Ocean Circulation Kinetic

Energy—Reservoirs, Sources and Sinks

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Abstract  The ocean circulation is a cause and consequence of fluid scale interactions from millimeters to 10,000km. Although the wind field produces a large energy input to the ocean, all but about 10% appears to be dissipated within about 100m of the sea surface, rendering very difficult observations of the energy divergence necessary to maintain the full water column flow. Attention thus shifts to the physically different kinetic energy reservoirs of the circulation, and their maintenance, dissipation, and possible influence on the very small scales representing irreversible molecular mixing. Oceanic kinetic energy is dominated by the geostrophic eddy field, and depending upon the vertical structure (barotropic versus low-mode baroclinic), direct and inverse energy cascades are possible. The pathways towards dissipation of the dominant geostrophic eddy kinetic energy depend crucially on the direction of the cascade, but are difficult to quantify because of serious observational difficulties for wavelengths shorter than about 100-200km. At high frequencies, kinetic energy is dominated in the internal wave band by inertial motions (frequencies near the local Coriolis parameter), whose shears appear to be a major source of wave breaking and mixing in the ocean interior.

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1 Introduction

That turbulent mixing processes in the ocean are extremely important in determining the oceanic general circulation, and a major limiting factor in the ability to calculate future climate states, are clichés in oceanography and climate dynamics. Unlike some other hackneyed statements, this one retains much of its validity. Examination of various attempts at predicting how the ocean will change under global atmospheric warming (or any other climate hypothesis), makes it apparent that a great many sources of error are present. Predictions running from decades to thousands of years must be regarded as being physically conceivable scenarios and not as true forecasts (in the weather sense). Some idea of the divergence of models under nominally fixed conditions or the limited understanding of their known biases, can be obtained from, among a number of sources e.g., Houghton et al. (2001), Large and Danabasoglu (2006), and the IPCC 4th Assessment Report (2007). Much, but definitely not all, of that divergence is readily attributed to differing inferences about how the ocean mixes, and consequently, very great uncertainty about how those processes would change under modified climate conditions.

The tight relationship between the large scale circulation and small scale mixing is a consequence of the turbulent nature of oceanic flows, with energy continuously exchanged among all scales of motion. Large scale structures cannot be analyzed in isolation from smaller scale swirls and billows. Study of the process of the mixing of scalar fields in the ocean goes back many decades, and begins with the representation of very large-scale temperature and salinity distributions, written generically as $C(r,t)$, in terms of the turbulent diffusion governed by “eddy-
“Ferrari and Wunsch coefficients” in forms such as,

\[ \mathbf{v} \cdot \nabla C = \nabla (\mathbf{K} \nabla C), \]  

with \( \mathbf{v} \) the three-dimensional velocity field, and \( \mathbf{K} \) a mixing tensor. It is only comparatively recently that major attention shifted from attempts to simply determine \( \mathbf{K} \) from observed large-scale distributions of property \( C \) through various forms of inverse calculation (e.g., Hogg, 1987) toward understanding the fundamental turbulence giving rise to \( \mathbf{K} \).

Knowledge of the origin of the turbulence, and the power requirements necessary to sustain it were reviewed by us comparatively recently (Wunsch and Ferrari, 2004; hereafter WF2004). Thorpe's (2005) book conveniently covers much of the wider background. A number of interesting and important developments in the interim lead us to partially update the earlier review, focussing particularly on the kinetic energy (KE) budget of the ocean. Total oceanic energy necessarily includes the internal and potential (PE) energies as well, but the kinetic component is most directly related to the displacement of fluid parcels, and hence the provision of the shear necessary to bring about irreversible mixing at molecular scales\(^1\). A loose emphasis on the KE also permits us to limit the scope of this review, while at the same time justifying ranging across a wide variety of oceanographic phenomena and without making any claim to being comprehensive.

It is convenient to work in a framework of a qualitative separation of oceanic motions—KE—by frequency band. To this end, we present in Figs. 1-2 power

\(^1\)Irreversible mixing happens only at molecular scales where KE is converted into disorganized molecular agitation. Turbulent stirring can twist and fold tracer and momentum into convoluted patches, but it cannot irreversibly mix them. In this review mixing will be used to refer to molecular mixing, while turbulent mixing will be used to describe the stirring and folding.
density spectral estimates of horizontal KE from a few reasonably representative locations in the deep open ocean; location maps are in Fig. 3. Various data bases contain over 2000 such records, and an exhaustive study of the archives is not intended. Qualitatively, however, there are some nearly universal features of such records that are useful for organizing a discussion.

The diagrams are discussed further below, but all of the spectral density estimates display: (1) A low frequency, nearly “white” (flat) band at periods longer than about 1000 hours (40 days). (2) This band then falls in an approximate power law $\sigma^{-q}$, where $\sigma$ is radian frequency, and $q$ an empirical constant, and which we will call the “geostrophic eddy” range. (3) A conspicuous inertial peak, $\sigma \approx f$ where $f = 2\Omega \sin \theta$ is the Coriolis parameter or frequency equal to twice the Earth’s rotation period times the sine of the latitude, $\theta$, and separates the geostrophic eddy band from higher frequency non-geostrophic motions.$^2$ (4) At frequencies $\sigma > f$, one has another approximate power law band usually identified as “internal waves.” A number of other features, especially tidal lines, appear in most of the records, but discussion is postponed. Note that in all cases, there is a sample time average velocity $\bar{u}, \bar{v}$, with a KE, $\frac{1}{2}(\bar{u}^2 + \bar{v}^2)$, which is commonly indistinguishable from zero in open ocean records.

Frequencies $\sigma < f$ are thought to be almost completely geostrophically balanced at least below the surface boundary layers$^3$, while those $f < \sigma < N$ are

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$^2$In this paper, as in the oceanographic literature, “inertial waves” denotes those waves in a stratified rotating fluid with radian frequency $\sigma \approx f$. They should be distinguished from the alternative use in rotating non-stratified fluids as waves with $0 \leq \sigma \leq f$ (e.g., Chandrasekhar, 1968). Here “internal waves” denote those motions $f \leq \sigma \leq N$ which include inertial waves as a special case. Analogous motions exist in fluids for which $N \leq \sigma \leq f$, including $N = 0$, but such conditions are almost nonexistent in the ocean.

$^3$Geostrophy results from near-exact balance between the Coriolis force and the pressure
controlled by gravity wave dynamics. The transitional inertial peak is dominated by gravity wave physics strongly modified by rotation, and with important effects from the latitudinal variation, $\beta = R^{-1} df/d\theta$ where $R$ is the Earth’s radius. Frequencies $\sigma > N$ are thought to be primarily in small-scale turbulent motions resulting from instabilities leading to breaking of the lower frequency internal waves.

Much fluid physics is known in the context not of frequency, but of wavenumber spectra. Only a few wavenumber spectral estimates of ocean variability exist (e.g., Katz, 1975; Stammer, 1997) and tend, qualitatively to be “red” \(^4\) without the distinctive features seen in the frequency domain. Theory suggests a major overlap in the wavenumber domain of the very different time-scales seen in the displayed frequency spectra. Frequency-wavenumber spectra are required to delineate space scales, and thus much theoretical discussion, taken up later, of energy transfers in wavenumber cascades, tends to be highly speculative because measurements capable of producing frequency-wavenumber separation are very expensive and rarely available. What few estimates there are (see Munk, 1981 and Zang and Wunsch, 2001), require assumptions about separability, have restricted ranges, and are geographically highly localized. A recurring theme here is the essential need for frequency-wavenumber separation capabilities in oceanography, particularly at length scales shorter than about 200km, where several distinct physical processes are present.

A schematic of the frequency-wavenumber spectrum is depicted in the supplementary material. How are these spectra maintained? Where does the energy gradient force. At the ocean surface where air-sea fluxes are strong and velocities become large, frequencies $\sigma < f$ are still nearly balanced, but the condition involves more forces.

\(^4\)That is, with energy generally increasing with wavelength.
come from? Is it redistributed in frequency and/or wavenumber space? How is it dissipated, and how much of the dissipated energy contributes to oceanic mixing processes? Are the seemingly different dynamical ranges coupled?

## 2 The Oceanic Energy Budget

To set the stage, it is useful to briefly recapitulate and update the discussion of the total energy budget of the ocean. WF2004 attempted an order of magnitude estimate of the major reservoirs and energy transfers towards dissipation in the global ocean. A modified and updated version can be seen in the supplementary material. Many of the numbers remain highly uncertain, and some are missing entirely. It is characteristic of the ocean circulation that important energy reservoirs differ by orders of magnitude in how much energy is resident in each. To a great extent, the size of the reservoirs is irrelevant; the amount of energy (a few tens of exajoules per year) required to sustain the oceanic general circulation is so slight compared to the magnitude of the energy reservoirs, that determining the pathways of flow through the system is very difficult.

The geostrophic eddy field\(^5\) dominates the energy content at subinertial frequencies, while surface turbulence is the largest reservoir at superinertial frequencies. The overlap in wavenumber space, illustrated in Fig. 2 of the supplementary material, is very difficult to observe and proves an important obstacle to understanding the behavior of the circulation.

The stress applied by the atmospheric winds at the ocean surface provides

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\(^5\)Commonly called “mesoscale” eddies, which is a perhaps misnomer (in the atmospheric literature they are referred to as “synoptic scale” eddies), but we will sometimes use the terminology.
the energy to keep the oceanic KE spectrum in equilibrium in the upper ocean. The situation is less clear in the abyssal ocean. The value of about 2TW of work required to maintain the abyssal KE spectrum was estimated by Munk and Wunsch (1998) by using the observed deep stratification and an assumption of how much bottom water had to be returned vertically to about 1500m. Whitehead and Wang (2008) produced a laboratory demonstration of this balance in a salinity stratified fluid, where the rate of upwelling of abyssal waters was linearly proportional to the energy expended in turbulent mixing of the fluid.

Hughes and Griffiths (2006) claimed that much of the bottom water is entrained at great depths and only a fraction of the fluid is returned to 1500m so that the power required might be as little as 0.2TW. Some reduction from 2TW seems reasonable, but e.g., North Atlantic Deep Water appears to be fully formed by 1000m depth, with little or no entrainment below that. St. Laurent and Simmons (2006) made an independent estimate by calculating the power requirement to sustain the “observed” (inferred) vertical mixing in the ocean interior from a great variety of methods and places. Their summary total is that about 3±1TW are required for the whole water column. Possibly important upper ocean dissipation regions are ignored (convective regions, continental margins). Of their total, somewhere between 1/3 and 1/2 of the total energy would be required above about 1500m, thus requiring about 1.5-2TW for the abyssal ocean. Estimates of oceanic mixing, mixing efficiencies, and the power required warrant full reviews of their own.

Toggweiler and Samuels (1995) speculated that some fraction of North Atlantic Deep Water is pulled towards the surface by the strong winds blowing along the Antarctic Circumpolar Current: the winds drive a divergent Ekman flow that
results in upwelling of subsurface waters. This plausible scenario has support from some observations (Speer et al., 2000) and suggests that a possibly large fraction of the power necessary to close the overturning circulation of the ocean is supplied by surface winds. Hence the 2TW estimate is likely an upper bound.

2.1 Upper Ocean/ Lower Ocean

A difficulty with global energy budgets is the lumping into crude global integrals of pathways and relative magnitudes of regional processes that may be markedly different from the volume average. In the context of the global energy budget, and as alluded to above, it is clear that the upper ocean behaves qualitatively differently from the abyssal one. As early as Defant (1961), the ocean was divided into a “troposphere” and “stratosphere”, and it was clear that the dynamics of the (roughly) upper 1000m of the ocean had to be quite different from that below. Separation between upper and abyssal oceans is here deliberately a bit vague, but depending upon position, lies somewhere between 1000 and 2000m depth. The separation is more pronounced at low and mid latitudes, while it is blurred in polar regions where deep convective events can mix the whole water column. The surface mixed-layer represents a third region requiring separate treatment.

WF2004 derive the KE budget for the global ocean with the KE per unit mass defined as $E = \frac{1}{2}u \cdot u$, which is, in steady state,

$$
\iint [pu + \rho \nu \nabla E] \cdot \hat{n} \, dA = - \iiint g \rho w dV + \iiint p \nabla \cdot u dV - \iiint \rho \varepsilon dV, \quad (2)
$$

where $\nu$ is the kinematic viscosity, $V$ the total volume of the ocean, $A$ the surface of that volume, $\varepsilon$ the turbulent dissipation, and $\hat{n}$ the unit outward normal. In deriving equation (2), trivial air-sea momentum exchanges due to evaporation and precipitation are neglected. Surface forcing is composed of the work done by
differential pressure and viscous stresses acting on the moving free surface. The three terms on the right-hand side represent the conversion of KE into potential energy, into compressive internal energy and viscous dissipation. Viscous dissipation is the irreversible conversion of kinetic energy into heat and for nearly incompressible fluids like seawater it takes the form,

\[ \varepsilon = \frac{1}{2}\nu \sum_{i=1}^{3} \sum_{j=1}^{3} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2, \]

where \((x_1, x_2, x_3) = (x, y, z)\) and \((u_1, u_2, u_3) = (u, v, w)\).

WF2004 show that generation and destruction of the oceanic potential and internal energy is confined to the surface mixed layer, because the vertical fluxes of heat and freshwater vanish when integrated on a level surface in the ocean interior.

At equilibrium, the KE generated by surface stresses and forces is balanced by viscous dissipation—a contrast to the atmospheric situation where solar heating is absorbed and radiated throughout, resulting in interior sources and sinks of potential and internal energy. The vanishing of the sum of the conversions to potential and compressive internal energies is not trivial, because it is the result of several much larger conversion terms. WF2004 show that these two terms can be lumped into a single conversion term, \(g\rho_\theta w\), representing the net kinetic energy required to lift a water parcel, once reversible adiabatic effects have been subtracted (\(\rho_\theta\) is potential density, i.e. the density of a water parcel brought to the ocean surface to eliminate compressive effects),

\[ g\rho_\theta w \approx g\rho w - p \nabla \cdot \mathbf{u}. \]

This relation is valid only in the ocean interior, where expansion by diabolic effects is weak.

The two variables \((\rho_\theta, w)\) contributing to the \(g\rho_\theta w\) conversion term can be
decomposed into three components due to mean motions, subinertial and super-inertial fluctuations,

\[
(r_\theta, w) = (\bar{r}_\theta, \bar{w}) + (r_{\theta g}, w_g') + (r_{\theta t}, w_t').
\] (4)

where the subscripts are a reminder that subinertial fluctuations are typically geostrophic, and superinertial ones include internal waves and small-scale turbulence. Because the sum of the gravitational and compressive works vanishes when integrated on a level surface, the integral of \( g\bar{r}_\theta w \) at any level in the ocean interior must also vanish,

\[
\int \int g\bar{r}_\theta w \, dS = \int \int g\bar{r}_\theta \bar{w} \, dS + \int \int g\bar{r}_{\theta g} w_g' \, dS + \int \int g\bar{r}_{\theta t} w_t' \, dS \approx 0. \tag{5}
\]

Geostrophic fluctuations represent a release of large scale PE through subinertial instabilities, without any irreversible mixing, and can be thought of as advection by a generalized Stokes drift \( \bar{w}^\uparrow \) (Plumb and Ferrari, 2005), i.e. \( \bar{r}_{\theta g} w_g' \approx g\bar{r}_\theta \bar{w}^\uparrow \). Superinertial turbulent fluctuations are the pathway to irreversible mixing through small scale instabilities.

Theories of the upper ocean, not ruled out by observation, suggest that turbulent fluctuations leading to irreversible mixing are confined to the surface mixed layer, but are negligible in the interior; that is, the circulation is nearly adiabatic in character (e.g., Pedlosky, 1996; Webb and Suginahara, 2001; Vallis, 2006). The balance in (5) is therefore between the first two terms. The mean contribution represents the transfer from KE to PE which occurs when the wind-driven Ekman flux raises the center of gravity of the ocean by pushing down light fluid in the subtropical regions and pulling up dense fluid in the subpolar regions. The subinertial contribution is through the release of PE by baroclinic instabilities, i.e. the slumping of lateral density gradients.
In the abyss, the sum of the mean and subinertial eddy contributions balances the generation of PE by turbulent mixing through internal wave breaking. Mean and subinertial components oppose each other as in the upper ocean, but they do not compensate to the same degree. Hence the energy expended by turbulent mixing supports the upwelling of buoyancy by the sum of the Eulerian mean and the generalized Stokes drift velocities, i.e. by the generalized Lagrangian mean velocity. Many authors seem to have confused the Eulerian and Lagrangian mean velocities in this budget.

3 External Sources of Kinetic Energy

External forces acting to set the ocean into motion on any scale are restricted in number and overall there is not a great deal of new insight available since WF2004. An update of the literature is given in the supplementary material.

The wind field is by far the dominant energy source to the ocean and can be regarded, oceanographically, simply as a reservoir of atmospheric KE directly transferable into the ocean. The coupling of generation of different forms of energy in the dynamics (in either balanced or gravity dominated motions), means one cannot generate or dissipate kinetic without also generating (dissipating) potential (and often internal) energy as well. Thus wind work on the ocean can be regarded either as directly producing KE (e.g., the large-scale ocean circulation), or instead producing its PE (as in the Gill et al., 1974, picture of Ekman pumping/suction working against the mean stratification) and the conversion of atmospheric KE cannot be regarded as going solely into one type of oceanic energy.

The sum of the two terms on the left hand side of the KE budget in (2)
represents the total working rate of the wind on the sea surface. Wang and Huang (2004) have calculated the total, paying specific attention to the generation of the wave field, and conclude that the net value is close to 60TW. Csanady (2000) shows that the terms on the right hand side of equation (2) are typically an order of magnitude larger in the near-surface ocean than in the interior: the oceanic energy budget is closed to better than 10% in the surface mixed layer alone. The interior circulation is driven by the small residual of energy that fluxes through the mixed layer base, making quantitative calculations and observations very difficult.

3.1 Stress Acting on the Geostrophic Circulation

Consider first the work done on the geostrophic circulation. Geostrophic motions are primarily horizontal and the working rate is approximately,

\[ W_{\text{wind}} \approx \int \int_{\text{ocean}} \rho \nu \frac{\partial}{\partial z} \left( \frac{1}{2} v^2 \right) dA = \int \int_{\text{ocean}} \tau \cdot v_g dA, \]

where \( v_g \) is the surface geostrophic flow in the ocean (the equatorial band, 1-2° of latitude on either side of the equator, requires special treatment) The wind stress acting on the ocean has typically been computed from the turbulent drag formula,

\[ \tau_1 = C_D \rho_{\text{air}} |v_a| v_a, \]

where \( C_D \) is an empirical coefficient dependent upon air-sea temperature differences and the windspeed itself, \( \rho_{\text{air}} \) is the air density, and \( v_a \) is the vector atmospheric wind, usually at 10m elevation. Wunsch (1998) estimated a net work of 0.8TW with about 80% entering in the Southern Ocean. von Storch et al. (2007) re-calculated the rate of work done on the subinertial circulation, i.e. they substituted the full subinertial surface velocity for \( v_g \), using a much
higher resolution model and estimated that a net work rate of 3.8TW was done on the upper ocean, but that only the 1.1TW associated with work done on the geostrophic velocity reached the ocean beneath the very surface layer. All the power input to the surface Ekman layer was dissipated there.

These and similar calculations likely have significant positive biases: Dewar and Flierl (1987), Cornillon and Park (2001), Chelton et al. (2004), Dawe and Thompson (2006), Duhaut and Straub (2006), Zhai and Greatbatch (2007), Hughes and Wilson (2007) demonstrate that $\tau$ in Eq. (7) is significantly inaccurate—because it must depend upon the water velocity, $v_o$, relative to the air, probably in the form,

$$
\tau_2 = C_D \rho_{air} |v_a - v_o| (v_a - v_o). 
$$

Wind velocities are so much higher than water velocities and the spatial structure of the wind field is so much larger than that in the ocean, that $\tau_2$ is conventionally approximated by $\tau_1$. The interesting difficulty is that the modification to the stress by small scale oceanic motions is a negative-definite systematic correction. When $|v_o| \ll |v_a|$,

$$
\tau_2 \cdot v_o = \tau_1 \cdot v_o - C_D \rho_{air} \frac{|v_a \cdot v_o|^2}{|v_a|} - C_D \rho_{air} \frac{|v_a|^2 |v_o|^2}{|v_a|^2}.
$$

Because of their differing space/time scales, ocean eddy perturbations are weakly correlated with the major atmospheric wind patterns, but do contribute to the reduction in Eq. (9). More than 90% of the surface KE in the ocean lies in the eddy field (Wunsch, 2007, Plate 6), and the reduction in Eq. (9) scales as $v_{\text{eddy}}^2/v_{\text{mean}}v_{\text{mean}}$. Spatially smoothing ocean currents leads to an increase in the power input by the wind compared to that for an unsmoothed flow (Hughes and Wilson, 2007)—that is, at high wavenumbers, stress and oceanic surface flows are negatively correlated.
An analogous effect arises (Dewar and Flierl, 1987, Behringer et al., 1979) from the dependence of $C_D$ upon air-sea temperature differences, which also has an eddy-scale effect, although not necessarily of one sign. The temperature sensitivity may be most important in its systematic effects on the potential vorticity distribution in strong western boundary currents. For winds in excess of 60m/s (Jones and Toba, 2001), the drag coefficient $C_D$ increases quadratically with wind speed. At these extreme values, the ocean surface becomes a mixture of air and water—“sea spray.” The two-phase mixture modifies the transfer of momentum between the atmosphere and the ocean and conventional bulk formulas fail. No estimates exist of such effects on the oceanic energy budget, although the impact of the sea spray regime on the energy budget of hurricanes has been examined (Emanuel, 2003).

Chelton et al. (2004) found that some correlation develops at the eddy scale between $v_o$ and $v_a$, because the ocean heat anomalies associated with eddies modify the stability of the atmospheric boundary layer and hence the surface winds. These feedbacks probably contribute little to (9), because they represent a minor modification of $v_a$ and they can contribute both positive and negative biases. Nevertheless a careful quantification from data is lacking to make firm conclusions.

Furthermore, temporal and spatial averages of quadratic (atmospheric variables) or cubic (oceanic ones) variables can be markedly higher than the quadratic or cubic values of the averages, especially because the statistics of stress are significantly non-Gaussian and skewed towards poorly sampled high values (Gille, 2000).

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6The general choice of $C_D$ as a function the velocities, air-sea temperature differences, and sea state is a disputed subject in its own right.
The eddy structure of the wind work will arise again when we turn to the issue of the dissipation of oceanic KE.

### 3.2 Stress-Generated Inertial Waves

The winds have long been known to excite inertial motions at the sea surface and they are a part of the so-called Rossby adjustment problem in which any non-geostrophically balanced motion in a rotating system will tend toward balance by radiating, among other internal waves, inertial ones. Pollard and Millard (1970) represented the upper ocean through a “slab” model, one which has been widely used to calculate the rate of generation of inertial waves by the wind field (D’Asaro, 1985). Alford (2003), using a slab, calculated a wind power deposit into inertial waves of about 0.5 TW.

Deriving the slab model of the flow field from the equations of motion is dependent upon a series of assumptions whose validity is obscure, and thus the true uncertainty of the rates of direct energy input into the inertial wave band is not known. Plueddemann and Farrar (2006) analyzed the effects of shear at the base of the mixed layer, which acts as a damping term, and showed that Alford’s estimates of the power input to the ocean by generation of inertial frequencies is likely being overestimated by about a factor of two; see also Stockwell et al. (2004) and Elipot and Gille (2007). Alford and Whitmont (2007) have produced the latest compilation of the generation of inertial waves by wind; they try to account for results such as that of Plueddemann and Farrar (2006) by doubling their estimate of the friction coefficient.
4 Oceanic Kinetic Energy—The Spectrum

We return now to some of the details in the KE spectral estimates in Figs. 1, 2, with the expectation that a closer analysis of the differing physics of the various frequency bands can shed light on their generation, dissipation, and inter-reservoir energy transfers.

4.1 The Overall Behavior

Figs. 1 shows spectral estimates from records at 27°N, 41°W in the North Atlantic. One instrument was at 128m, the second near 1500m and the other near 3900m over the eastern flank of the Mid-Atlantic Ridge (Fu et al., 1982). The top two spectra are representative of many open ocean records, while the bottom instrument is surrounded by complex three-dimensional topography, rendering the spectral density measurably differently shaped. In all three results, one sees a distinct peak at the frequency of the principal lunar semi-diurnal tide; a second peak is marked “inertial”. These measurements were made at a latitude where the frequencies of the principle diurnal tides $K_1, O_1$ nearly coincide with $f$, although North Atlantic diurnal tides are comparatively weak (compared both to the semi-diurnal tides and their amplitudes in other oceans). At frequencies above $M_2$, all estimates display a near power-law behavior which has been fit by least-squares to $A \sigma^{-q}$ where $q$ is typically between about 1.5 and 2 in most parts of the world. This oceanic internal wave band is discussed in an immense literature, much of which can be located starting with Munk (1981), or Thorpe (2005). Superimposed on this power law, and visible in both spectra, are harmonics of the tides and of the inertial peaks. The extent to which the spectra are real and the result of fluid wave-interaction non-linearities, as opposed to non-linearities in the
instruments or advection by larger scale flow, is not obvious. The deepest instrument on this mooring shows a suppressed inertial band energy—unsurprising when one recognizes that the motions would necessarily reflect from steep nearby topography—generating velocity nodes at the topography (here “steep” means, crudely, with slopes significantly greater than that of the internal waves group velocity vector). That inertial motions are suppressed near topographic features has been known for a long time (e.g., Wunsch, 1976) and they are missing in canyon-like features. Whether steep topographic features are inertial motion energy sources, simple adiabatic reflectors, or energy sinks through instabilities and mixing, cannot be determined from isolated measurements of the energy level.

As one of a large number of variants, Fig. 2a comes from the tropical abyssal plane at $15^\circ N$ at about 500m. Diurnal tides appear above the inertial frequency, and inertial and $M_2$ band energies are nearly equal. Whether the reduced energy in the inertial band relative to the $M_2$ band is the result of displacement of the diurnal tides, of the inability of the $M_2$ tides to resonantly generate inertial motions (taken up below), or the ambient background oceanography, meteorology and topography, is unknown.

Many spectral estimates exhibit overtones of the inertial and/or tidal bands, and sometimes their sum and difference frequencies. The equations of motion are non-linear and such secondary peaks are not unexpected (e.g., Niwa and Hibiya, 1999), but as with the power laws, instrumental issues mean overtones need to be viewed as suspicious.

A significant drop in the spectral estimates occurs at frequencies below $f$. At periods longer than $50 - 100$ days, the spectra rise again in another approximate power law in the geostrophic eddy range, until the flattening into the low
frequency “white noise” band occurs. Motions in the latter band are not well described, but are often dominated by near-zonal currents. The transition between the white noise band and the spectral rolloff determines the scale at which the motions become decorrelated. A flat spectrum at scales longer than the decorrelation times suggests that low frequency motions in the ocean behave like an unpredictable white noise. Frequency spectra from Lagrangian time series have a similar shape, but the spectral slope in the geostrophic eddy range is substantially steeper, exhibiting a $\sigma^{-4}$ dependence. Small-scale (10 km scale) features are advected by the energy containing eddies (the spectral peak) leading to a Doppler shift into the high frequency region of the Eulerian spectrum, but no such Doppler shift appears in the Lagrangian version. The white noise band is also evident in Lagrangian spectra, but extends to higher frequencies (periods of ten days). A shorter decorrelation time scale in Lagrangian measurements is consistent with Corrsin’s (1959) conjecture: Ocean velocities decorrelate both in space and time, but Eulerian ones experience only the temporal part, because they are taken at a fixed spatial position. A Lagrangian observer, by drifting, experiences both the temporal and spatial decorrelation simultaneously and will generally measure a shorter decorrelation time. Middleton (1985) summarizes theory and evidence from drifters for a shorter Lagrangian decorrelation time scale.

A flat spectrum at low frequencies has important implications for estimating the lateral stirring by eddy motions. Taylor (1921) showed that the dispersive power of a turbulent velocity field, i.e. the magnitude (eigenvalues) of the mix-

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7LaCasce (2007) gives an up to date review of the current technology and observations for oceanographic Lagrangian-type of measurements.
ing tensor in Eq. (1), is proportional to the integral of the Lagrangian frequency spectrum. The integral of an eddy kinetic energy (EKE) spectrum with a white noise low frequency band and a steep rolloff at high frequencies is dominated by the energy content at the frequencies where the rolloff begins. These frequencies are also proportional to the inverse Lagrangian decorrelation time (LaCasce, 2007). Hence the lateral spreading of tracers in the ocean is mostly associated with geostrophic eddies having decorrelation time scales of order ten days.

The absence of a cutoff at $N$, when resolved by the sampling, is usually explained as either that the instrument noise level is too great to show the very low energy levels in what can be regarded as a “turbulent” band or because small nonlinearities in the instruments produce spurious high frequencies. Detailed information of the spectral transition at $N$ can be obtained with Lagrangian-like measurements. D’Asaro and Lien (2000) estimated Lagrangian spectra from neutrally buoyant floats over and near the sill of Knight Inlet. In the internal wave range of frequencies, $f \leq \sigma \leq N$, the energy spectra were consistent with a broad continuum of energy with a power law behavior $A\sigma^{-2}$ except for tidal and inertial peaks. At higher frequencies, $\sigma > N$, beyond the internal wave range, the spectra rapidly transitioned to a steep spectral roll-off consistent with an inertial subrange of stratified turbulence (Moum, 1996). The high frequency field of fluctuating motions apparently can be described as a sum of internal waves, $\sigma < N$, and turbulent motions with $\sigma > N$, and little interaction.

The figures show the fraction of the energy for frequencies $\sigma \geq f$ that lies in the inertial peak and in the $M_2$ band. The energy content in the peaks was based upon a subjective visual estimate of where the excess energy in the peak falls to the background continuum. Peaks near $f$ extend below that frequency some
distance into the eddy band to capture the spectral structure (and the present
definition of the internal wave energy starts at that point.) In the upper ocean
record, about 50% of the internal wave KE is inertial, and about 20% is in the
tides. In the lower record, the ratios are nearly reversed. An analysis of 138
North Atlantic records produced approximately 50% of the internal wave band
variance in the inertial peak, with about 20% lying in the $M_2$ band (C. Wortham,
personal communication, 2007). No global inventory exists of the relative energies
of inertial motions and tides, but the perception that the KE of the inertial band
is dominant, rather than the tidal one, has some rudimentary support.

The broad energy continuum between $f$ and $N$ in the spectra is well described
by the heuristic model (GM) of Garrett and Munk (1972) and its subsequent
modifications (Munk, 1981). The most surprising and significant aspect of the
GM spectrum is its near universality. Important (e. g., larger than a factor of
three) deviations from the reference form are observed only in special places such
as the Arctic Basin or submarine canyons. The spectrum, however, does not ac-
count for the great variance found in the tidal and inertial peaks. In atmospheric
measurements, no conspicuous inertial peak appears and what oceanographers
regard as internal waves, are instead interpreted as arising from Doppler shifting
of the low frequency energetic wind fields (the Taylor hypothesis), with little con-
tribution from wavelike phenomena (Gage and Nastrom, 1986). By contrast, the
oceanographic interpretation is that advection dominates at frequencies below $f$
(Hua et al., 1998), while at higher frequencies advection is ignored altogether.
The extent to which the moored oceanic spectra are “contaminated” by Doppler
shifting to high frequencies, and the meteorological ones by internal gravity waves,
is unclear, but it seems unlikely that either extreme view can be justified (e. g.,
What do these spectra have to do with oceanic mixing? At frequencies below $f$ the motions are very energetic (see energy diagram in supplementary material) but apparently of very large vertical scale (Wunsch, 1997), and hence of low shear and little direct role in driving instabilities leading to irreversible mixing. Attention focuses on the internal wave band where vertical shears can become very large (Munk, 1981) and are thought to control open ocean background mixing. We return below to the question of why KE is transported most efficiently to small scales in the inertial band.

4.2 Tides in the Internal Wave Band

Tidal peaks in the spectral estimates are conspicuous but not dominant. Tidal mixing is essentially the study of internal waves of tidal period—their generation by barotropic motions, their interaction with the background internal wave field, the coupling to topography both as generator and dissipater, and their stability in the most general sense. Note that Toole (2007) withdraws the conclusion that there was significant tidal modulation of the Brazil Basin mixing rates—undermining one of the inferential pillars. Much of the progress in understanding internal tide generation and consequent mixing comes from the recently concluded Hawaii Ocean Mixing Experiment (HOME; see Rudnick et al., 2003). For details in understanding the generation, breakdown, and mixing from tides, the reader is referred to the papers in *Journal of Physical Oceanography*, June, 2006 and Xing and Davies (2006), among others. Klymak et al. (2006), for example, conclude that $3\pm1.5$ GW of energy is dissipated by the tides near the Hawaiian Ridge with $4\text{-}7$ GW of energy unaccounted for, but apparently lost by the barotropic tide.
If one seeks 2TW to power the abyssal general circulation, then if, in round numbers 5GW is available at Hawaii, 400 Hawaiis can do it all. Whether such an extrapolation worldwide is a reasonable one is unknown. Internal tidal generation at topography has been reviewed by Garrett and Kunze (2007), and no more is said here.

The nature of the boundary layers formed on slopes by tidal and other internal wave motions has received comparatively little attention (see e.g., Nash et al., 2007 for measurements). Laboratory experiments are quite striking (e.g., Ivey et al., 2008, and references there), but field measurements at the much higher Reynolds numbers present in the open ocean would be welcome, as would laboratory studies of the boundary layers in the presence also of strong rotation.

4.3 Inertial Waves

Despite their ubiquity, energy, and many years of study, much about their behavior remains obscure. How inertial oscillations are generated by a time varying wind stress was described above. In common, however, with many wave boundary value problems of Sturm-Liouville type, direct, linear, wind generation tends to excite primarily low modes (barotropic and the first few baroclinic ones) and this inference is supported by the analytical model of Gill (1984) and the numerical one of Zervakis and Levine (1995). Thus although there is a great deal of energy sometimes present in directly driven motions, little shear exists. Observations on the other hand, show that there are strong inertial motions in the open-sea with high shear values. The difference in vertical structure is crucial, because only waves with strong shears undergo Kelvin-Helmholtz-like instabilities leading to three dimensional turbulence and irreversible mixing. Low mode
waves are very stable and do not affect the ocean circulation much. Balmforth and Young (1999) and Mohelis and Llewellyn Smith (2000) show that interaction with the geostrophic field, the planetary vorticity, and topography can pump inertial energy into high shears. No quantitative estimates of these effects exist.

Understanding of the rates of generation of inertial waves is greatly complicated by the propagation of energy poleward in the ambient internal wave field as depicted by Munk and Phillips (1968) and Fu (1981). Wave motions at low latitudes at frequency $\sigma$ can reach a latitude where $\sigma \approx f$ generating a wave caustic with considerable structure. Fu (1981) concluded that most of the subsurface inertial energy arose from this “turning latitude”. These motions would then be generated from processes producing internal waves at all frequencies—including the ambient high modes—rendering the inertial wave generation problem identical to that of understanding the origin of internal waves generally. Linear wind-generation within the caustic has apparently been discussed only by D’Asaro (1989), who finds a strong increase in horizontal wave numbers relative to the $f$–plane.

Garrett (2001) noted that the spectral density in the range of frequencies, $f \leq \sigma \leq 2f$, could acquire energy from internal waves above this band through the parametric subharmonic instability (PSI) and indeed the power law at high frequencies seen in the figures is much flatter than it is in the rise into the inertial peak maximum. Hibiya et al. (2002) discussed the effects of PSI, and Mackinnon and Winters (2005, 2007) found it permits the $M_2$ tide to efficiently transfer energy into the inertial peak at the latitude where $\sigma_{M_2} \approx 2f$. Some field observations (van Haren, 2005) seem supportive of the idea while others (Rainville and Pinkel, 2006) are ambiguous. van Haren (2007) reports almost no shear in the
The significance of inertial waves for ocean mixing is unclear. A number of studies have examined their shear profile (e.g., Leaman, 1976). Open ocean shear (Kelvin-Helmholtz) instability of internal waves generally appears capable of providing the background mixing (values near $10^{-5} \text{m}^2/\text{s}$) but not the much higher values that are seen near complex topography and that appear to dominate ocean mixing. A considerable body of literature exists concerning the interaction of high frequency internal waves with topography (e.g., Eriksen, 1985, 1998), but little of it is applicable to motions occurring where $\sigma \approx f$ and the meridional gradient in $f$ (the $\beta$-effect) has to be accounted for.

As in the discussions of tides in mixing the ocean, there are several complementary, but overlapping, problems. The Poincaré equation which governs the inviscid limit of $f$-plane internal waves permits the rapid generation of high shear regions from low shear disturbances when topography is encountered. Laboratory studies have focussed on the dissipative and instability processes, the so-called critical slope, when the internal wave characteristics (direction of the group velocity vector) slope, $c = (\sigma^2 - f^2)^{1/2} / (N^2 - \sigma^2)^{1/2}$ is approximately equal to the bottom slope $\gamma$, can produce intense motions, and in the laboratory at least, intense mixing (Ivey et al., 2000, among many others).

On a $\beta$-plane and in related approximations (Munk and Phillips, 1968; Fu, 1981), the group velocity is finite as $\sigma \rightarrow f$ from above, and it is unclear what happens. In observations at sea, Lai and Sanford (1986) concluded that they were seeing hurricane-generated low-vertical mode inertial motions emanating from sloping topography to their north in what one infers to be a reflection process for barotropic motions; see especially Xing and Davies (2002) for a numerical study.
of wind-generated inertial motions and shear over a continental margin. The intensified inertial motions near Caryn Seamount close to Bermuda in the North Atlantic reported by Kunze and Sanford (1986) were interpreted by them as being the result of an interaction with a nearby eddy rather than from topographic interaction. Sensitivity of inertial motions both to bottom slopes and to ambient fluid vorticity perturbations to $f$ renders delicate the problem of understanding their life-cycle. Young and Ben Jelloul (1997), Klein et al. (2004), and Zhai et al. (2007) show that anticyclonic geostrophic eddies focus near—inertial motions. In GCMs this leads to a spatially heterogeneous vertical mixing strongly related to the eddy field properties, but little appears known of the realism of inertial/internal waves in GCMs. Xing and Davies (2002) used 60 vertical levels and a horizontal grid of 0.6 km spacing (roughly 0.005° resolution) to capture the important non-linearities of the internal wave band. Hibiya et al. (1996) employed a resolution of 10 m horizontally and 1.25 m vertically. Such resolutions are far beyond the capability of any existing ocean GCM. But how the accuracy degrades with reduced resolution is not known. To further complicate the story, Gerkema and Shrir (2005) have proposed that the so-called traditional approximation, in which the Coriolis force due to the locally horizontal component of the Earth’s angular velocity is neglected, is invalid when $\sigma \approx f$ and use a modified $\beta$—plane with various predicted consequences at the inertial latitude that have not been well tested.

5 Oceanic Kinetic Energy — Turbulent Cascades

The ocean is forced at the surface by momentum, heat, and freshwater fluxes, on a wide range of temporal and spatial scales, while energy dissipation occurs at
molecular scales in short and sudden bursts. To achieve a steady state, energy must be transferred from the forcing to the dissipation scales. This transfer is achieved through nonlinear turbulent interactions among oceanic motions at different lengthscales (recall again the paucity of wavenumber spectra). Energy is not simply moved from large to small scales, but also from locations where forcing acts, mostly at the sea surface, to regions where dissipation is most intense, the surface and bottom boundary layers and coastal areas. The oceanic energy spectrum is therefore the result of turbulent transfers of energy among different regions and different spectral bands. Nonlinear transfers within the subinertial and superinertial frequency bands are very effective, while the transfers between them are apparently weak.

5.1 Internal-Gravity Waves

Nonlinear interactions in the internal wave band play an important role in the ocean circulation, because they transfer energy from large to small scales and provide a link between climatological forcing and small-scale dissipation. The link between the gravity wave band and geostrophic motions is particularly important: there is a scale gap of roughly three orders of magnitude between the geostrophic band and the isotropic, small-scale turbulence that completes the forward energy cascade to dissipation. This gap needs to be bridged to achieve an equilibrium KE balance in the ocean.

Internal waves are generated mostly as large-scale waves and radiate into the global oceans. As they propagate in physical space, nonlinear wave-wave interactions and other scattering processes cascade their energy through wavenumber space to small scales where they break and dissipate.
The hydrodynamic equations that describe all fluid motions contain advective nonlinearities that produce energy exchange among waves of different frequencies and wavenumbers. In strongly turbulent flows, energy is continually exchanged among different scales of motions through “strong, promiscuous interactions” (Phillips, 1966) which affect all wavenumbers and all frequencies. For internal waves, the interactions are through resonant triads of waves.

Internal waves are a stochastic field capable of undergoing resonant triad interactions, and are sometimes labelled “wave turbulence” (Zakharov et al., 1992). Theoretical studies have been made to examine the way in which internal waves interact with each other in the hope of investigating the apparently stable and universal form of the oceanic internal wave continuum spectrum: Müller et al. (1986) reviewed the early literature, Caillol and Zeitlin (2000) and Lvov and Tabak (2001) provided a new perspective, and Lvov et al. (2008) showed that all approaches are equivalent. Idealized distortions from the GM spectrum relax rapidly to its universal form and the equilibrium spectrum displays a steady cascade of energy from large to small vertical scales, where energy is dissipated through a process of wave breaking. The cascade is driven by energy transfer among waves undergoing resonant interactions (McComas and Bretherton, 1977). The interactions are efficient at transferring energy to frequencies close to \( f \) and to smaller vertical scales where they can break.

Numerical simulations of energy transfer by interactions within the internal wave field show a net flux towards small vertical scales that increases the shear variance \((|\partial u/\partial z|^2)\) until it overcomes the stratification and the waves break. When the wave field is statistically steady, the rate at which wave breaking dissipates energy approximately equals the rate at which the energy is transferred
from large to small scales. This equivalence was used by Gregg (1989) to derive a semi-empirical relation in which the dissipation rate caused by internal waves \( \varepsilon_{INT} \) is expressed in terms of parameters in the GM spectrum,

\[
\varepsilon_{INT} = 7 \times 10^{-10} \langle \frac{N^2}{N_0^2} \rangle \langle \frac{S^4_{10}}{S^4_{GM}} \rangle \text{Wkg}^{-1},
\]

where \( N_0 = 5.2 \times 10^{-3} \text{s}^{-1} \) is a reference buoyancy frequency, \( S_{10} \) is the observed shear variance at scales greater than 10 m, and \( S_{GM} \) is the corresponding variance in the GM spectrum. Are internal waves a primary pathway to energy dissipation in the global ocean? Equation (10) and its subsequent modifications (Gregg et al., 2003) match within a factor of two observed dissipation rates away from ocean boundaries and this link strongly suggests that internal waves are such a major pathway.

### 5.2 Geostrophic Motions

The oceanic KE at subinertial frequencies in mid and high latitudes is dominated by geostrophic eddies on scales of 50 to 100 km. The dominance of EKE at the oceanic mesoscale was first documented in the early seventies from ship-going and mooring observations in the Western North Atlantic (MODE Group, 1978, Hua et al., 1986, etc.). However a full characterization of the properties and distribution of the mesoscale eddy field has only been possible in the last twenty years when satellite altimeters provided the first global pictures of the geostrophic circulation at the ocean surface. Three main features emerge from altimetric maps such as the one shown in Fig. 4. (1) Mean flows are strongly inhomogeneous; most of the mean KE is concentrated in narrow intense currents like the Gulf Stream along the Eastern US coast, or the Agulhas Current down the east coast of Africa. (2) EKE is spatially inhomogeneous, decreasing by an order of magnitude as one
moves from the swift mean currents into the interior of the ocean basins. (3) The ratio of eddy to time-mean geostrophic KE is large everywhere, often by a factor of 100 or more, the exception being the Southern Ocean where it is only a factor of 10 larger.

In the tropical oceans by contrast, EKE is dominated by seasonal oscillations of the equatorial currents in response to shifts in the intertropical convergence zone and its associated wind patterns. At smaller scales, subinertial variability is associated with instability waves with wavelengths of $O(1000)$ km propagating zonally along the mean equatorial currents.

The coincidence of vigorous EKE and strong currents suggests that the geostrophic eddy field arises from instabilities of more directly forced, persistent large-scale currents. Gill et al. (1974) computed the stability of density and velocity profiles and concluded that baroclinic instability is the dominant generator of the large-scale ocean currents in mid and high latitudes. These instabilities occur in rotating, stratified fluids with strong currents in thermal wind balance having steeply sloping density surfaces. The energy of the growing perturbations is extracted from the PE stored in the density fronts. Eddies pinch off from the fronts—resulting in flattening of the density surfaces and a release of PE$^8$. PE associated with strong ocean currents is about 1000 times larger than the KE of the gyre-scale circulation.

A distinguishing feature of a baroclinically unstable flow is its intrinsic length

$^8$Another type of instability, known as barotropic, dominates in the tropics. This instability occurs in the presence of strong horizontal shear and derives its energy from the kinetic energy of the mean currents.
scale—known as the “Rossby deformation radius,”

\[ R_d \equiv \sqrt{\frac{g}{f^2 \rho}} \Delta \rho D, \]

where \( \Delta \rho \) is the density change over the vertical distance \( D \), and which is the distance a disturbance of that vertical scale propagates in the ocean before reaching geostrophic balance. If \( D \) is the depth of the ocean, then among all unstable baroclinic modes, the ones with the largest growth rate have a scale proportional, but somewhat smaller, than \( R_d \). Fig. 5 shows the zonally-averaged Rossby deformation radius in the ocean as a function of latitude computed from the Gouretski and Koltermann (2004) global ocean hydrography.\(^9\) The decrease of \( R_d \) with latitude is mostly due to the increase of \( f \) with latitude. (Fu et al. (1981), using current meter records, and Gille et al. (2000) globally from altimeter data, inferred the geostrophic eddy band is suppressed over rough topography—an effect seen in theory for eddies subject to strong bottom friction (Treguier and Hua, 1988; Arbic and Flierl, 2004).

Smith (2007) recently examined the linear stability implied by the Gouretski and Koltermann (2004) hydrographic climatology, and confirmed that the ocean is everywhere baroclinically unstable. In his model, fastest growth occurred in regions where EKE is largest and quiet regions with reduced KE were only weakly unstable. Smith’s maps of eddy growth rates have patterns remarkably similar to those in Fig. 4. The horizontal scale at which the instabilities develop is proportional, but somewhat smaller than, \( R_d \) (see Fig. 5) and direct observations are consistent with the predictions of linear stability analysis. Scott and Wang

\(^9\)The literature is vague concerning the definition of “scale” e.g., whether it is \( R_d \) or \( R_d/2\pi \), and in some cases, we are unable to determine whether published results are inconsistent or not because of this fundamental ambiguity.
Ferrari and Wunsch (2005) find a source of KE at scales close to $R_d$ from a spectral analysis of altimetric observations and interpret it as the conversion of large-scale PE into EKE by baroclinic instability.

We can attempt an estimate of the total energy released through baroclinic instability of major currents. Because the source of eddy energy is through conversion of large-scale PE and associated with a spindown of the large-scale ocean circulation, the baroclinic release of PE, given by the $g\rho_\theta w$ term in Eq. (5), can be expressed as an effective eddy stress $\tau_e$ acting against the large-scale ocean circulation,

$$\int\int\int g\rho_\theta w\,dV = -\int\int \frac{\partial \tau_e}{\partial z} \cdot \bar{u} dV,$$

(11)

where the overbar denotes a long-term average. Ferreira et al. (2004) estimated $\tau_e$ and the mean circulation with a numerical model of the global ocean constrained with Levitus climatology leading (D. Ferreira, private communication, 2008), in Eq. (11) to a baroclinic release of 0.3TW, in the range 0.2 – 0.8TW obtained by WF2004.\(^{10}\) In summary, baroclinic instability seems capable of releasing 30%-100% of the wind power input to the large-scale circulation.

The approximate agreement between linear theory and observations supports the view that baroclinic instability of mean currents is the main source of the energy in the geostrophic eddy field. Ocean eddies are, however, observed to be two to ten times larger than the scale of the most unstable waves (Fig. 5) and their vertical structure is not as surface intensified as predicted by linear stability analysis (Smith, 2007), but note the ambiguity of scale is order $2\pi$.

These discrepancies are likely the result of nonlinear turbulent interactions that

\(^{10}\)The total is the residual of a positive release of PE in the upper ocean, and a negative value (creation of PE) below the thermocline.
redistribute energy across spatial scales and maintain an equilibrium between
generation and dissipation of geostrophic eddies. Details of these interactions
are not fully understood, but a first order description of oceanic turbulence is
emerging thanks to the growing number of observations.

In the eddy-band, rotation and stratification strongly suppress vertical ve-
locities and motions are constrained to be quasi two-dimensional. Eddy-eddy
interactions in two dimensional flows result in an “inverse cascade” of energy to
progressively larger scale motions (Salmon, 1998) in contrast to the three dimen-
sional “direct cascade” to smaller scales. An inverse cascade of energy in the
geostrophic eddy band might therefore be expected. Oceanic flows, however, are
quasi-two-dimensional in the sense that horizontal velocity dominates the vertical
velocity, but the horizontal velocity varies with depth in contrast to its behav-
ior in purely two-dimensional flows. The direction of the energy cascade is very
sensitive to these vertical variations.

Vertical structure of the ocean velocity at the mesoscale is well described by an
orthogonal set of basis functions given by the eigenfunctions of a Sturm-Liouville
problem involving the water depth, the vertically dependent buoyancy frequency,
and the Coriolis parameter (see e.g., Vallis, 2006). The resulting eigenfunctions
are called the “baroclinic modes”: the \( m \)-th mode, \( \phi_m \), has \( m \)-zero crossings in
the vertical. When, \( m = 0 \), the mode is barotropic and represents the depth-
average, \( \phi_0 = \text{constant} \). The eigenvalues \( \lambda_m \) are the inverse of the deformation
radii (length\(^{-1}\)) of the corresponding modes. The Rossby deformation radius,
\( R_d \), is a good approximation to \( \lambda_1^{-1} \). Theories of geostrophic turbulence (Rhines,
show a direct (downscale) energy cascade for the total baroclinic energy, kinetic
and potential, at scales larger than the corresponding deformation radius and an inverse (i.e. upscale) cascade of energy for the barotropic KE (the barotropic mode has almost no PE). A schematic of these energy pathways is shown in Fig. 6.

Geostrophic turbulence theory predicts that when baroclinic energy in vertical mode $m$ reaches its corresponding deformation radius $\lambda_m$, a vertical transfer of energy occurs to lower baroclinic modes and eventually to the barotropic one (Charney, 1971). For a fluid with strongly surface-intensified stratification, such as the ocean, the baroclinic modes interact inefficiently with the barotropic mode, and thus energy from higher baroclinic modes collects in the first mode and converges toward the first deformation radius before it finally barotropizes (Flierl, 1978; Fu and Flierl, 1980; Smith and Vallis, 2001). The inverse cascade is the final stage whereby energy in the barotropic mode near the deformation scale moves toward even larger scales. Scott and Arbic (2007) recently showed that the inverse cascade is not confined to the barotropic mode—in numerical simulations KE associated with the first baroclinic mode also fluxes upscale. In summary, the energy in the mesoscale field moved upscale in deep barotropic and first baroclinic eddies. Numerical simulations suggest that the ratio of energy in the two modes is very sensitive to the strength of bottom dissipation (Scott et al., 2007). This paradigm does not apply at the ocean surface, where energy appeared to cascade downscale in surface trapped modes (Klein et al., 2008).

The presence of a vertical shear due to the large-scale geostrophic currents supports surface trapped modes in addition to free interior modes. These surface modes tend to extract energy from the interior modes and transfer energy to small horizontal scales where they become unstable to three dimensional instabilities.
and dissipate their energy (Capet et al., 2008a,b; Klein et al., 2008). Whether the surface modes transfer a substantial amount of KE out of the interior geostrophic eddy field is unknown.

The geostrophic turbulence scenario is broadly supported by observations. Analysis of velocity measurements from mooring data confirms that most of the subinertial EKE resides in equal parts in the barotropic and first baroclinic modes with a very little residual in higher ones (Wunsch, 1997). Sea surface height measurements reveal a source of EKE at scales near to or larger than the first deformation radius (Scott and Wang, 2005). Most of this energy source is likely associated with the development of baroclinic eddies at the expense of the large-scale currents, but some fraction could arise from the nonlinear conversion of energy from high baroclinic modes into the first mode. Scott and Wang (2005) estimated the direction of the energy fluxes from a spectral analysis of sea surface data. At scales larger than $R_d$, energy appeared to flow upscale, consistent with the inverse cascade paradigm. The result is a bit ambiguous because the altimeter slope signal reflects only surface velocities, which are dominated by low baroclinic modes with little barotropic contribution, and therefore provide no information on the barotropic energy flux. However an inverse cascade is also visible by eye in sea surface height pictures: mesoscale eddies tend to progressively increase in size downstream of their formation region (Stewart et al., 1996; Kobashi and Kawamura 2002). An increase in eddy size is consistent with the shift from the scale at which they are generated to the larger one where they equilibrate (Fig. 5). Altimetric power-law results are, however, fragile: Stammer (1997) and Scott and Chambers (2008) had to make strong assumptions about the noise contributions to wavelengths shorter than 100-200km.
Additional support for the geostrophic turbulence paradigm has come from observational (Maximenko et al., 2005) and computational (Nakano and Hasumi, 2005; Qiu et al., 2008) evidence of multiple alternating zonal jets primarily, but not only, in the abyssal ocean, although Huang et al. (2007) show the real difficulties of obtaining a clear interpretation of data. Zonal jets are believed to result from an interplay between the inverse cascade of energy in the barotropic mode and planetary potential vorticity gradient $\beta$ (Rhines, 1975; Vallis, 2006). The hypothesis is that the inverse cascade is nearly arrested meridionally by angular momentum constraints, but proceeds in the zonal direction resulting in oblongated eddies and eventually in alternating jets. Jets are visible in the atmosphere of giants planets where there is a large gap between the first deformation radius, the scale at which eddies are generated, and the Rhines scale, i.e. the scale at which angular momentum constraints apply. Similar gaps exist in the world oceans, but not in the atmosphere where the first deformation radius is of O(1000) km and the inverse cascade can span a limited wavenumber range (Boer and Shephard, 1983; Schneider and Walker, 2006).

A serious limitation in that geostrophic turbulence paradigm is that it ignores interactions with bottom topography. The ocean has topography at all scales, particularly prominently so in the vicinity of mid-ocean ridges and fracture zones. This structure provides a potential route to shortcut the cascade, because energy can be exchanged among different scales via coupling of different vertical modes in the presence of topography. The presence of an inverse energy cascade and alternating zonal jets at mid-latitudes seems to be good evidence for the plausibility of ignoring (at least to first order) topographic interactions at those latitudes. However, zonal jets are conspicuously absent from the Southern Ocean - the re-
gion where the mean flow is most clearly influenced by topography. This suggests that at high latitudes the interaction with topography is ubiquitous and the idea of a cascade defined in terms of a horizontal wavenumber spectrum is incomplete. One expects stronger topographic response at high latitudes because of the weaker overall stratification, the strong atmospheric synoptic forcing, and in the Southern Ocean, the need to dissipate the very large input of KE and to balance the corresponding momentum injection.

Müller and Frankignoul (1978) and Frankignoul and Müller (1979) pointed out that geostrophic eddies can be generated by rapidly fluctuating winds in addition to arising from free instabilities of mean currents. Stammer and Wunsch (1999) found that significant correlations between observed variations in EKE and wind stress are confined to the high latitudes in the North Atlantic and North Pacific, and even there only a small fraction of the eddy energy could be attributed to direct wind generation. It does appear, however, that the very strong, large-scale, barotropic fluctuations seen in high latitude altimetric records (Fukumori et al., 1998; Stammer et al., 2000, etc.) are not properly accounted for (the associated KE is small, but their bottom interactions may be very strong). Elipot and Gille (2007) found that rotary spectra of surface winds and upper ocean velocities at subinertial frequencies are predominantly anticyclonic and interpreted this asymmetry as evidence of wind generation of surface eddies, because upper ocean velocities respond preferentially to anticyclonic rotating winds. Whether these surface eddies, with scales of O(10) km, are dynamically coupled to the larger and more energetic geostrophic eddies that populate the ocean interior is unknown.
5.3 Dissipation of Geostrophic Motions — The End of the Cascade

In summary, large-scale wind and surface fluxes maintain the large-scale reservoir of PE. Baroclinic instability and nonlinear interactions transfer energy to the first baroclinic mode and from there to the barotropic mode, where an inverse cascade of barotropic KE to larger horizontal scales occurs (Charney, 1971; Salmon, 1998; Scott and Arbic, 2007). Absent a dissipation mechanism, the inverse cascade would result in unseen barotropic eddies the size of the ocean basins (Fig. 5). Some process must drain energy from geostrophic eddy motions and arrest the cascade.

In the energy diagram of WF2004, the largest uncertainty was associated with the essentially unknown dissipation of the energy contained in the eddy field, which contains over 90% of the KE of the flow (but a minute fraction of the PE). A few candidates exist: (1) bottom drag; (2) loss of balance (e.g., Molemaker et al., 2005; Williams, et al., 2008; see Marshall and Naveira Garabato, 2007, for a list of earlier references); (3) interactions with the internal wave field; (4) continental margin scattering/absorption; (5) suppression by wind work. Other possibilities (e.g., lateral dissipative stresses, damping of eddy motions through air-sea buoyancy fluxes) appear unimportant.

Dissipative interaction with bottom topography can result in an arrest of the cascade. As mentioned above, both observations and theory suggest strong spatially variable bottom friction. Arbic and Flierl (2004) found that moderate bottom friction is required for the modeled geostrophic turbulence to reproduce the observed amplitudes, vertical structure, and horizontal scales of mid-latitude eddies.
WF2004 argued that drag in bottom boundary layers appeared to be too weak to represent the dominant eddy energy sink. From observations, Sen et al. (2008) inferred that between 0.2 and 0.8 TW are dissipated by quadratic bottom boundary layer drag. The uncertainty in the estimate is very large, because the dissipation depends on the cube of the bottom geostrophic velocity and is dominated by poorly sampled eddies with exceptionally large velocities. Their upper bound is close to the total estimated power input to the geostrophic field. The lower bound is equal to the estimate reported in WF2004 and would account for about 30% of the total power input into the eddies.

An additional mechanism for damping geostrophic motions is through generation and radiation of gravity waves from small-scale topographic roughness. Geostrophic flows can radiate waves by flowing over topographic features with scales between $u/f$ and $u/N$, where $u$ is the magnitude of the geostrophic flow normal to the topographic relief. (Topographic scales outside this range generate evanescent waves.) For typical ocean parameters the relevant scales lie between 100m-10km. Numerical experiments show that these short topographic waves tend to break within a kilometer from the ocean bottom and dissipate their energy locally (Chapman and Haidvogel, 1993; Nikurashin and Ferrari, 2007). Radiation and breaking of internal gravity waves are reported in observations from two regions of the Southern Ocean characterized by rough topography and energetic barotropic eddies (Polzin and Firing, 1997; Naveira Garabato et al., 2004). Marshall and Naveira Garabato (2007) compute an upper bound on the mixing rates induced by topographic wave generation and breaking in the Southern Ocean of $\kappa \leq 5 \times 10^{-3} \text{ m}^2/\text{s}$ (a large number). If one uses Eq. (8) of WF2004, assigns an area of the Southern Ocean of $0.6 \times 10^{14} \text{ m}^2$, assumes this mixing takes
place over the bottom 1000m, in a region where \( N \approx 2 \pi/1/\text{hr} \), an upper bound of about 1TW is found for the energy loss by the mesoscale in the Southern Ocean alone (the crudity of this estimate will be obvious), already somewhat larger than the estimate of the rate of eddy production from Eq. (11).

Molemaker et al. (2007) analyzed high resolution numerical simulations of an idealized ocean current and found that loss of geostrophic balance can occur in the ocean, but mostly at the upper ocean boundary as result of frontogenesis (Hoskins and Bretherton, 1972). Surface frontogenesis describes the formation of sharp density gradients as a result of eddy—stirring at the ocean boundaries. These fronts have characteristic horizontal scales of a few kilometers, much smaller than mesoscale eddies, and so loss of balance can take place. From the perspective of geostrophic turbulence theory, surface frontogenesis represents a direct cascade of baroclinic energy to scales smaller than the deformation radius. A cascade is possible only at the surface because the surface modes at small scales do not interact efficiently with the interior modes and do not barotropize. Observations show that KE is surface intensified at scales smaller than the first deformation radius (Ferrari and Rudnick, 2001; LaCasce and Mahadevan, 2006) and support the notion that the ocean surface is a primary location for transfer of energy to small scales and hence dissipation. Molemaker et al. (2007) found that in idealized ocean simulations of geostrophic turbulence, two thirds of the eddy energy was fluxed upscale into the barotropic mode and one third cascaded to smaller scales through surface frontogenesis. Surface dynamics plausibly have an important role in the dissipation of geostrophic eddy EKE.

Other ways exist, beyond spontaneous loss of balance, by which geostrophic energy can be lost to superinertial motions. Müller (1976), and more recently
Bühler and McIntyre (2005), showed that the deformations of internal wave packets propagating through the strain due to eddies result in an irreversible extraction of energy from the geostrophic field. Characterization of the energy exchange as a viscous damping of the mesoscale field gives a horizontal viscosity of $\nu_h = 50 \text{m}^2/\text{s}$ and a vertical viscosity of $\nu_v = 2.5 \times 10^{-3} \text{m}^2/\text{s}$ (Polzin, 2007).

Watson (1985) estimated similar viscosities as representative of the exchange of energy between mesoscale motions and near-inertial frequency waves through a form of triad interaction. His result showed that the inferences of Bühler and McIntyre apply also in situations where there is no scale separation between the mesoscale and the internal wave field. Evidence of energy loss from geostrophic motions at horizontal scales of $O(30) \text{km}$, characteristic of the internal wave field, has been reported from an analysis of altimetric data by Scott and Wang (2005). It is premature (because of questions about the data) to conclude that the observed energy loss is associated with coupling between geostrophic eddies and internal waves, but the possibility is tantalizing.

One can attempt an estimate of the eddy energy loss to internal waves based on the horizontal and vertical viscosity coefficients, respectively $\nu_h$ and $\nu_v$, proposed by Watson (1985) and Polzin (2007),

$$\varepsilon \approx \int \int \left[ \nu_h \left( \frac{\partial u}{\partial x} \right) \cdot \left( \frac{\partial u}{\partial x} \right) + \nu_h \left( \frac{\partial u}{\partial y} \right) \cdot \left( \frac{\partial u}{\partial y} \right) + \nu_v \left( \frac{\partial u}{\partial z} \right) \cdot \left( \frac{\partial u}{\partial z} \right) \right] \text{d}V. \quad (12)$$

Altimetric velocities reflect mostly the first baroclinic mode (Wunsch, 1997) and in most of the ocean, that mode has a characteristic vertical and horizontal scales, $H = 1000 \text{ m}$ and $\lambda_1$. Typical vertical and horizontal velocity gradients squared are then $(u^2 + v^2)/2H$ and $(u^2 + v^2)/2\lambda_1$, respectively. Altimetric data (Le Traon et al., 1998) provide velocities; from $\lambda_1$ (Chelton et al., 1998) and assuming that the gradients are confined to the upper 1000 m, the net conversion...
is about 0.35 TW. Although the uncertainty is very large, the energy exchange
with the background internal wave field remains a potentially important sink of
eddy energy. This estimate does conflict with two properties of the observed
internal wave field: (A) The energy flux into the background internal wave field,
as predicted by the GM spectrum, is an order of magnitude smaller than 0.35
TW. (B) The energy content in the internal wave field is believed to be nearly
constant throughout the ocean interior, and this scenario suggests an unobserved
strong modulation by the geostrophic eddy field. This story remains unfinished.

Finally, the effects of eddy-like motions on the wind-stress act to “spin-down”
geostrophic eddies as described Sec. 4. The energy diagram in the supplementary
material proposes that about $13 \times 10^{18} \text{J}$ (13 exajoules, EJ) are contained in the
geostrophic eddy field, and that the rate of generation by instability mechanisms
is such that all the energy could be renewed every 6 months (Thorpe, 2005,
estimated a one year renewal time). Here it is roughly estimated that 2.6EJ are
KE, the majority being potential. Supposing, from the fragmentary literature
now existing, that 10% of the estimated rate of wind work on the ocean goes to
spinning down the geostrophic eddy field. Taking that rate as again very roughly
1TW, then 0.1TW is available for the stated purpose, and the time to spin down
would be $13 \times 10^{18} \text{J}/10^{11} \text{W} = 1 \times 10^8 \text{s}$ or approximately 3 years. Given the crudity
of all of the numbers, it appears that the mechanism is important. Calculations
by Niiler (1969) and Dewar and Flierl (1987), accounting for the distortion of
Ekman pumping by the vorticity of the ambient motions, appear to be relevant,
but no quantitative result exists for the global problem. We have gone from the
situation described in WF2004 of being unable to account for the dissipation of
geostrophic turbulence to the opposite case—there are now too many candidates.
6 Discussion

At subinertial frequencies, the ocean circulation is dominated by large-scale currents and geostrophic eddies. The currents are driven by atmospheric winds, while the eddy field results from instabilities of the mean currents. Geostrophic eddies accounts for about 90% of the total kinetic energy of the oceans, while the large-scale circulation is the largest reservoir of potential energy. Eddy kinetic energy acts as a stirrer of tracers and momentum along density surfaces: geostrophic motions have less shear than internal waves and do not generate overturns and mixing across density surfaces. This eddy—stirring is a key element of the meridional overturning circulation of the ocean. For example the heat budget of the Southern Ocean is dominated by eddies that transport heat poleward against the equatorward heat transport by the wind driven circulation (Speer et al., 2000).

At equilibrium the energy flux into the geostrophic eddy field must be balanced by dissipation. It is an open and important question whether some of the dissipation is associated with mixing in the ocean interior. The problem is not well understood because it requires a complete description of the energy pathways from the 100 km scale at which eddies are generated down to the molecular scales at which irreversible energy dissipation occurs. Theory and altimetric observations suggest that once eddies are generated, eddy-eddy interactions transfer energy primarily to large-scale barotropic and first-baroclinic motions and to small-scale surface trapped eddies. If the barotropic and surface eddies are completely dissipated in the bottom and surface turbulent boundary layers, then geostrophic energy does not affect mixing in the ocean interior. However there is evidence that vigorous internal waves are radiated by barotropic eddies impinging over to-
pography. These waves supposedly radiate and break, resulting into irreversible mixing in the ocean interior. We are not yet in a position to determine whether eddy driven generation of waves support a large fraction of the observed abyssal mixing.

KE in the oceanic internal wave band is dominated by inertial waves, frequency $\sigma \approx f$, and often carrying high shears. Maintenance of this peak is possible through a great many energy sources, including resonant interactions of all higher frequency internal waves, coupling to geostrophic eddies, and the direct generation by local winds. The last mechanism, while known to be effective, in linear theory tends to produce low vertical wavenumbers and hence little shear. Even less is known about how this nearly universal inertial peak is dissipated, and thus it is unclear whether it is a major element in mixing the oceanic abyss, particularly as interactions with topographic features near the critical value, are unexplored. It is possible that mixing by the internal tide has been overemphasized recently, but the last word has not been said.

A major obstacle to progress remains with the difficulty of obtaining direct estimates of frequency and wavenumber spectra in the ocean. Each is separately reasonably well known, but the overlap of space scales corresponding to radically different time scales and hence physics, precludes any simple understanding of the origin of the KE, and hence shear in the open ocean. Observations on wavelengths of 200km and shorter are particularly needed. Many processes within the near-surface boundary layers, and which are poorly observed, have the potential for much more effective coupling of deep and upper oceans than is widely appreciated.

Even such apparently well-established mechanisms as the transfer of momentum from the atmosphere to the ocean are seen to be both qualitatively and
quantitatively obscure. That the oceanic flow is important to calculation of the stress boundary condition has implications both for generation and dissipation of KE.

There is no shortage of problems remaining in the much discussed area of ocean mixing. The suggestion from 10 years ago that the oceanic energy budget was uncertain by about a factor of two remains valid. For KE, the comparatively huge reservoir represented by the eddy field still cannot be balanced in terms of inputs and outputs within even larger factors, although negative work by the wind, and strong bottom dissipation together probably dominate energy removal.

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Figure 1: Kinetic energy spectral estimates for instruments on a mooring over the
Mid-Atlantic Ridge near 27°N (Fu et al., 1982). The inertial, principal lunar
semi-diurnal $M_2$, and diurnal $O_1$, $K_1$ tidal peaks are marked, along with the percentage
of kinetic energy in them lying between $f$ and the highest frequency estimate.
Least-squares power law fits for periods between 10 and 2 hours and for periods lying
between 100 and 1000 hours are shown. The approximate percentage of energy of the
internal wave band lying in the inertial peak and the $M_2$ peak are noted. In most
records, the peak centered near $f$ is broader and higher than the one appearing at the
$M_2$ frequency. When $f$ is close to the diurnal frequency, it is also close to 1/2 the
frequency of $M_2$, when the parametric subharmonic instability (PSI) can operate; see
the text. Some spectra show the first overtone, $2M_2$ of the semi-diurnal tide. (a)
Instrument at 128m. (b) Instrument at 1500m, and (c) 3900m (near bottom). The
geostrophic eddy band is greatly reduced in energy near the bottom, as is the inertial
band, presumably because of the proximity of steep topography. Note differing axis
scales.
Figure 2: (a) Kinetic energy estimate for an instrument in the western North Atlantic at about 15°N at 500m. In this record, the diurnal tides are well-separated from the inertial frequency. This record was described by Fu et al. (1982). (b) Power density spectral estimate from a record at 1000m at 50.7°S, 143°W, south of Tasmania in the Southern Ocean (Phillips and Rintoul, 2000). Now the diurnal tides are below $f$ in frequency, but whether the apparent peaks represent dominantly barotropic or baroclinic motions is not known.
Figure 3: (a) Location chart for the North Atlantic current meter records used here. Depths in km. One mooring lies in the complicated three-dimensional topography of the Mid-Atlantic Ridge, the other lies over an abyssal plain. (b) Position of the Southern Ocean current meter whose kinetic energy spectral estimate is displayed in Fig. 1b (Phillips and Rintoul, 2000). Depths in km.
Figure 4: (From Wunsch and Stammer, 1998). Estimate of the geostrophic kinetic energy (cms/s)$^2$ of oceanic variability at the sea surface, here multiplied by $\sin^2 \phi$ where $\phi$ is the latitude, to avoid the equatorial singularity in noisy data. Note the very large spatial changes of kinetic energy. [Note to Editor: only lower panel should be used.]
Figure 5: Zonally averaged scales (in km) of: maximum growth rate of baroclinic instability of the main thermocline estimated from hydrography (Smith, 2007); the spectral peak of eddy kinetic energy from the analysis of satellite observations by Stammer (1997b) (dash-dotted line); and the first deformation radius estimated from Levitus climatology (Chelton et al., 1998). The estimate of the spectral peak of eddy kinetic energy is very uncertain. First the altimetric signal is dominated by noise at scales below 100-50 km and the spectral energy is largest at the smallest wavenumbers. Hence the spectral peak estimate is not independent of the choice of filter. Second, the spectral peak is evident only in a small fraction of satellite tracks that cross well defined coherent eddy structures: the spectral peak does not characterize the typical background eddy kinetic energy spectrum which is close to white.
Figure 6: Schematic of the energy pathways in geostrophic turbulence. The horizontal axis represents the horizontal wavenumber, and the vertical variation are decomposed into the barotropic mode (lower line) and the sum of all baroclinic modes (upper line). Large-scale forcing maintains the available potential energy, and so provides energy to the baroclinic mode at very large scales. At these large scales, baroclinic energy is transferred to smaller horizontal scales. At horizontal scales comparable to the deformation radius energy is transferred to the barotropic mode and thence to larger barotropic scales. Some fraction of the baroclinic energy leaks to smaller scale through surface-intensified baroclinic modes.