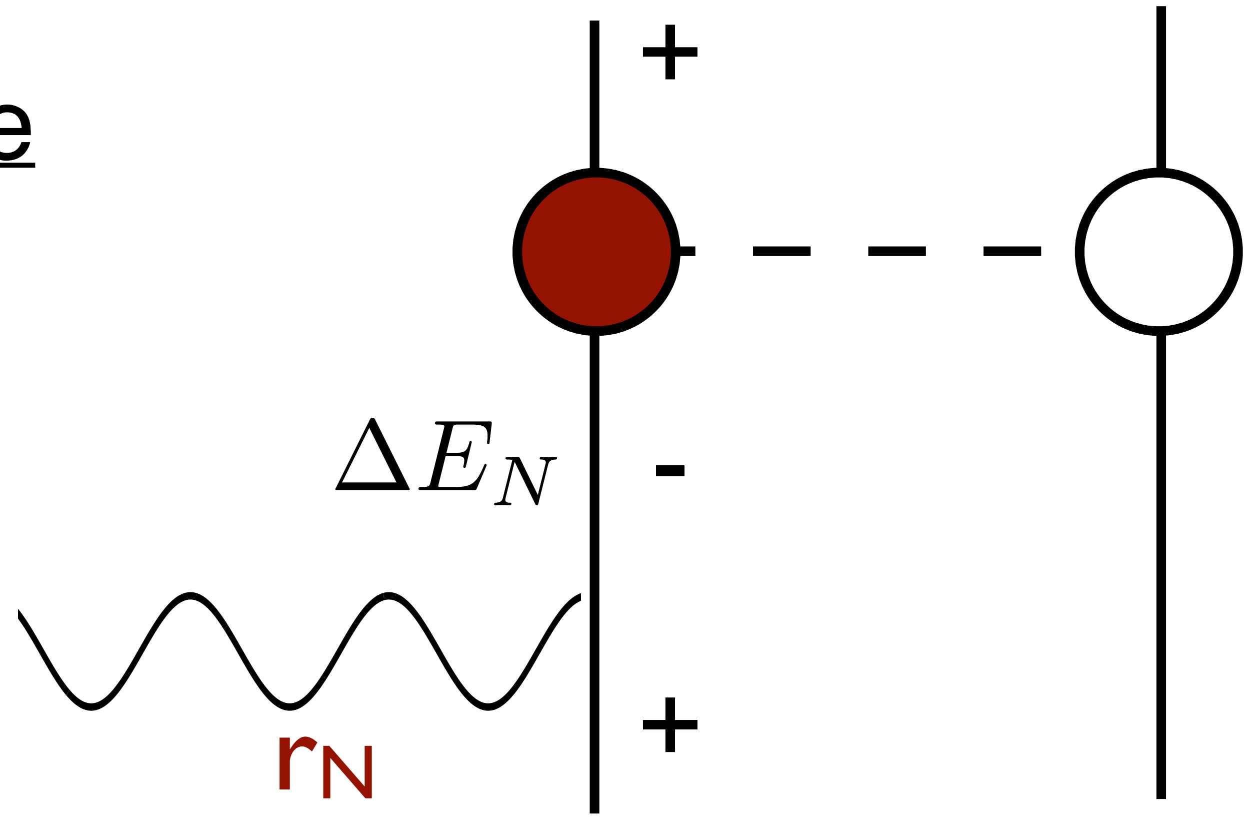


Polarization due to a P- and T-odd interaction

nuclear edm = 1-body + polarization + exchange current

~ 1 ~ 10 $\sim 1/10$

Dimensional estimate



$$d_{\text{Nuclear}} \sim \frac{m_\pi}{\hbar\omega} d_n \sim 10 d_n \quad \text{typically}$$

$$\Rightarrow 10 d_n \, \frac{\hbar\omega}{\Delta E_N} \quad \text{for a favorable parity doublet}$$

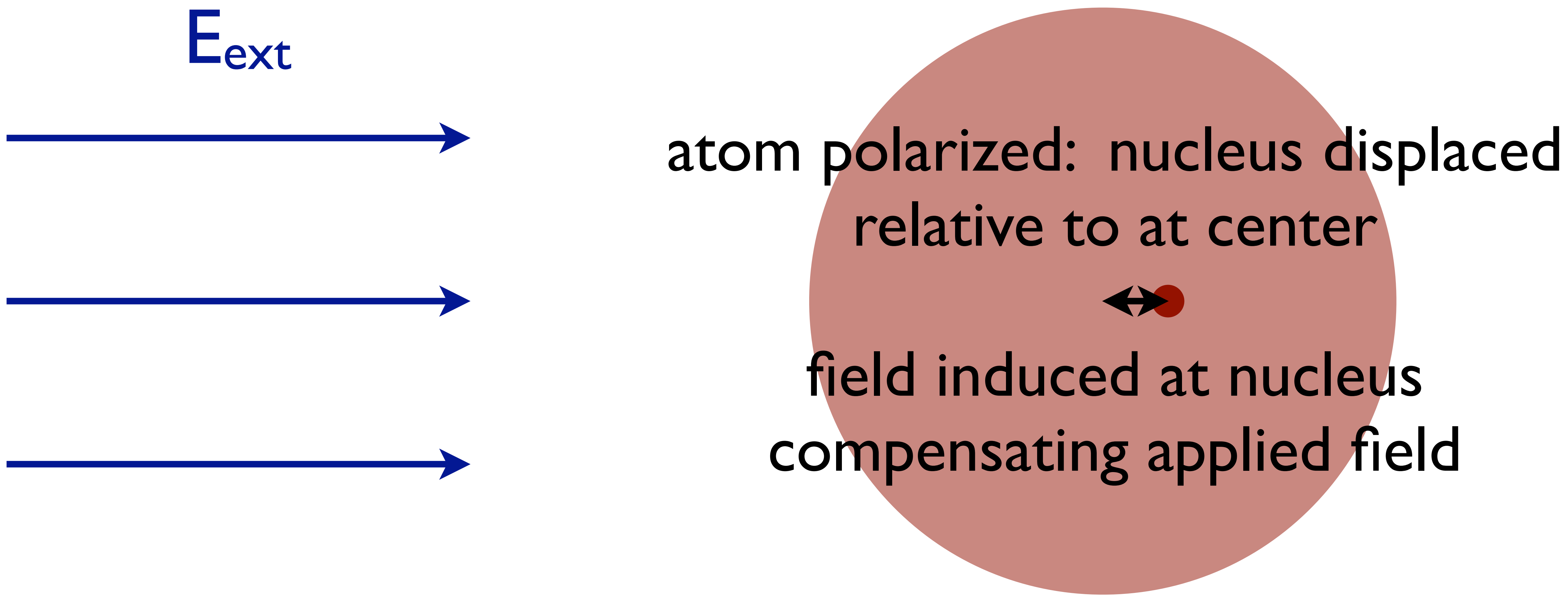
maximize the charge separation by deforming the valence orbital over the full nuclear size (good!)

Schiff screening (bad!)

Interaction energy of a nonrelativistic point nucleus carrying an edm, inside a neutral atom, is zero

reduction in edm sensitivity

$$\sim 10 Z^2 \, (R_N/R_A)^2 \sim 10^{-3}$$



Back-of-the-envelope sensitivity estimates for 199Hg

$$d_A^1(^{199}\mathrm{Hg}) \sim 5 \cdot 10^{-4} d_n$$

generic Schiff screening + nucleon edm

$$d_A(^{199}\mathrm{Hg}) \sim d_A^2(^{199}\mathrm{Hg}) \sim \frac{m_\pi}{\hbar\omega}\, d_A^1(^{199}\mathrm{Hg}) \sim 5 \cdot 10^{-3} d_n$$

add polarization

$$[d_A(\mathrm{Hg}^{199})]_{\mathrm{exp\ limit}} \sim \; 4.1 \cdot 10^{-4} [d_n]_{\mathrm{exp\ limit}} \qquad \begin{cases} 7.4 \cdot 10^{-30} \text{ e cm} & ^{199}\mathrm{Hg} \quad \text{Graner et al.} \\ 1.8 \cdot 10^{-26} \text{ e cm} & \text{neutron} \quad \text{PSI} \end{cases}$$

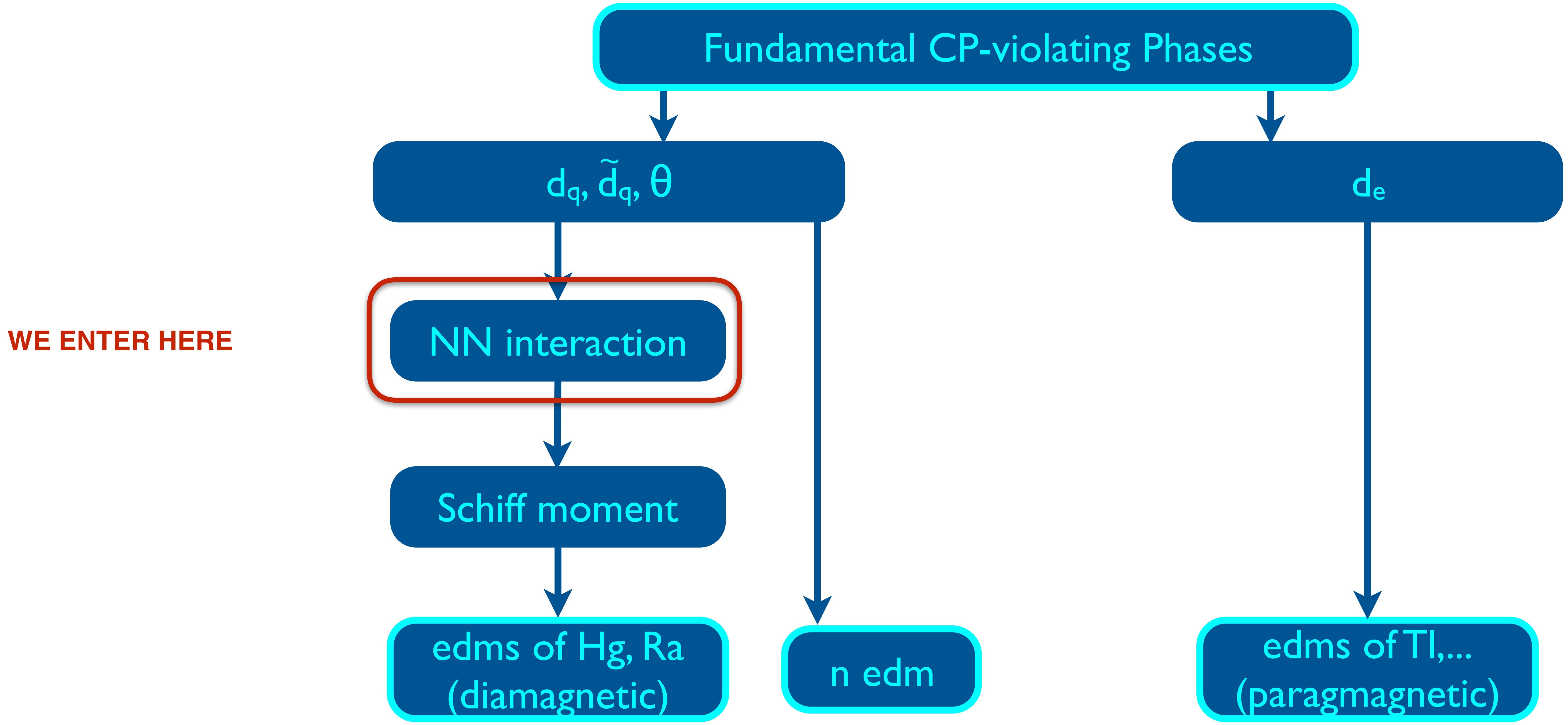
so one would expect ^{199}Hg limits on underlying sources of CP violation to be about a factor of 10 more stringent than those derived from the neutron

Cases of nuclear enhancement similar to the two-level PNC example of ^{18}F where a factor of 10^3 was found, one might be able to probe

$$d_n \sim 10^{-30} \text{ e cm}$$

First effort to find nuclear ground states with enhanced CPNC responses was done a long time ago, and uncovered a rather interesting case   WH and Henley, PRL 51 (1983) 1937

To start we needed nucleon-level operators from which we could derive the nuclear edm: QCD θ parameter was a convenient SM choice



Sketch of the QCD θ parameter example

Underlying coupling in the QCD Lagrangian: $\bar{\theta}\frac{g^2}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$

Generates a low-energy CP-odd coupling to the nucleon*

$$L_{\pi NN} = L_{\pi NN}^{CPNC} = \vec{\pi}\cdot\bar{N}\left(i\gamma_5 g_{\pi NN} + \bar{g}_{\pi NN}\right)N \qquad |\bar{g}_{\pi NN}| \sim 0.027\ |\bar{\theta}|$$

which happens in this case to be isoscalar - the isovector coupling arises in relative order m_π/m_N

from which then generates a CP-odd nuclear interaction

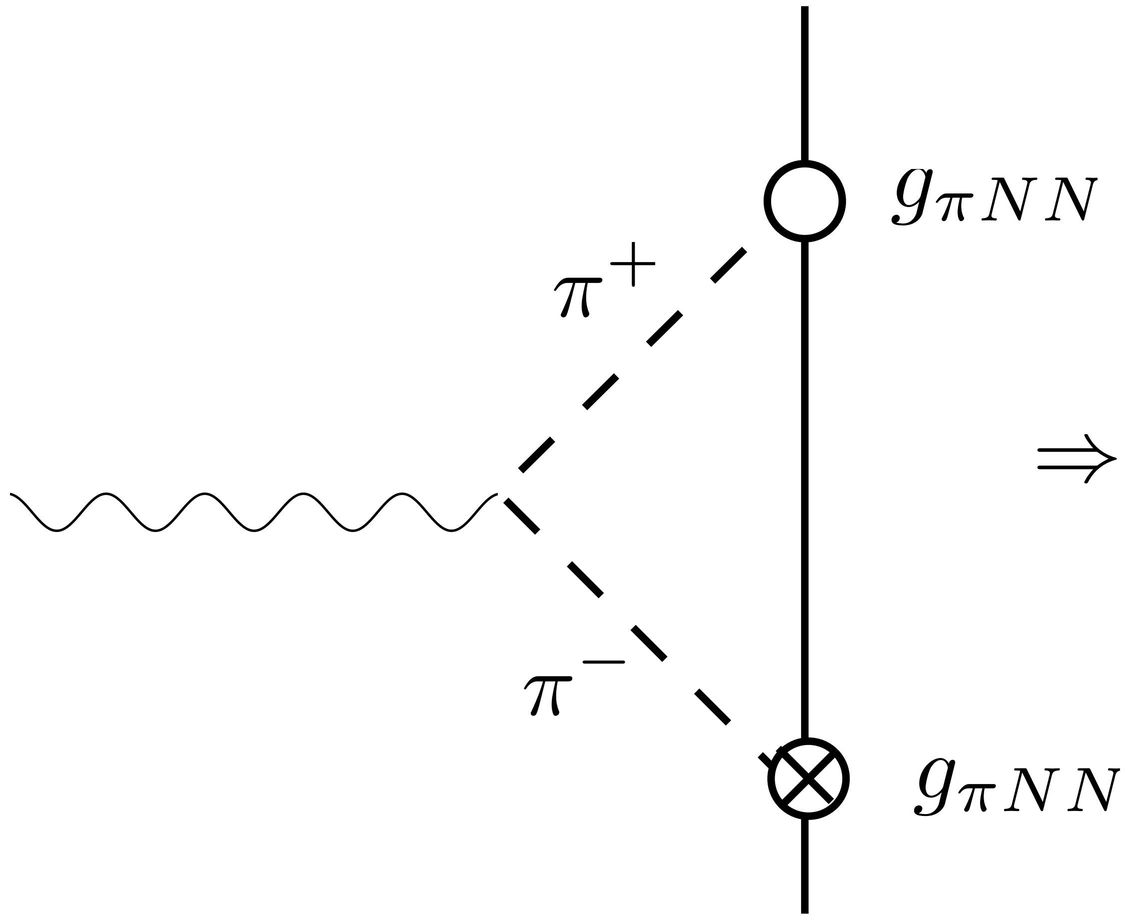
$$\overline{V}_{12} = -0.9\,\frac{d_n}{e}\;m_\pi^2\;\tau(1)\cdot\tau(2)(\vec{\sigma}(1)-\vec{\sigma}(2))\cdot\hat{r}\;\frac{e^{-m_\pi r}}{m_\pi r}\left[1+\frac{1}{m_\pi r}\right]$$

that can mix valence S and P orbitals of valence nucleons in an odd-A nucleus

** Crewther, Di Vecchia, Veneziano, Witten, Phys. Lett. 88B (1979) 123 and 91B (1980) 487*

A nucleon edm is similarly generated

(again, maximize charge separation)



$$\Rightarrow \quad d_n \sim \frac{e g_{\pi NN} \bar{g}_{\pi NN}}{4\pi^2 M} \, ln\left(\frac{M}{m_\pi}\right)$$

$$\sim 3.6 \times 10^{-16} \; \bar{\theta} \text{ e cm} \;\; \Rightarrow \;\; \bar{\theta} < 10^{-11}$$

which fixes the one-body electromagnetic current operator

$$\langle p'|J^{(1)}_\mu|p\rangle = e\bar{U}(p') \left[F_1\gamma^\mu + F_2\sigma^{\mu\nu}\frac{q_\nu}{M_N} + d_n\sigma^{\mu\nu}q_\nu\gamma_5\tau_3 \right] U(p)$$

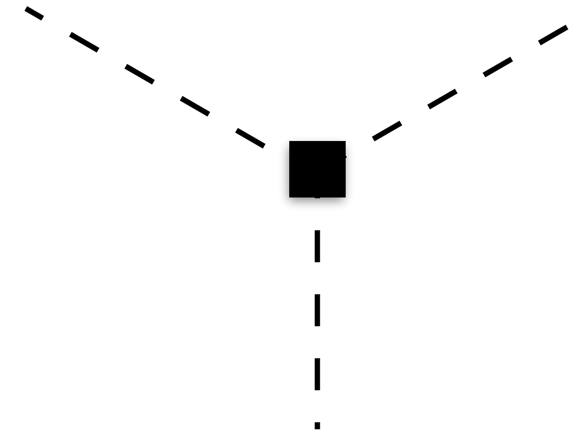
multipoles:	C1, C3, … M2, M4, …	$\overline{\text{C1}}$, $\overline{\text{C3}}$, … $\overline{\text{M2}}$, $\overline{\text{M4}}$, …
contributes:	via polarization	via unperturbed wf

So if we evaluate nuclear $\not{C}\!P$ moments, each moment has two contributions, e.g.,

$$\langle 0|\overline{C1}|0\rangle + \sum_n \left[\langle 0|C1|n\rangle \frac{1}{E_0 - E_n} \langle n|\frac{1}{2}\sum_{i\neq j}\overline{V_{ij}}|0\rangle + \langle 0|\frac{1}{2}\sum_{i\neq j}\overline{V_{ij}}|n\rangle \frac{1}{E_0 - E_n}\langle n|C1|0\rangle \right]$$

with the polarization term involving the ordinary C1 operator generically dominant.

One can elaborate the nuclear physics: current conservation demands two-body currents. The θ-term QCD Lagrangian generates a three-pion vertex that would induce three-body potentials in nuclei



which in this case arises only at order m_π^2/m_N^2

Similar reductions to the nuclear scale can be performed for other sources of $\not{C}\!P$*

* see KITP talks by Andreas Wirzba, Jordy de Vries, and Vincenzo Cirigliano from the 2016 Frontiers in Nuclear Physics program)

Search for enhanced T-odd nuclear responses

polarization term : one-body : exchange currents 10 : 1 : 1/10

looked ground-state parity doublets in heavy nuclei connected by strong C1 amplitudes

TABLE I. Nuclear electric dipole and magnetic quadrupole moments.

Nucleus	$[Nn_Z\Lambda,K^\pi]_{g.s.}$ [a]	$[Nn_Z\Lambda,K^\pi]_{e.s.}$ [a]	ΔE (keV)	$\langle 1\|V\|0\rangle/\bar{g}$ (keV) [b]	$\langle 0\|\|GT\|\|0\rangle$ [b]	$\langle 0\|\|C1\|\|1\rangle$ [c]	D_N/d_n	$M2/m2$
^{153}Sm	[651, $\frac{3}{2}^+$]	[521, $\frac{3}{2}^-$]	35.8	−170	−0.65	>3.74	>86.1	>10.1
^{161}Dy	[642, $\frac{5}{2}^+$]	[523, $\frac{5}{2}^-$]	25.7	−237	−1.21	0.39	10.3	−541
^{165}Er	[523, $\frac{5}{2}^-$]	[642, $\frac{5}{2}^+$]	47.2	213	1.03	0.64	9.6	664
^{225}Ac	[532, $\frac{3}{2}^-$]	[651, $\frac{3}{2}^+$]	40.0	180	−0.56	<−0.74	>19.3	<−610
^{227}Ac	[532, $\frac{3}{2}^-$]	[651, $\frac{3}{2}^+$]	27.4	187	−0.56	−0.21	8.7	−926
^{229}Pa	[642, $\frac{5}{2}^+$]	[523, $\frac{5}{2}^-$]	0.22	39	1.05	−4.58	2390	12400

C1 matrix elements derived from lifetimes and internal conversion rates, using standard IC tables

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C1 matrix elements derived from lifetimes and internal conversion rates, using standard IC tables

Results are generally somewhat disappointing: but not surprising as C1s and E1s tend to be weak at low E

The ^{229}Pa case

Polarization enhancements of the nuclear edm $D_N/d_n \sim 2400$ and M2 moment $M_2/m_2 \sim 12400$ exceed other cases

This is definitively a “two-level” system, analogous to cases like ^{18}F, ^{19}F, and ^{21}Ne in HPNC

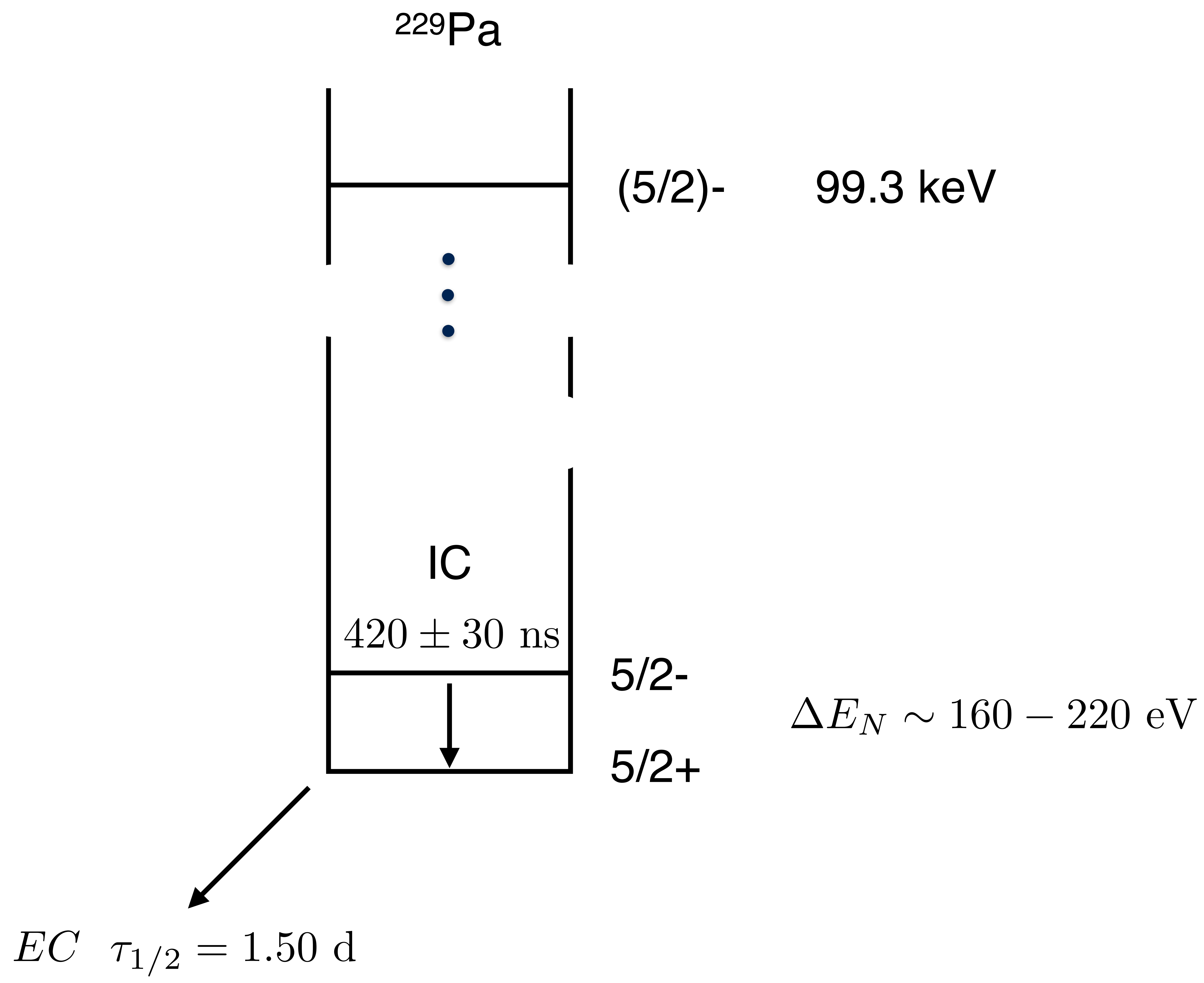
The ^{229}Pa parity doublet, originally assigned a 220 eV splitting (!) had been identified just before our study, in an ANL experiment by Ahmed et al.

In the simple Nilsson model for the canonical deformation used for this mass region,
the nearly degenerate doublet has K=5/2 and labels $[Nn_z\Lambda, K^\pi] = [642, \frac{5}{2}^+]$ $[Nn_z\Lambda K^\pi] = [523, \frac{5}{2}^-]$

Unstable to EC: $\tau_{1/2} \sim 1.50$ d

We were intrigued by this case, but no motivation to pursue it (at the time)

- no examples of edm experiments with unstable isotopes
- ^{229}Pa source?
- connections between octupole deformation and enhanced edms unexplored



^{229}Pa
(5/2)-
99.3 keV
IC
420 ± 30 ns
5/2-
$\Delta E_N \sim 160 - 220$ eV
5/2+
$EC \quad \tau_{1/2} = 1.50$ d

$$D_N(^{225}\text{Ra}) < 1.4 \times 10^{-23}\ \text{e cm}$$

significant improvements expected

RADIUM SETUP

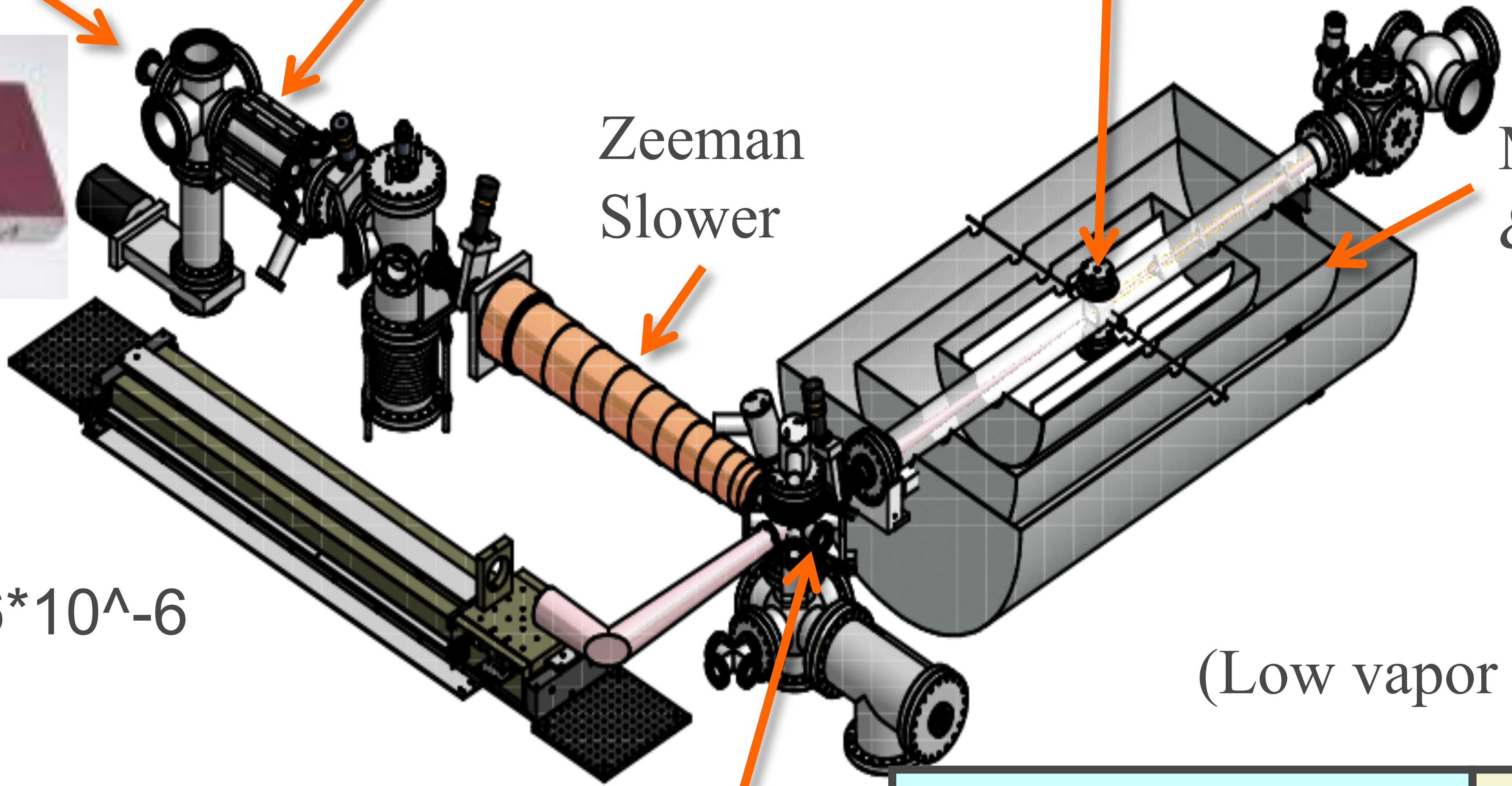
$Ra(NO_3)_2$+Ba Oven

Transverse Cooling

HV Electrodes

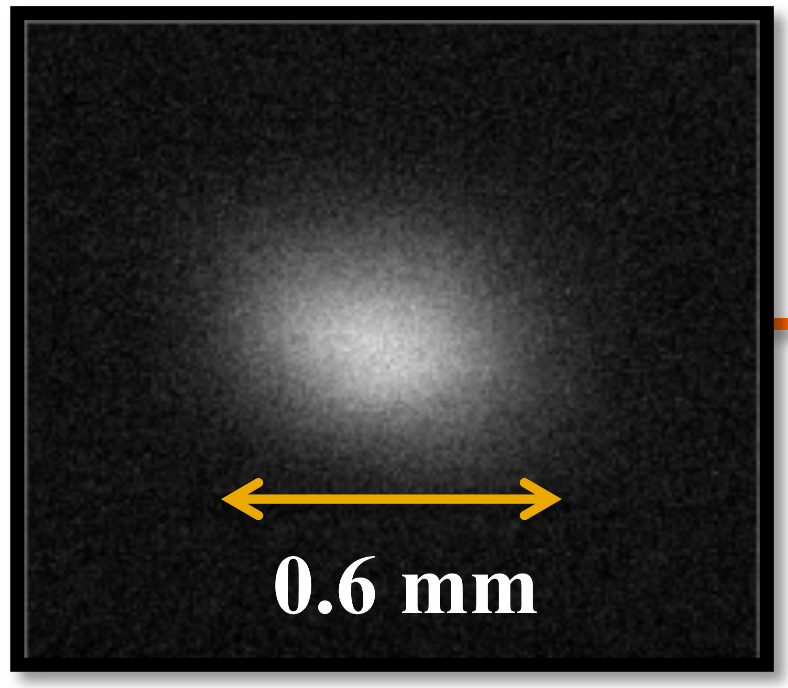
Zeeman Slower

Magnetic Shielding & Magnet Coils



η ~ 1.6*10^-6

(Low vapor pressure)



^{226}Ra MOT
200,000 atoms
40 μK

For EDM:	For Testing:
Ra-225	Ra-226
I = 1/2, J = 0	I = 0, J = 0
$t_{1/2}$ = 15 days	$t_{1/2}$ = 1600 yrs

J. R. Guest et al., PRL 98 093001 (2007)

Isotope sources

FRIB is planning an isotope harvesting program
^{238}U beam program will yield large productions of ^{225}Ra, ^{229}Pa, ^{221}Rn, ^{223}Rn

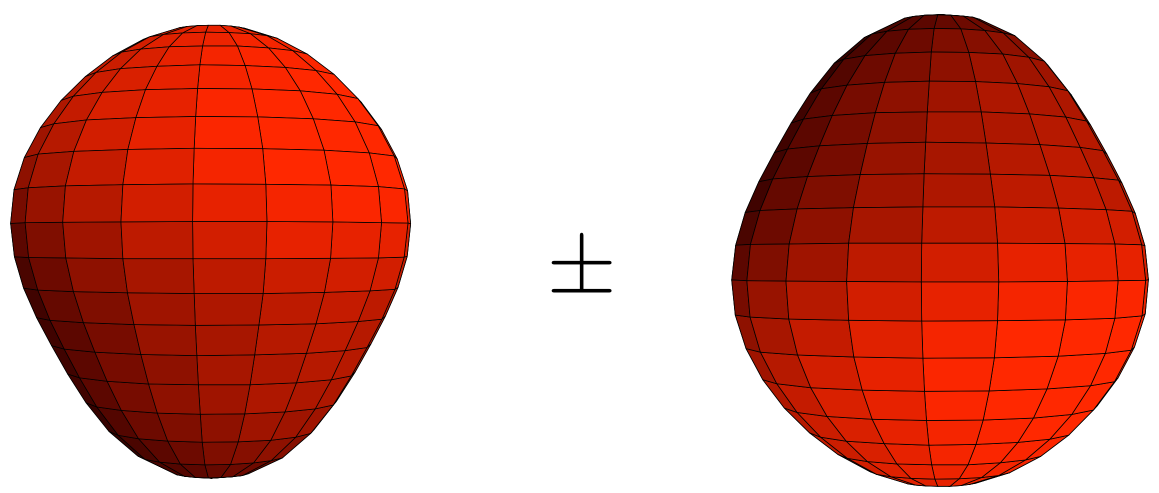
Ra	225	15 d	EDM	EDM	^{238}U	4.9E+00	mCi/wk
Ac	225	10 d	medicine	generator for ^{213}Bi, or direct alpha therapy	^{238}U	4.4E+01	mCi/wk
Ac	227	21.7 y	medicine	impurity in ^{225}Ac / parent to ^{227}Th	^{238}U	3.4E-02	mCi/wk
Th	227	18.7 d	medicine	generator for ^{223}Ra	^{238}U	6.4E+01	mCi/wk
Th	228	1.9 y	medicine	generator ^{212}Pb/^{212}Bi	^{238}U	8.1E+00	mCi/wk
Pa	229	1.5 d	EDM	level splitting, octupole deformation, EDM	^{238}U	3.9E+02	mCi/d
Th	229	7.9 ky	medicine, EDM	nuclear clock, ^{225}Ra parent, ^{225}Ac parent	^{238}U	2.0E-03	mCi/wk

The productions of both ^{225}Ra and ^{229}Pa are both favorable, ^{229}Pa particularly so

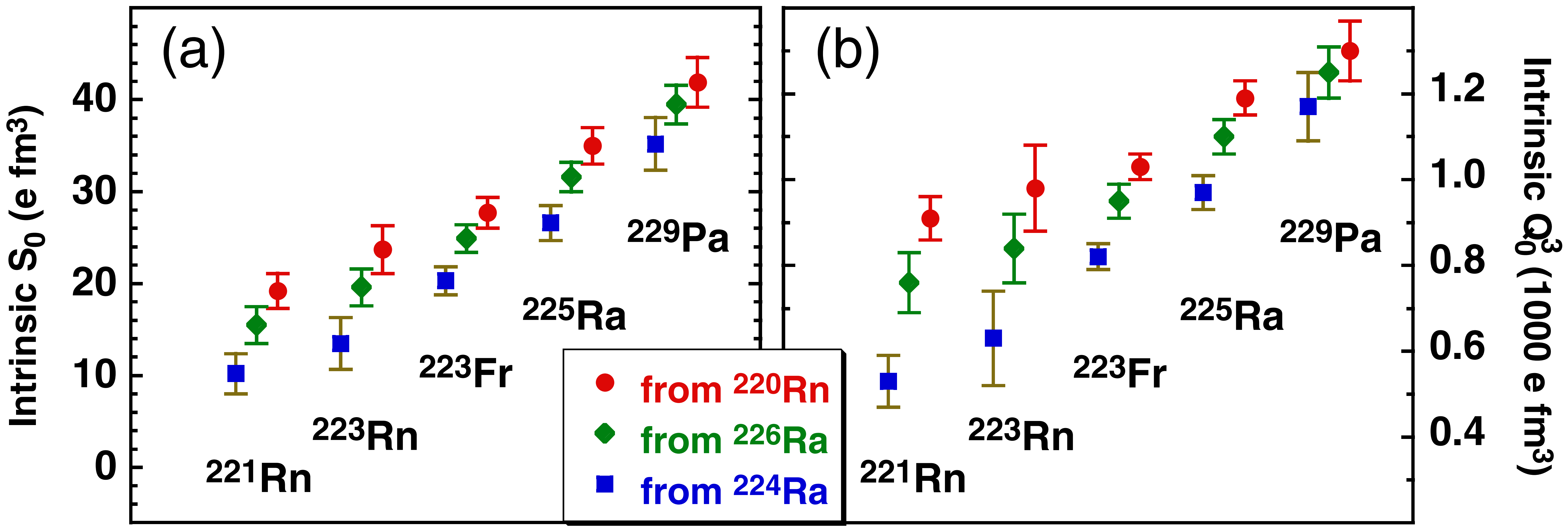
If harvesting can be done daily, the available number of atoms of ^{229}Pa available for trapping would be about 50 times that of ^{225}Ra

Isotope Harvesting at FRIB, G. W. Severin, editor

Octupole collectivity



Now appreciated this collectivity operates for ^{229}Pa

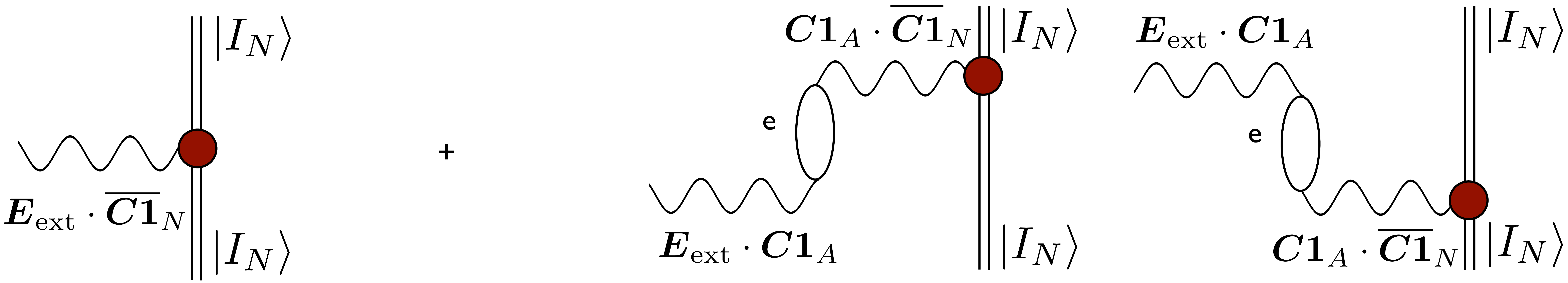


Dobaczewski, Engel, Kortelainen, and Becker, PRL 121 (2018) 232501

FIG. 3. Intrinsic Schiff moments S_0 in e fm^3 (a) and octupole moments Q_0^3 in units of 1000 e fm^3 (b) of ^{221}Rn, ^{223}Rn, ^{223}Fr, ^{225}Ra, and ^{229}Pa, determined from the experimental octupole moments of ^{224}Ra, ^{226}Ra, and ^{220}Rn.

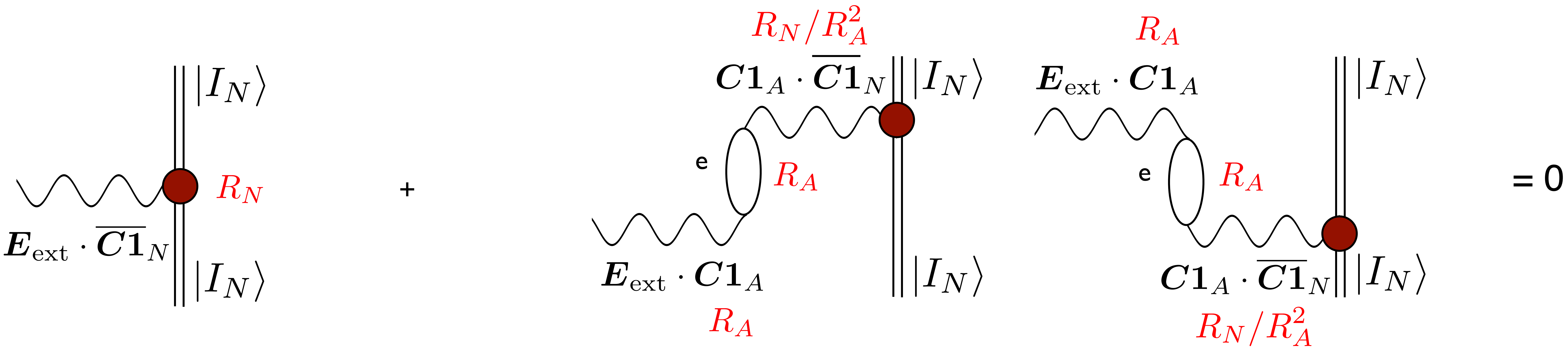
So some of the requirements for a viable experiment are established

Schiff moment in ^{229}Pa



Schiff moment in ^{229}Pa

Count in $\frac{R_N}{R_A}$ where $E_A \sim \frac{1}{R_A}$ $E_N \sim \frac{1}{R_N}$



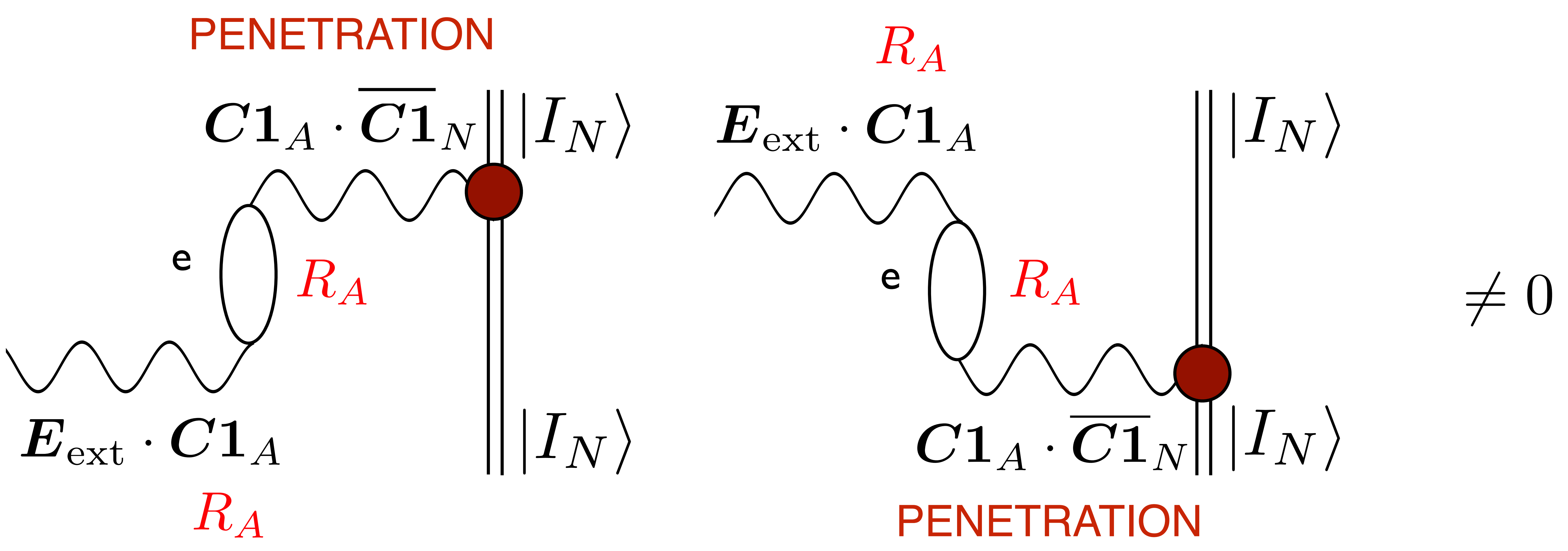
so leading order terms are all o(R_N) but cancel exactly in the point nucleus limit

$$H' = -\frac{4\pi\alpha}{R_A} \sum_{\ell=1}^{\infty} \frac{1}{2\ell+1} \left(\frac{R_N}{R_A}\right)^{\ell} \boldsymbol{C}_\ell^{\boldsymbol{A}} \cdot \boldsymbol{C}_\ell^{\boldsymbol{N}}$$

point Coulomb interaction used above: cancels

+ PENETRATION TERMS

Schiff moment in ^{229}Pa



so leading order terms are all o(R_N) but cancel exactly in the point nucleus limit

$$H' =$$

$$-4\pi\alpha \sum_{\ell=0}^{\infty} \left[\left(\sum_{i=1}^{Z} x_i^\ell \; Y_\ell(\Omega_i) \right) \cdot \int_{x_i}^{\infty} d\boldsymbol{y} \; \rho_N(\boldsymbol{y}) \frac{1}{y^{\ell+1}} Y_\ell(\Omega_y) - \left(\sum_{i=1}^{Z} \frac{1}{x_i^{\ell+1}} \; Y_\ell(\Omega_i) \right) \cdot \int_{x_i}^{\infty} d\boldsymbol{y} \; \rho_N(\boldsymbol{y}) y^\ell Y_\ell(\Omega_y) \right]$$

penetration terms generate l=1 Schiff moment: $o[R_N \frac{R_N^2}{R_A^2}]$

Dynamical shielding* in ^{229}Pa

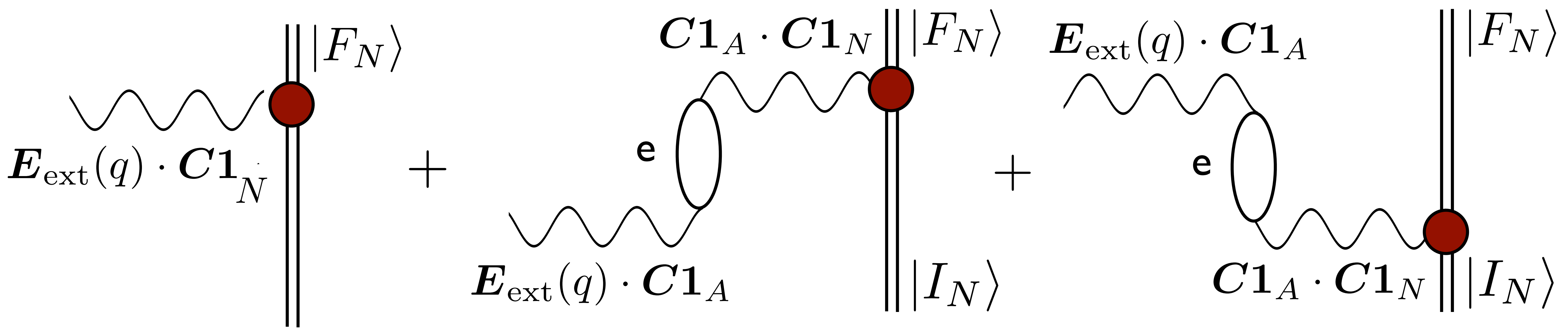
The $\frac{5}{2}^- \rightarrow \frac{5}{2}^+$ C1/E1 strength was determined from the measured IC rate

Ahmad et al. found a 420 ± 30 ns lifetime: using standard IC coefficients $\alpha(E1)$, extrapolated, to deduce a 0.025 W.u. E1 strength — unusually strong

Direct calculations were later done, yielding values of $\alpha(E1)$ somewhat larger, ameliorating the situation a bit

Dragoun et. al, PRC 47 (1993) 870

But consider the T- and P-conserving process



source of $\mathbf{E}_{\rm ext}(q)$ is the IC of a weakly-bound electron: Schiff-shielding argument goes through

(up to a correction $\sim \Delta E / \langle E1 \rangle_{\rm atomic} \sim 1/10$)

*M. Leon and R. Seki, NP A298 (1978) 333

This physics would be accounted for in an atomic RPA calculation. Find

Energy	shell	shell	shell	shell	IPA $\alpha(E1)$	RPA $\alpha(E1)$
220 eV		5p3/2	5d3/2	5d5/2	2635	67.7
220 eV		5p3/2	5d3/2	5d5/2	2580	67.7
220 eV		5p3/2	5d3/2	5d5/2	2610	68.8
170 eV			5d3/2	5d5/2	6352	222.5
195 eV		5p3/2	5d3/2	5d5/2	4698	126.1
245 eV	5p1/2	5p3/2	5d3/2	5d5/2	2127	61.4
270 eV	5p1/2	5p3/2	5d3/2	5d5/2	1315	49.2

The IPA range roughly encompasses previous results: 1300-6400

Turning on RPA corrections reduces the range: 49-222

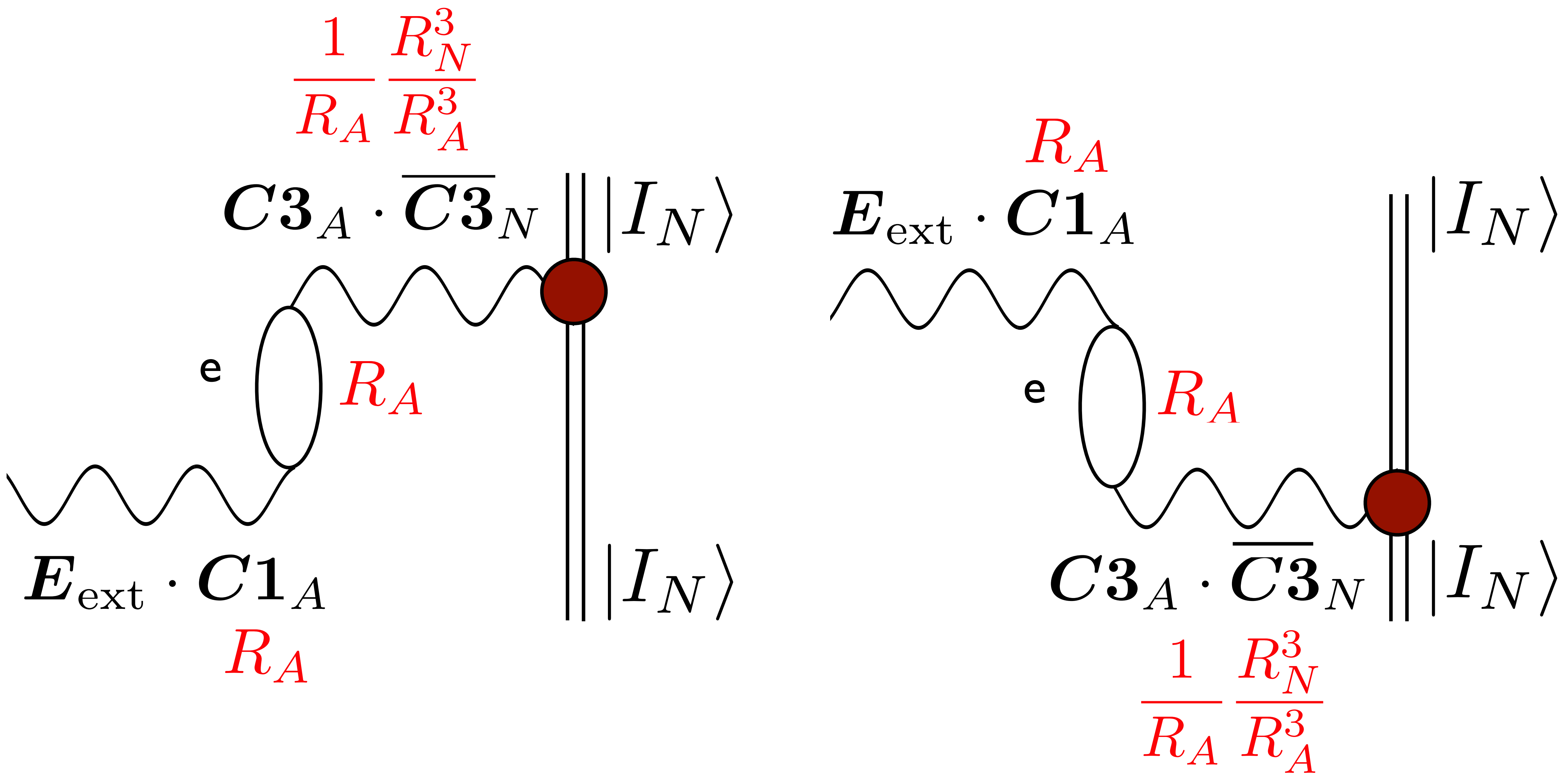
Implies nuclear E1 strength of 0.18-0.37 W.u. !! Is ^{229}Pa this exceptional?

Unique in that it has both an extreme energy splitting and a very strong C1

Other moments for ^{229}Pa (5/2-): C3

so dimensional of the same order as the shielded C1 response

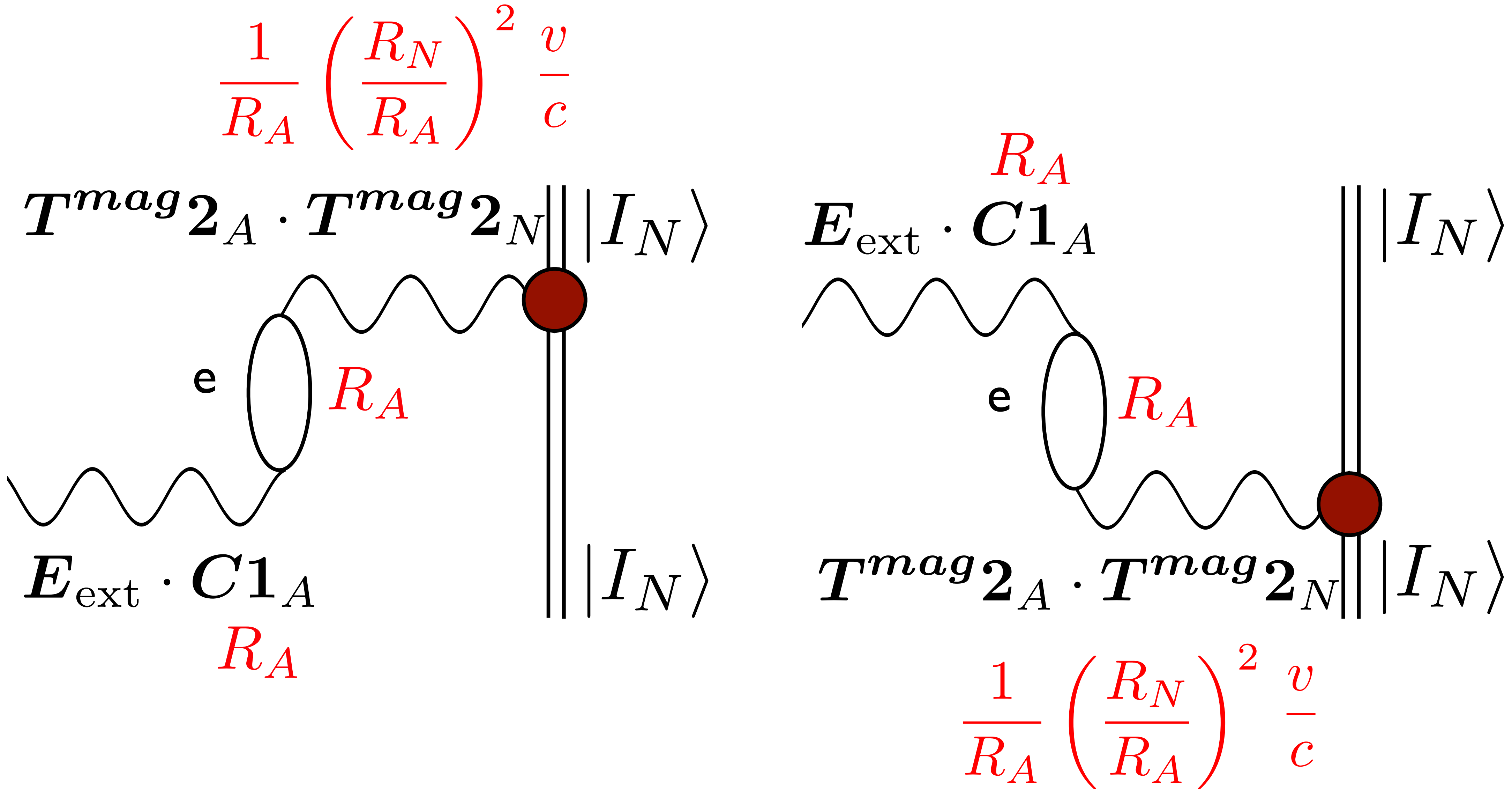
depends explicitly on the enhanced nuclear octupole moment



$$\text{NNLO}: \quad R_N \left(\frac{R_N}{R_A}\right)^2$$

Other Schiff moments for ^{229}Pa: M2

$$\mathrm{T}_L^{mag\,N}(\boldsymbol{\mu}) \xrightarrow[\text{T-odd P-odd}]{} \mathrm{T}_L^{mag\,N}(\boldsymbol{\mu}) + \mathrm{T}_L^{mag\,N}(\bar{\boldsymbol{\mu}}) \longrightarrow \mathrm{T}_L^{mag\,N}(\boldsymbol{\mu}) \qquad \boldsymbol{\mu} \equiv \tfrac{1}{2}\boldsymbol{r} \times \boldsymbol{j_N}$$



$NLO: \quad R_N \left(\frac{R_N}{R_A}\right) \frac{v}{c}$

but suppressed by relativity

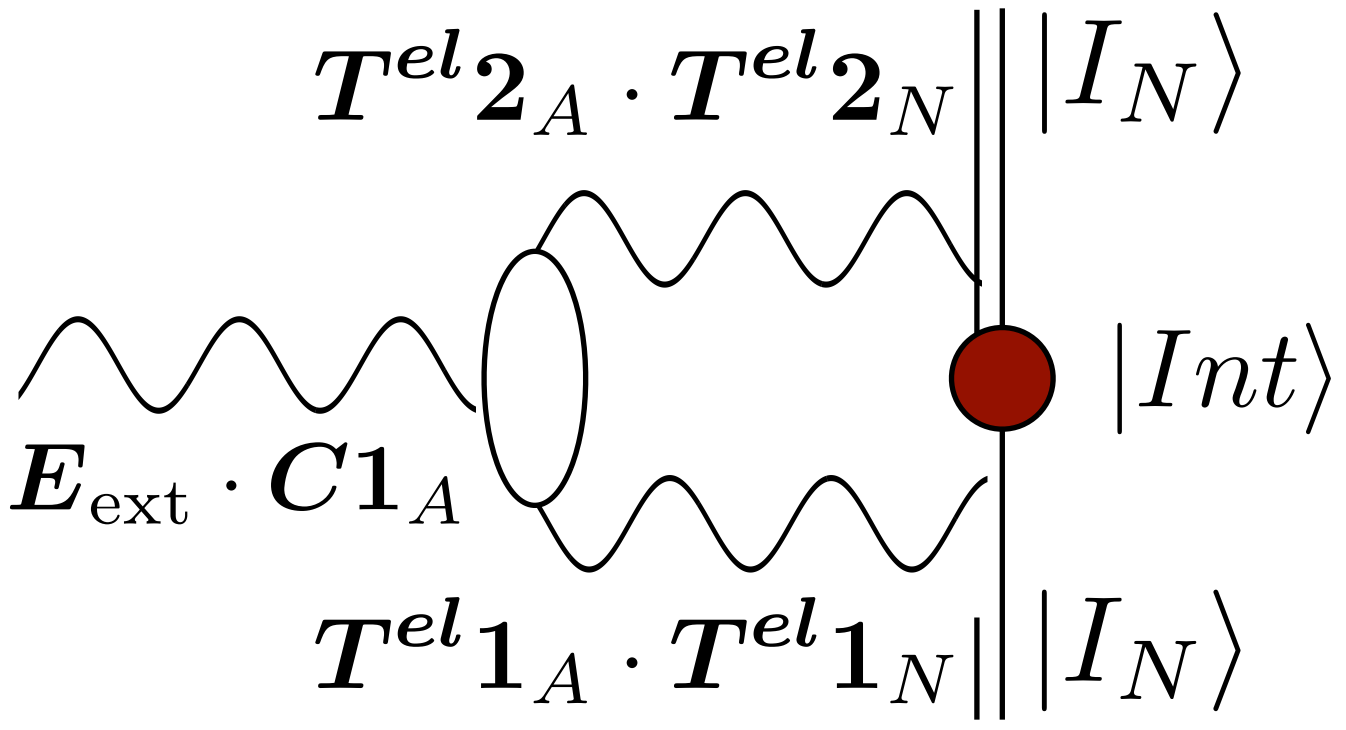
effectively $\;NNLO$

M2 nuclei moments were evaluated in WH/EH, found especially enhanced by nuclear polarization

Other Schiff moments for ^{229}Pa: Generalized Siegert's theorem contributions

$$\mathrm{T}_L^{el\,N}(\hat{\boldsymbol{q}}\cdot\boldsymbol{j},\boldsymbol{\mu}) \quad\longrightarrow\quad \mathrm{T}_L^{el\,N}(\hat{\boldsymbol{q}}\cdot\boldsymbol{j},\boldsymbol{\mu})+\mathrm{T}_L^{el\,N}(\hat{\boldsymbol{q}}\cdot\bar{\boldsymbol{j}},\bar{\boldsymbol{\mu}}) \quad\longrightarrow\quad \mathrm{T}_L^{el\,N}([H^N,\rho],\boldsymbol{\mu})$$

Friar and Falleros



$220\ \mathrm{eV} \ll E1_{atomic} \sim 2\ \mathrm{keV}$

Satoru Inoue

The simple counting available in other cases breaks down because there is a nuclear energy denominator that is small on atomic scales

Concluding comments

- The success with ^{225}Ra, the extraordinary energy denominator, FRIB production abilities, and the predicted size of the octupole collectivity all seem to motivate work on ^{229}Pa

- The IC screening arguments presented here are intuitive, and backed up by detailed calculations. Requires an unusual, enhanced E1/C1 — again very positive for edm goals

- ^{229}Pa theory is complicated because C1, M2, C3 multipoles will all contribute, and because of the breakdown in the usual R_N/R_A counting - but ought to be done in detail

- Progress will require a fair amount of coordination between atomic and nuclear physics, because each nuclear contribution is associated with a distinct atomic polarizability, that would have to be evaluated to better understand the overall atomic response