

8 Lateral diffusivities are computed from synthetic particles and tracers advected by a
9 velocity field derived from sea surface height measurements from the South Pacific, in a
10 region west of Drake Passage. Three different estimates are compared: (1) the tracer-based
11 “effective diffusivity” of Nakamura (1996), (2) the growth of the second moment of a cloud of
12 tracer and (3) the single and two-particle Lagrangian diffusivities. The effective diffusivity
13 measures the cross-stream component of eddy mixing, so the manuscript focusses on the
14 meridional diffusivities for the others, as the mean flow (the ACC) is zonally-oriented in the
15 region.

16 After an initial transient of a few weeks the effective diffusivity agrees well with the
17 meridional diffusivity estimated both from the tracer cloud and from the particles. This
18 proves that particle and tracer based estimates of eddy diffusivities are equivalent, despite
19 recent claims to the contrary. Convergence among the three estimates requires that the
20 Lagrangian diffusivities are estimated via their asymptotic values, not their maximum values.
21 The former are generally much lower than the latter in the presence of a mean flow.

22 **1. Introduction**

23 Quantifying tracer transport by geostrophic eddies is one of the outstanding problems in
24 large-scale ocean dynamics. Geostrophic eddies set the rate at which tracers like heat and
25 carbon are mixed laterally and vertically in the ocean and play a central role in determin-
26 ing large scale circulation patterns, such as the meridional overturning circulation (see for
27 example the reviews by Olbers et al. (2004) or Marshall and Speer (2011)).

The mixing induced by geostrophic eddies is often quantified in terms of an eddy diffusivity. The component of the diffusivity across the mean currents is most relevant, because tracer transport along currents is generally dominated by the current itself. The cross-current eddy diffusivity K is defined as,

$$\overline{v'C'} = -K\bar{C}_y, \quad (1)$$

where $\overline{v'C'}$ is the cross-current tracer flux and \bar{C}_y is the cross-current tracer gradient. Overbars denote large-scale averages and primes are departures from those averages. As noted hereafter, there are discrepancies in the literature about both the magnitude and spatial patterns of K , and it is a goal of this paper to help resolve these discrepancies.

Estimates of K can be obtained from Eulerian data, from tracer distributions and from Lagrangian (particle) data. Early attempts to estimate K with Eulerian data relied on mooring time series of velocity \mathbf{v} and tracer C (e.g. temperature) to construct $\overline{\mathbf{v}'C'}$ and its relationship to \bar{C}_y (e.g. Bryden and Heath 1985). However, the time series are often too short for significant results (Wunsch 1999). In addition, the estimates are hampered by the presence of large, non-divergent eddy fluxes that do not contribute to mixing (Marshall and Shutts 1981).

Another example of Eulerian estimates is the use of satellite measurements of sea surface height (SSH) in conjunction with mixing length theory (Prandtl 1925) to estimate diffusivities, as proposed by Holloway (1986) and Ferrari and Nikurashin (2010). This approach has the advantage that time series at fixed locations lend themselves well to the determination of mean and residual quantities. The weakness is that the eddy diffusivity can be determined up to undetermined constants that appear in the mixing length theory.

45 Tracer-based estimates can be made from hydrographic data. Armi and Stommel (1983)
46 studied the oxygen distribution in the Eastern North Atlantic in the context of an advective-
47 diffusive equation and inferred an eddy diffusivity of approximately $500 \text{ m}^2\text{s}^{-1}$ at 800 m
48 depth. Similarly, Ferrari (2005) and Zika et al. (2010) used temperature and salinity obser-
49 vations in the same region to estimate diffusivities from $O(1000 \text{ m}^2\text{s}^{-1})$ at the surface and
50 $O(100 \text{ m}^2\text{s}^{-1})$ below 1500 m . Jenkins (1998) studied tritium and ^3He distributions in the
51 same region and estimated $K \sim 1200 \text{ m}^2\text{s}^{-1}$ at a depth of 300 m .

52 Such estimates assume a steady state of advection and diffusion. The evolution of the
53 eddy diffusivity can only be deduced with observations of a time-evolving tracer distribution,
54 as in the North Atlantic Tracer Release Experiment (NATRE; Ledwell et al. (1998)). The
55 diffusivity can then be estimated from the dispersion of the patch. The NATRE release (of
56 sulphur hexafluoride) yielded an estimate of $K \sim 1000 \text{ m}^2\text{s}^{-1}$ at 300 m depth, consistent
57 with the steady-state estimates of Jenkins (1998).

58 Lagrangian estimates on the other hand derive from the trajectories of freely-drifting
59 instruments, like surface drifters and subsurface floats (e.g. Davis 1991; LaCasce 2008). The
60 diffusivity is proportional to the derivative of the mean square separation of the particles from
61 their starting positions, as first proposed by Taylor (1921). Early diffusivity estimates from
62 drifting buoys in the North Atlantic were made by Freeland et al. (1975), Colin de Verdiere
63 (1983) and Krauss and Böning (1987). Colin de Verdiere (1983) obtained a meridional
64 diffusivity of $K \sim 1700 \text{ m}^2\text{s}^{-1}$ at the surface and Freeland et al. (1975) obtained $K \sim$
65 $700 \text{ m}^2\text{s}^{-1}$ at 1500 m depth.

66 However, some subsequent estimates were much larger. Zhurbas and Oh (2004) obtained
67 values in excess of $4000 \text{ m}^2\text{s}^{-1}$ at the surface in the Western Atlantic, reaching values of

68 20,000 m^2s^{-1} in the vicinity of the Gulf Stream. Lumpkin et al. (2002) and McClean et al.
69 (2002) found comparable values.

70 Diffusivities have also been measured in the Southern Ocean, which is the focus of this
71 paper, and here too the results vary. Eulerian-based estimates, from SSH, yield values of
72 2000 – 4000 m^2s^{-1} in the Antarctic Circumpolar Current (ACC), with smaller values in
73 the subtropical gyres to the north (Keffer and Holloway 1988; Stammer 1998). However
74 the tracer-based estimates differ. Using a technique proposed by Nakamura (1996) (Sec.
75 2), Marshall et al. (2006) obtained values of $O(1000 m^2s^{-1})$ near the core of the ACC and
76 *larger* values of $O(2000 m^2s^{-1})$ to the north. This method yields only the cross-stream
77 component of the diffusivity and thus in principle does not suffer from “contamination” by
78 the mean flow. Lower diffusivities in the ACC are consistent with the mean flow suppressing
79 cross-stream mixing (e.g. Ferrari and Nikurashin 2010). The ACC weakens with depth,
80 suggesting the diffusivity should have a subsurface maximum (as the eddy field is intensified
81 near the current). Abernathey et al. (2010) found this to be the case, using the tracer-based
82 diagnostic.

83 Lagrangian estimates on the other hand tend to be larger. Lumpkin and Pazos (2007) and
84 Sallée et al. (2008) obtained values in the vicinity of the ACC of approximately 4000 m^2s^{-1} ,
85 using surface drifters, with smaller values north and south of the jet. The exception was
86 near the western boundary currents where the diffusivities were large. Consistent with
87 some earlier studies in the North Atlantic (e.g. Lumpkin et al. (2002) and refs. therein),
88 Sallée et al. (2008) found that K is correlated with the eddy kinetic energy; the largest
89 values were near the ACC, where eddy variability is most pronounced. However, not all the
90 Lagrangian estimates agree. Using synthetic float data from a numerical model, Griesel et al.

91 (2010) found much smaller cross-stream diffusivities ($O(750 m^2 s^{-1})$) in the polar frontal zone,
92 consistent with the tracer-derived estimates. However, they did not observe a suppression
93 of eddy mixing in the ACC itself, nor did their diffusivities vary with depth.

94 Thus there are discrepancies between the estimates particularly in the Southern Ocean.
95 This is a significant challenge for ocean and climate simulations, because coarse-resolution
96 models are sensitive to the magnitude and the lateral and vertical structures of eddy diffusiv-
97 ities in the Southern Ocean (Griffies et al. 2005; Danabasoglu and Marshall 2007). Indeed,
98 ocean models can reproduce reasonable stratification and transports with meridional eddy
99 diffusivities in the range of a few hundreds to one thousand $m^2 s^{-1}$ (e.g. Griffies et al. (2005)).
100 As such, it is important to understand why the tracer, Eulerian and Lagrangian diffusivity
101 estimates differ.

102 Several studies have addressed this question previously. Sundermeyer and Price (1998)
103 used an idealized ocean model to reproduce eddy statistics from the NATRE field cam-
104 paign (Ledwell et al. 1993) and simulated the motion of synthetic floats and tracers. Their
105 Lagrangian estimates of K were 2-6 times larger than the tracer-based estimates. Riha
106 and Eden (2011) used an idealized zonally periodic channel with eddy-driven zonal jets to
107 compare Lagrangian and Eulerian diffusivity estimates. Interestingly, their estimates were
108 comparable, with the Lagrangian ones being slightly smaller. Incidentally, both methods
109 indicated an increase of the eddy diffusivities with depth.

110 In the present work, we examine further the relationship between different diffusivity
111 estimates. To this end, we use a three-dimensional velocity field representative of the South-
112 ern Ocean, derived from satellite measurements of SSH, to estimate eddy diffusivities from
113 tracers and floats. We focus on a region in the Southern Ocean upstream from Drake passage

114 which is the site of a large field experiment, the Diapycnal and Isopycnal Mixing Experiment
115 in the Southern Ocean (DIMES). A major goal of DIMES is to infer eddy diffusivities from
116 tracer and float releases. In the present paper, we focus on diffusivity estimates; in a separate
117 paper (Klocker et al. 2011), we examine mixing suppression by the mean flow.

118 The results suggest that cross-stream estimates of eddy diffusivities from both Lagrangian
119 statistics and tracer releases converge if measured properly and if the statistics are adequate.
120 The comparison is hindered by the mean flow, which causes the Lagrangian diffusivity to
121 overshoot its asymptotic value. Correct estimates thus require integrating past this over-
122 shoot.

123 The paper is organized as follows. Section 2 examines the different methods for calcu-
124 lating the diffusivity and how they are related. Section 3 describes how we construct the
125 velocity field used to advect tracers and floats and section 4 presents and compares the diffu-
126 sivity estimates for a sector of the Southern Ocean upstream of Drake Passage. The results
127 are discussed in section 5.

128 2. Theory

Here we compare theoretical estimates of tracer-based and particle-based diffusivities. The former is described by Nakamura (1996), Shuckburgh and Haynes (2003) and Marshall et al. (2006). The term derives from the advection-diffusion equation for a passive tracer, $C(x, y, t)$:

$$\frac{\partial}{\partial t}C + \nabla \cdot (\mathbf{u}C) = \nabla \cdot (\kappa \nabla C) \quad (2)$$

where \mathbf{u} is taken to be a 2-D (horizontal) velocity, assumed non-divergent, and κ is a small scale diffusivity. The latter might be the molecular diffusivity or a numerical diffusivity imposed to make solutions stable. The equation can be written as a simple diffusion equation when expressed in coordinates defined by the tracer isolines:

$$\frac{\partial}{\partial t} C = \frac{\partial}{\partial A} (K_{nak} \frac{\partial C}{\partial A}) \quad (3)$$

where A is the area bounded by an isoline of the tracer C , assumed to be a monotonic function of C . Further,

$$K_{nak} = \frac{\kappa}{(\partial C / \partial A)^2} \frac{\partial}{\partial A} \int |\nabla C|^2 dA \quad (4)$$

129 is Nakamura's effective diffusivity. This is proportional to the small scale diffusivity, κ , but
 130 it also depends on the gradient of C , integrated over the area bounded by the isoline. Thus
 131 if stirring acts to increase the gradients, the effective diffusivity can be much larger than
 132 the small-scale value. Note that K_{nak} has units of $(length)^4/time$, in contrast to a usual
 133 diffusivity.

Nakamura's effective diffusivity is often written in terms of a ratio of squared length scales:

$$K_e = \kappa \frac{L_{eq}^2}{L_0^2} \quad (5)$$

134 where L_{eq} is greater than or equal to L , the length of the tracer contour, L_0 is the initial
 135 length of the contour and $K_e = K_{nak}/L_0^2$. This form shows that the effective diffusivity
 136 exceeds the small scale value if the tracer contour is elongated by the advecting flow.

A simple way to see the preceding relation is as follows. Consider the integral of (2) over a region bounded by an isoline of tracer:

$$\iint \frac{\partial}{\partial t} C dA = \kappa \oint \frac{\partial C}{\partial n} dl = \kappa \langle \frac{\partial C}{\partial n} \rangle L \quad (6)$$

where n is the vector normal to the contour and the brackets indicate the mean value along the length of the contour. Given two tracer contours, e.g. C_1 and C_2 , the mean gradient is:

$$\left\langle \frac{\partial C}{\partial n} \right\rangle = \Delta C \left\langle \frac{1}{\delta} \right\rangle \quad (7)$$

where $\Delta C = C_2 - C_1$ is the difference between the tracer values, assumed fixed, and δ is the spacing between the contours. The spacing is related to the area between the contours:

$$dA = \langle \delta \rangle L \quad (8)$$

so:

$$\left\langle \frac{\partial C}{\partial n} \right\rangle \geq \frac{\Delta C L}{dA} \quad (9)$$

137 because $\langle 1/\delta \rangle$ is greater than $1/\langle \delta \rangle$ as per the Cauchy-Schwarz inequality for positive-definite
 138 δ .

During the initial straining, before small scale mixing sets in, this area is approximately conserved (as the flow is non-divergent). Then the area difference, dA , can be found from the initial distribution. Assuming the contours are uniformly spaced initially, then $dA = \delta_0 L_0$.

Thus:

$$\left\langle \frac{\partial C}{\partial n} \right\rangle \geq \frac{\Delta C L}{\delta_0 L_0}. \quad (10)$$

Using this, we can re-write the integrated tracer equation:

$$\iint \frac{\partial}{\partial t} C \, dA \geq K_{el} \frac{\Delta C}{\delta_0} L_0 \quad (11)$$

where K_{el} is:

$$K_{el} = \kappa \frac{L^2}{L_0^2} \quad (12)$$

139 Thus K_{el} is the lower bound of the effective diffusivity divided by the initial contour length
 140 squared.

We can use this to make the connection with the Lagrangian diffusivities. We think of the tracer contour as being comprised of N discrete particles. Then the squared length of the contour is given by:

$$L^2 = \sum_{n=1}^{N-1} (\mathbf{r}_{n+1} - \mathbf{r}_n)^2 = \sum_{n=1}^{N-1} (x_{n+1} - x_n)^2 + (y_{n+1} - y_n)^2 \quad (13)$$

where $\mathbf{r}_n = (x_n, y_n)$ is the position of the n -th particle. Clearly:

$$L^2 = N \langle \Delta r^2 \rangle \quad (14)$$

where $\langle \Delta r^2 \rangle$ is the mean square spacing between the particles. This is the *relative dispersion* of particle pairs (e.g. Bennett (2006)). Thus if the area between tracer isolines is conserved, we have:

$$K_{el} = \kappa \frac{\langle \Delta r^2 \rangle}{\langle \Delta r_0^2 \rangle} \quad (15)$$

¹⁴¹ So the effective diffusivity is proportional to the relative dispersion.

But how is the effective diffusivity related to the particle diffusivity? Following Taylor (1921), the Lagrangian diffusivity is given by:

$$K_1 = \frac{1}{2} \frac{d}{dt} \langle (\mathbf{r} - \mathbf{r}_0)^2 \rangle \quad (16)$$

where \mathbf{r} is the position of a particle, \mathbf{r}_0 is its initial position and the brackets indicate an average over all available particles. This applies to *single* particles, and accordingly we refer to (16) as the *single-particle diffusivity*. However, one can also define a *relative diffusivity* (or *two-particle diffusivity*):

$$K_2 = \frac{1}{4} \frac{d}{dt} \langle \Delta r^2 \rangle \quad (17)$$

¹⁴² (e.g. Batchelor and Townsend (1953); Davis (1985); Babiano et al. (1990)). The factor of

143 1/4 is arbitrary but useful, as described below. An important point is that the relative
 144 diffusivity, unlike the single-particle diffusivity, is Galilean invariant.

Now if the stirring is *non-local* (dominated by larger scale eddies), the relative dispersion increases exponentially in time (Lin 1972; Bennett 1984; Babiano et al. 1990; LaCasce 2008):

$$\langle \Delta r^2 \rangle = \langle \Delta r_0^2 \rangle e^{t/\tau} \quad (18)$$

where τ is a time scale, usually determined by the root mean squared shear. From (15), the effective diffusivity will also grow exponentially in time, with the same e-folding time scale, τ . With exponential growth, the relative diffusivity is proportional to the relative dispersion:

$$K_2 = \frac{1}{4\tau} \langle \Delta r^2 \rangle \quad (19)$$

So the effective and relative diffusivities are proportional to one another:

$$K_{el} = \kappa \frac{4\tau K_2}{\langle \Delta r_0^2 \rangle} \quad (20)$$

145 But this equivalence follows from two assumptions: first, that the area is conserved be-
 146 tween tracer contours and second, that the stirring is non-local. The relation will presumably
 147 break down if either assumption is violated. So, for example, the diffusivities should differ
 148 under local stirring, as occurs in a 2-D inverse energy cascade or 3-D forward energy cascade.

149 In numerical studies, the effective diffusivity asymptotes to a constant value when the
 150 small-scale diffusion is actively mixing away the tracer gradients (Nakamura 2008). Area is
 151 no longer conserved between isolines, implying the lower bound in (12) is no longer useful.
 152 The relative dispersion on the other hand continues to grow. When particle separations are
 153 large enough, the pair velocities are uncorrelated and the relative diffusivity, defined with
 154 the prefactor of 1/4, asymptotes to the single particle diffusivity, $K_2 \rightarrow K_1$ (e.g. Babiano

155 et al. (1990); LaCasce (2008)). At this point, relative dispersion grows linearly in time, as
156 for a diffusive process. But as the effective diffusivity is constant, the two cannot be the
157 same at later times.

158 The Lagrangian diffusivities (the time derivative of dispersion) should also asymptote to
159 constant values at later times, provided there are no long-term correlations in the velocities
160 (Taylor 1921). Yet there is no guarantee that the particle diffusivities will agree with the
161 tracer-derived estimate. Indeed, the former derive purely from advective considerations
162 while the latter are based on an areal integration which effectively removes the advective
163 term from the tracer equation. While the effective diffusivity is proportional to the small-
164 scale diffusivity, the relative diffusivity is determined essentially by the scale and kinetic
165 energy of the energy-containing eddies. The two diffusivities are expected to converge, if
166 the rate at which tracers are homogenized is set by the large-scale eddies and not by small
167 scale diffusion. This likely happens in strongly turbulent fields where the large-scale eddies
168 set the rate at which tracer filaments are generated. Once generated, the filaments are
169 stretched and folded until they are homogenized by small scale diffusion. A reduction of
170 small-scale diffusion does not alter the rate of tracer homogenization, rather it results in
171 thinner filaments that can be irreversibly mixed faster to keep up with the rate of filament
172 generation.

173 Previous results suggest that the two diffusivities are the same, at least for some idealized
174 flows. A collection of particles constitutes a tracer, and thus an initial distribution should
175 evolve according to the same diffusion equation (3). Shuckburgh and Haynes (2003) examined
176 this, computing numerical solutions to this equation, using the effective diffusivity diagnosed
177 from the flow field, and comparing the solutions to time-evolving distributions of particles

178 advected by the same flow. The close agreement suggests the particles effectively obey the
179 same diffusion equation. So the diffusivity measured by the particles and the tracer ought
180 to agree.

181 We test this hereafter using numerical calculations of evolving tracer fields and particle
182 distributions using a velocity field representative of the surface Southern Ocean. We consider
183 that field next.

184 **3. The velocity field**

185 The velocity at the surface is estimated from the sea surface height slope measured by
186 altimeters from space. We use sea level anomaly maps from the combined processing of
187 Ocean Topography Experiment (TOPEX) and European Remote Sensing Satellite-1 and
188 2 (ERS-1 and ERS-2) altimetry data with a temporal resolution of 10 days and a spatial
189 resolution of $1/4^\circ$ longitude \times $1/4^\circ$ latitude. We focus on the region upstream of Drake
190 Passage between 103° - $78^\circ W$ and 30° - $66^\circ S$, broadly the same area chosen for the DIMES
191 campaign; a snapshot of sea surface height from this region is shown in Fig. 1. Conveniently,
192 the ACC is nearly zonal in this region, so the cross-stream direction is simply north-south.

193 The sea level anomalies are computed with respect to a 3-year mean (from January
194 1993 to January 1996). The mean streamlines are computed from the sum of the mean
195 geoid and the 3-year time-mean sea-surface height from altimetry. The geoid model is
196 described in Lemoine (1998) and the altimetric data are described in Marshall et al. (2006)
197 and Ferrari and Nikurashin (2010). The geostrophic relation is used to derive zonal and
198 meridional geostrophic velocities from sea surface height slopes, $(u_g, v_g) = f^{-1}g(-\partial_y h, \partial_x h)$

199 where f is the Coriolis parameter and g the acceleration of gravity. The velocities are then
 200 interpolated onto a $1/10^\circ$ resolution grid. We constructed two different velocity fields, an
 201 ‘eddy’ velocity field, in which we calculate the geostrophic velocities using only the sea-
 202 surface height anomalies and a ‘full’ velocity field in which we add the zonal mean flow
 203 shown in Fig. 3 (ignoring the very weak meridional mean flow) to the eddy velocity. The
 204 results described below do not change if we retain the longitudinal dependence of the mean
 205 flow.

The altimeter provides an estimate for the surface horizontal velocity, but one needs hor-
 izontal velocities at all vertical levels to estimate both the horizontal and vertical variations
 in the lateral eddy diffusivity. Killworth and Hughes (2002) argue that the mean and eddy
 flows in the ACC have an equivalent-barotropic structure, i.e. the horizontal velocities at all
 levels can be reconstructed by multiplying the surface values by a structure function that
 depends only on depth, $\mu(z)$, defined as

$$\mu(z) = \frac{|\mathbf{u}(x, y, z)|}{|\mathbf{u}(x, y, 300)|}. \quad (21)$$

206 The equivalent-barotropic structure applies only to the geostrophic velocity, hence the ve-
 207 locity field is scaled to its value at $z = 300$ meters below any Ekman flow. The structure
 208 function is computed from the output of the Southern Ocean State Estimate (SOSE; Mazloff
 209 et al. 2010), an eddy-permitting general circulation model of the Southern Ocean constrained
 210 to observations in a least-squares sense. The structure function $\mu(z)$ is calculated at each
 211 vertical level as a two-year average and a spatial average restricted to the area spun by the
 212 ACC in the region under study; the equivalent-barotropic model is meant to apply only in
 213 the ACC and not north of it (Killworth and Hughes 2002). The structure function $\mu(z)$,

214 shown in Fig. 2, is then used to scale the surface geostrophic velocity in the whole domain
215 (not just in the ACC latitude band) and produce a three-dimensional map of geostrophic
216 velocity. We tested the skill of the equivalent-barotropic model in reproducing the hori-
217 zontal velocity field in SOSE: the difference between the model horizontal velocity and the
218 equivalent-barotropic model horizontal velocity is typically less than 20% in the ACC lati-
219 tude band. The approach is less successful north and south of the ACC, where the error can
220 be as large as 100%. The focus of this paper is mainly on dispersion in the ACC latitude
221 band, so the lack of skill in reconstructing the velocity in other regions is less important.
222 Moreover, our goal is to compare different diffusivity estimates using the same velocity field
223 and it is not essential that the field be completely realistic.

224 The eddy diffusivities are computed by advecting tracers and floats along horizontal
225 planes. All diffusivity estimates converge in less than a year, as shown below. We focus on
226 dispersion in the meridional direction, as noted; fluid parcels experience meridional displace-
227 ments of 100-200 km during the first year.

228 The advection of tracers and floats is purely horizontal, i.e. vertical velocities are ignored.
229 The vertical displacements experienced in a year are less than 10 – 100 *m*. Thus the lateral
230 dispersion is hardly affected by the additional vertical displacement. The vertical velocities
231 are nevertheless important because they permit horizontal divergence ($\partial_x u + \partial_y v = -\partial_z w \neq$
232 0). Over long times fluid particles would artificially cluster in regions of convergence and
233 drift away from regions of divergence. We therefore make the velocity field non-divergent
234 by adding a divergent correction, $\Delta\chi$, to the altimetric velocity, i.e. we create a velocity
235 field $\mathbf{u} = \mathbf{u}_g + \Delta\chi$, where \mathbf{u}_g is the geostrophic velocity. The divergent correction further
236 imposes no-normal flow conditions at the continental and north-south boundaries of the

237 computational domain. Periodic conditions are instead enforced at the east-west boundaries.
238 The implementation of the non-divergent correction is described in detail in Marshall et al.
239 (2006). In practice the adjustment leads to very small changes of the geostrophic velocity
240 field and primarily at the boundaries. The estimates of dispersion are hardly sensitive to
241 these corrections, except close to the meridional boundaries.

242 4. Methods

243 We now compare the effective diffusivity of Nakamura (1996) with the single-particle
244 and two-particle diffusivities. We also estimate the diffusivity from the dispersion of tracer
245 patches. As the effective diffusivity measures mixing across the mean flow, we focus solely
246 on the meridional components of the diffusivities, as the mean flow in the region is primarily
247 zonal.

248 We compute the effective diffusivity K_e by advecting a tracer with the equivalent-
249 barotropic velocity field shown in Fig. 1 and using a constant numerical diffusivity κ . The
250 approach is identical to that described in Marshall et al. (2006) and Ferrari and Nikurashin
251 (2010) and the reader is referred to those papers for technical details. This requires careful
252 consideration of the implicit mixing due to the numerical scheme which changes the value
253 of κ to be used in Eq. (4) – we show in Appendix A that the implicit mixing changes with
254 depth and these variations must be taken into account.

The meridional component of the single-particle diffusivity is defined as:

$$K_{1y}(y_0, t) = \frac{1}{2} \frac{d}{dt} \langle (y(t) - y_0)^2 \rangle, \quad (22)$$

255 where $y(t)$ is the meridional position of a particle released at y_0 at $t = 0$ and $\langle \cdot \rangle$ is the average
 256 over all particles. We anticipate the diffusivity will vary with latitude, so we express it as a
 257 function of the initial position, y_0 .

The eddy diffusivity is equal to the integral of the Lagrangian autocorrelation function:

$$K_{1y}(y_0, t) = \int_0^t \langle R_{vv}(y_0, \tau) \rangle d\tau \quad \text{where} \quad R_{vv}(y_0, \tau) = \langle v_L(y_0, \tau) v_L(y_0, 0) \rangle. \quad (23)$$

258 Assuming that the correlation goes to zero after a given period of time and that its integral
 259 is finite, the diffusivity $K_{1y}(y_0, t)$ will asymptote to a constant value (Taylor 1921).

The relative (two-particle) diffusivity is defined analogously:

$$K_{2y}(y_0, \Delta r_0, t) = \frac{1}{4} \frac{d}{dt} \langle (y_i(t) - y_j(t))^2 \rangle, \quad (24)$$

260 where $y_i(t)$ and $y_j(t)$ are the meridional positions of particles i and j released within a
 261 distance Δr_0 of each other (y_0 is the initial condition of either the i or j particle); the
 262 ensemble average is taken over many such pairs. As noted, we use the pre-factor of $1/4$ to
 263 ensure that the two-particle diffusivity asymptotes to the single-particle value at long times
 264 (e.g. Davis (1985)).

265 Trajectories are generated by advecting numerical particles with the velocity field de-
 266 scribed above. The floats are initialized on a regular grid with an initial spacing of 0.2°
 267 and advected for one year using a 4^{th} -order Runge-Kutta scheme. Float positions are saved
 268 every 0.6 days. We impose a periodic boundary condition in the zonal direction, so that
 269 floats leaving the eastern or western boundary re-enter from the opposite side. This ensures
 270 that floats do not leave the computational domain, increasing the length of the particle time
 271 series. A no-normal flow condition is imposed at the northern and southern boundaries
 272 (Sec. 3), so the floats are unable to leave there.

273 A few sample trajectories illustrate the different kinematic regimes in the subtropical
 274 gyre and in the ACC (Fig. 4). In the subtropical gyre, the floats have a random motion.
 275 In the ACC, on the other hand, the particles exhibit coherent, meandering motion. This
 276 meandering plays an important role in the subsequent particle statistics. Note that after
 277 150 days many floats in the ACC would have left the domain from the east, were they
 278 not reintroduced from the west (re-entering floats are not shown in the figure).

Lastly, there is the diffusivity derived from the dispersion of a cloud of tracer. The center
 second moment of the cloud is equivalent to the relative dispersion of all pairs of particles
 in the cloud, as long as the spreading of the tracer cloud is dominated by stirring and not
 by small scale diffusion (LaCasce 2008). When the pair motions become uncorrelated, the
 centered second moment of the cloud increases linearly in time, as for a diffusive process;
 at this point the diffusivity can be obtained from the time derivative of the centered second
 moment (Garrett 1983). The meridional component is defined:

$$K_{ty} = \frac{1}{2} \frac{\partial \sigma_y^2}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} \frac{\iint (y - y_c)^2 C(x, y) dx dy}{\iint C(x, y) dx dy}, \quad (25)$$

where σ_y^2 is the second moment of the tracer concentration in latitude and y_c is the meridional
 position of the baricenter of the tracer patch,

$$y_c = \frac{\iint y C(x, y) dx dy}{\iint C dx dy} \quad (26)$$

279 We advected tracer patches with the same velocity field and numerical scheme described
 280 before. We used two different initial conditions: zonally uniform stripes with a meridional
 281 Gaussian profile and a standard deviation of 0.5° latitude, and isotropic Gaussian blobs with
 282 a 0.5° standard deviation in longitude and latitude. The zonal stripes are most appropriate

283 for estimating meridional diffusivities for the whole Pacific sector, while the blobs are an
284 attempt to reproduce an anthropogenic tracer release in the real ocean.

285 5. Results

286 *a. Initial behavior*

287 We consider first the initial behavior of the effective diffusivity, and test the relation to
288 the relative dispersion given in Eq.(15). To do this, we calculate the relative dispersion of a
289 set of particles deployed along latitude lines and compare that to the effective diffusivity for
290 the same latitude (technically equivalent latitude, see the Appendix).

291 Examples at two latitudes are shown in Fig. 5. The particle diffusivity, defined as
292 $\kappa\langle\Delta r^2\rangle/\langle\Delta r_0^2\rangle$, is shown by the solid and dashed curves and the effective diffusivity by
293 the dash-dot curve. The solid curve is for pairs with an initial separation of 0.1° and the
294 dashed curve for an initial separation of 0.02° .

295 The upper panel shows the results from 45°S , north of the ACC. The three curves track
296 one another closely for the first 10 days and the growth is nearly exponential. Exponential
297 growth is to be expected (e.g. Bennett (1984)), as the initial particle spacing is below the
298 resolution of the velocity field, implying that the velocity spectrum is very steep at these
299 scales (see also Poje et al. (2010)). After the initial period, the effective diffusivity grows
300 slower (and saturates, soon after); the relative dispersion on the other hand continues to
301 increase. Of these two, the dispersion with the smaller initial separation (the dashed curve)
302 increases fastest.

303 The latter effect can be understood as follows. All the curves have the same initial value,
304 equal to κ . Imagine that the exponential growth proceeds to a scale, L ; this could be the
305 deformation radius, for example. At that scale, the particle estimate of the dispersion from
306 (12) is $\kappa L^2 / \Delta r_0^2$. So the smaller the initial spacing, the larger the diffusivity at scale L .
307 In addition, the exponential growth is clearest with the $\Delta r_0 = 0.02^\circ$ spacing, as the pairs
308 experience the interpolated velocity differences for a longer period.

309 The comparison at 59°S , in the ACC latitude band, is not as good. The particle dispersion
310 increases exponentially and the two curves with different initial separations agree with one
311 another at early times, as before. But both grow more rapidly than the effective diffusivity.
312 The latter asymptotes to a value near $500 \text{ m}^2 \text{ s}^{-1}$ by day 25, while the particle-based estimates
313 are much larger.

314 This suggests that the mean flow causes the effective diffusivity to diverge faster from
315 the area-conserving estimate given in (12). This is due in part to the zonal dispersion of
316 the particles by the mean flow. Rhines and Young (1983) show that tracers are rapidly
317 homogenized along mean streamlines, so after a very short transient the tracer contours
318 reflect only the the cross-flow diffusivity. But the relative dispersion used to evaluate L^2 also
319 includes the zonal dispersion, and this is increasing faster due to the mean shear.

320 In summary, the effective diffusivity behaves like the relative dispersion of particle pairs
321 initially. But at later times, when the effective diffusivity asymptotes, the similarity no
322 longer holds. This is due in part to the non-conservation of area between tracer contours
323 and because the effective diffusivity only measures the cross-flow diffusivity. Thus having a
324 mean zonal flow worsens the agreement and it is more sensible to focus on the cross-stream

325 particle diffusivity at later times.

326 *b. Asymptotic behavior; effective diffusivity*

327 Now we turn to the diffusivities at later times. Shown in Fig. 6 is the asymptotic value
328 of the effective diffusivity, K_e , as a function of latitude at the surface and at 1500 m. The
329 asymptotic value is obtained by averaging the diffusivity over the period from 150-365 days,
330 after the measure has leveled off. At the surface the eddy diffusivity is close to $1000 \text{ m}^2\text{s}^{-1}$
331 north of the ACC, while it drops to $500 \text{ m}^2\text{s}^{-1}$ in the core of the current. In contrast, the
332 eddy diffusivity at depth is largest in the core of the ACC where it reaches $1000 \text{ m}^2\text{s}^{-1}$ and
333 is smaller to the north. These patterns are consistent with those of Marshall et al. (2006)
334 and Abernathey et al. (2010).

335 The observed latitudinal and vertical variations in the diffusivity support the notion that
336 the mean flow suppresses mixing (Ferrari and Nikurashin 2010). To test this, we advected the
337 particles with a velocity field with the zonal mean flow (Fig. 3) removed: the ‘eddy’ velocity
338 field introduced in Sec. 3. The resulting estimates of K_e are shown in Fig. 6. Without
339 a mean flow, K_e peaks in the core of the ACC, where the eddy kinetic energy (EKE) is
340 largest, both at the surface and at depth. Such behavior is consistent with mixing length
341 arguments in which the diffusivity is proportional to the eddy length scale times the square
342 root of the kinetic energy (Holloway 1986; Keffer and Holloway 1988). The eddy length
343 scale varies little with depth and latitude, so that $K_e \propto \sqrt{EKE}$ as shown in Klocker et al.
344 (2011). However, the addition of a mean flow breaks the scaling at the surface (at depth the
345 mean flow is very weak). The suppression is more pronounced in the ACC where the mean

346 flow is largest. The suppression of mixing by the mean flow is the focus of a separate paper
347 (Klocker et al. 2011).

348 Fig. 7a shows the full depth-latitude map of K_e for the Pacific sector under study. In the
349 ACC latitude band, between 55°S and 65°S , the diffusivity is suppressed by the mean flow
350 at the surface and has a subsurface maximum around 1500 m where the mean flow becomes
351 weak. To the north, where the mean flows are weaker, the diffusivity is largest at the surface
352 where the EKE is largest.

353 *c. Moment method: dispersion of a tracer patch*

354 Next we consider the diffusivities as estimated from the growth of a patch of tracer.
355 Fig. 6 shows the eddy diffusivity for a series of zonal “stripes” of tracer, released at different
356 latitudes at the surface and at 1500 m . The growth rate of σ_y^2 becomes linear in time after
357 150 days. We estimate the eddy diffusivity K_{ty} as a least square fit of the change in σ_y^2
358 versus time between 150 and 365 days. The K_{ty} estimated from Eq. (25) agree well with
359 the effective diffusivities K_e at all latitudes, despite some discrepancies at the northern and
360 southern boundaries where edge effects suppress the dispersion.

361 Results from the Gaussian patch releases are also shown in Fig. 6. Patches were released
362 at 100°W at different latitudes, at the surface and at 1500 m . The initial transient is much
363 more variable in these simulations, but after 150 days the dispersion is increasing linearly,
364 allowing the eddy diffusivity to be computed over the interval from 150 - 365 days. Agreement
365 with the previous estimates is very good. Again discrepancies are visible at the north and
366 south edges of the domain, and also between $53^\circ - 54^\circ\text{S}$. The latter is the result of the rapid

367 zonal variations in eddy diffusivity at the northern edge of the ACC.

368 Note that while the effective diffusivity and the zonal stripes yield a zonal average of
369 the eddy diffusivity, the Gaussian patches at early times can be used to detect longitudinal
370 variations. Averaging the results from patches released across the domain in the zonal
371 direction yields the same diffusivity as with the stripe (not shown here). Note that agreement
372 between different methods is achieved because we can sample all of the tracer (periodic
373 boundary conditions ensure that the tracer never leaves the domain). Such sampling cannot
374 be done in field experiments.

375 *d. Particle dispersion*

376 Lastly, we consider diffusivities derived from particle trajectories. We begin with the
377 single-particle diffusivity, as this is the measure used most frequently in observational stud-
378 ies. We calculated $K_{1y}(y_0, t)$ using Eq. (22)—estimating the diffusivity by integrating the
379 velocity autocorrelation yielded identical results. We averaged the diffusivities for all parti-
380 cles deployed in regions 25° in longitude (the width of the domain) by 2° in latitude. This bin
381 size is substantially larger than the typical eddy scale in the Southern Ocean of $O(100)$ km.
382 As the mean velocity is purely zonal, we do not need to separate mean and eddy velocities
383 in the meridional direction.

384 Following Davis (1991), we treat every point at which a float position is saved as a starting
385 position for a new trajectory, substantially increasing the number of pseudo-trajectories. The
386 method is illustrated graphically in Fig. 8: every orange dot shows a point at which a float
387 position is saved and it is treated as the starting position for a new trajectory. Both forward

388 and backward trajectories are used. The technique is described by Griesel et al. (2010) and
 389 represents a slight modification of the technique used in Colin de Verdiere (1983) and Poulain
 390 and Niiler (1989). The single-particle eddy diffusivities converge to a constant within half a
 391 year in the subtropical gyres and sooner elsewhere. Hence we consider all trajectories with
 392 a minimum length of 160 days and estimate the diffusivity as an average over the last 20
 393 days of all available pseudo-trajectories. The averaging window is different from tracer-based
 394 estimates because particles drift too far from their release latitude after 160 days to allow
 395 local in latitude estimates of diffusivities.

396 Examples of single-particle diffusivities are shown in Fig. 9 for (a) particles released in a
 397 2° bin in the subtropical gyre ($45^\circ S \pm 1^\circ$) and (b) particles released in a 2° bin in the ACC
 398 ($59^\circ S \pm 1^\circ$). In line with the previous calculations of effective diffusivity, we use both the full
 399 velocity field (solid lines) and the eddy velocity field (dashed lines). These bins were chosen
 400 because they are representative of regions with (a) weak mean flow and (b) strong mean
 401 flow (see Fig. 4 for the different behavior of trajectories in these regions). The shaded areas
 402 around the single-particle diffusivities show the 2σ errors estimated using a bootstrapping
 403 technique.¹ For comparison, the corresponding Lagrangian autocorrelation functions, R_{vv} ,
 404 are shown in Fig. 10.

405 In the northern-most domain, the diffusivities based on particles advected by the eddy
 406 velocity field asymptote to a constant value of $\simeq 2000 \text{ m}^2\text{s}^{-1}$ after 20 days (Fig. 9a). The

¹Bootstrapping consists of first sub-sampling the approximately 1000 floats deployed in every bin 100 times, allowing for duplicates, and then using each of these subsets of floats to calculate a diffusivity. The 2σ errors are the standard deviations of the K 's estimated from the 100 realizations. This is the same approach to calculate confidence intervals used by Griesel et al. (2010).

407 diffusivity for the particles advected with the full velocity behaves similarly initially, but
408 only reaches a value of $\simeq 1700 \text{ m}^2\text{s}^{-1}$; thereafter it decreases slowly. This decrease is related
409 to a negative lobe in the velocity autocorrelation, R_{vv} (Fig. 10a).

410 The difference between the diffusivities based on the eddy and mean velocity fields is more
411 evident in the southern domain (Fig. 9b). The velocity autocorrelations for the experiments
412 with and without a mean flow have negative lobes, but the addition of a mean flow results
413 in a much more pronounced lobe. As a result, both diffusivities decrease after first reaching
414 a maximum, at roughly $4000 \text{ m}^2\text{s}^{-1}$. But, the diffusivity for the mean velocity falls much
415 more, eventually asymptoting to a value between $500\text{-}1000 \text{ m}^2\text{s}^{-1}$. The diffusivity for the
416 eddy velocity is 2-3 times larger. The difference is again linked to a negative lobe in the
417 autocorrelation, which is most pronounced with the mean flow.

418 The link between mean flows and negative lobes is explained in detail in a separate paper
419 (Klocker et al. 2011), but it can be easily understood. The mean flow in the ACC is directed
420 eastward flowing at 10 cm s^{-1} , while the eddies drift much slower at $1 - 2 \text{ cm s}^{-1}$. Hence
421 the mean flows advects particles around nearly stationary eddies, resulting in back and forth
422 meandering motions. The negative lobe in the velocity autocorrelation is the signature of
423 trajectories that meander back and forth from their release latitude. These meanders do
424 not contribute any irreversible mixing and must not be included in the calculation of the
425 diffusivity. This is achieved by integrating the velocity autocorrelation past the negative
426 lobe.

427 Now we use Eq. (22) to compute meridional profiles of eddy diffusivities for velocities at
428 the surface and at 1500 m depth. We employ the 25° longitude \times 2° latitude bins described
429 above, but with one bin every 1° of latitude (i.e. the bins overlap). Again, the deep velocity

430 field is obtained by multiplying the surface velocity by the equivalent-barotropic structure
431 function $\mu(1500\text{ m})$ (see Fig. 2).

432 The results are plotted in Fig. 11, along with the effective diffusivities found previously.
433 The estimates agree within the errors at all latitudes, at both depths. Both estimates capture
434 the suppression by the strong ACC flow at the surface. We therefore can answer positively
435 to the question we posed in the introduction. For the flow regimes observed in the Southern
436 Ocean, estimates of eddy diffusivity based on tracers and particles are equivalent. This is
437 very encouraging, because numerical models need the eddy diffusivity that describes the rate
438 at which tracers are homogenized, while direct estimates of diffusivity in the ocean rely on
439 float dispersion (especially below the surface). Agreement between the two estimates means
440 that we can compare models and observations.

441 In a number of previous studies, the single-particle diffusivity was estimated either from
442 the maximum value achieved by K_{1y} at short times or, equivalently, by integrating to the
443 first zero crossing of the velocity autocorrelation (e.g. Freeland et al. 1975; Poulain and Niiler
444 1989). The reasoning is that the error in the diffusivity grows in time, so that the estimates
445 at longer lags are less certain. Alternatively some studies fit the velocity autocorrelation
446 with an exponential curve and estimate the asymptotic value from that (e.g. Garraffo et al.
447 2001; Sallée et al. 2008). Both methods depend on the initial behavior of the diffusivity and
448 miss the contribution from the negative lobe.

449 Integrating our velocity autocorrelations to the first zero crossing yields much larger
450 diffusivities (not shown). Moreover, there was little difference between the diffusivities found
451 using the full velocity and the eddy velocity. Thus using the first zero crossing method
452 effectively eliminates the suppression of eddy diffusivity by the mean flow.

453 How many particles are required to obtain convergent estimates, i.e. to fully resolve
454 the negative lobes? In the calculations above, we used approximately 1000 floats per bin, a
455 quantity which is unrealistic by observational standards. From theory (e.g. Davis (1991)), the
456 error in the diffusivity is expected to decrease as $n^{-1/2}$. We checked this by recalculating the
457 diffusivities using smaller numbers of particles, both for particles released in 2° bins in the
458 subtropical gyre ($45^\circ S \pm 1^\circ$) and for particles released in 2° bins in the ACC ($45^\circ S \pm 1^\circ$). Errors
459 were calculated by the same bootstrapping technique used for Fig. 9. The relative errors
460 defined as $2\sigma/K_{1y}$ are shown in Fig. 12, where σ is the standard deviation calculated from
461 bootstrapping, plotted against the number of particles used. The results are consistent with a
462 decrease proportional to $n^{-1/2}$ (although the decrease is somewhat faster with large numbers
463 of particles). Thus the errors for realistic deployments decrease less than linearly with the
464 number of particles with daunting implications for observational programs as discussed in
465 the next section.

466 Lastly, we consider the diffusivities derived from pairs of particles. These are calculated
467 as with the single-particle diffusivities, but now at every point at which a float trajectory is
468 saved we find all other trajectories which are within a specified radius. We then calculate
469 two-particle diffusivities using the meridional distance between those floats according to
470 Eq. (24). This method thus involves using “chance pairs”, or pairs of particles not deployed
471 together (Morel and Larcheveque 1974; LaCasce 2008). We use particle trajectories with a
472 minimum length of 60 days and average over the last 20 days. The resulting two-particle
473 diffusivities, together with the single-particle diffusivities from the same bin, are shown in
474 Fig. 13.

475 As noted previously, the two-particle diffusivity (defined with a pre-factor of 1/4) should

476 asymptote to the single-particle diffusivity after the pair velocities become decorrelated.
477 This is the case (Fig. 13). However, the two-particle diffusivity differs at early times. This
478 can be seen in Fig. 14 in which we changed the initial spacing of the particle pairs between
479 0.2° and 0.8° . At 45°S (Fig. 14a), the closer the initial pairs, the more slowly the diffusivity
480 asymptotes to the final value. This is because it takes longer for pairs to separate to a
481 distance at which their motion is no longer correlated. The effect is more dramatic at 59°S
482 where there is a strong mean flow (Fig. 14b); we see that the overshoot is greatly diminished
483 for closer pairs. For the smallest initial separation of 0.2° , there is barely any overshoot at
484 all.

485 The lack of overshoot in K_{2y} is that relative dispersion, unlike single-particle dispersion,
486 is Galilean invariant. Thus a pair of particles moves with the meandering flow (Fig. 7), and
487 the diffusivity measures only their separation. The closer the particles are initially, the more
488 faithfully they track one another through the meanders.

489 The pair diffusivities as a function of latitude are shown in red in Fig. 11. We see that
490 the estimates are consistent with both those from single particles and with the effective
491 diffusivities. Thus all three methods yield consistent values.

492 6. Summary and discussion

493 We have calculated lateral diffusivities for synthetic particles advected with a proxy
494 velocity field. The latter derives from sea surface height measurements from the South
495 Pacific, in a region west of Drake Passage. The geostrophic surface velocities were adjusted
496 to yield a non-divergent field, and the velocities were projected downward in the water

497 column assuming an equivalent-barotropic structure.

498 We calculated the diffusivities in three ways: 1) by using a tracer to estimate the “effective
499 diffusivity” (Nakamura 1996), 2) from the second moment of the tracer distribution and 3)
500 via single and two-particle diffusivities. The effective diffusivity measures the cross-stream
501 component, so we focused on the meridional diffusivities for the others, as the mean flow
502 (the ACC) is zonally-oriented in the region.

503 We show that the effective diffusivity at early times is proportional to the relative dis-
504 persion (between pairs of particles). At later times, the relation breaks down as small scale
505 mixing violates area conservation between tracer isolines. However, at the later times the
506 effective diffusivity agrees well with the meridional diffusivity estimated both from the tracer
507 cloud and from the particles. While our analysis focussed on the Southern Ocean, the agree-
508 ment is expected to hold in other parts of the world’s oceans.

509 The effective diffusivity is most useful for satellite-derived velocity fields, such as this, or
510 for model fields; it is probably not practical with *in situ* tracer deployments. The moment
511 method has been used previously with deployed tracer (e.g. Sundermeyer and Price (1998)),
512 but obtaining sufficient sampling is also an issue. Thus float/drifter based studies are still
513 the most desirable. The present results demonstrate that the float-derived estimates are
514 consistent with the others, as long as they are calculated correctly.

515 A critical issue, seen clearly here, is that a mean flow suppresses the cross-stream dif-
516 fusivities (e.g. Ferrari and Nikurashin (2010); Naveira Garabato et al. (2011)). One finds
517 that the particle-based diffusivity reaches a maximum value before decreasing to a lower
518 asymptotic limit. This reduction, often seen in observations, reflects a negative lobe in the
519 velocity autocorrelation. Physically, this corresponds to “meandering” motion — particles

520 being advected first across the mean flow and then returning some distance back toward their
521 starting location. Thus the initial maximum is a transient advective feature, and does not
522 reflect the true mixing. Indeed, a similar overshoot is not seen with the effective diffusivity.
523 Given this, using the maximum diffusivity, or equivalently integrating the autocorrelation to
524 the first zero crossing, yields artificially large diffusivities.

525 There are two ways around this. One is to average the diffusivity after the initial tran-
526 sient phase, i.e. after it has reached the lower asymptotic limit. This means choosing an
527 “intermediate” averaging period, after the initial transient but before the measurement er-
528 rors have grown too large (e.g. Koszalka and LaCasce (2010)). The second is to use the
529 two-particle diffusivity, which relates to the separation between pairs of particles. As both
530 particles in the pair experience the initial meander, the relative diffusivity is devoid of the
531 initial maximum provided the initial pair separation is small enough. However, this requires
532 having a sufficient number of pairs to obtain reliable statistics.

533 A motivation for the present work was to see how many floats are required to distinguish
534 lateral and vertical variations in eddy diffusivities in the real ocean. This is a relevant point
535 for field campaigns such as the Diapycnal and Isopycnal Mixing Experiment in the Southern
536 Ocean (DIMES). In DIMES, both floats and an anthropogenic tracer have been released to
537 infer eddy diffusivities. The tracer was released at $58^{\circ}\text{S}/107^{\circ}\text{W}$ between the Polar Front
538 and the Subantarctic Front on the 27.9 kg m^{-3} neutral density surface, which is at a depth
539 of approx. 1500 m (see Ledwell et al. (2011) for details on the tracer release). Due to the
540 filamentation and the sparse sampling of the tracer in time (i.e. every 6-12 months) and in
541 space (i.e. only part of the tracer is found using shipboard CTDs due to coarse sampling),
542 it is impossible to calculate eddy diffusivities from the tracer using the methods described

543 above. In addition to the tracer, 150 isopycnal RAFOS floats have been released between
544 54°S - $60^{\circ}\text{S}/105^{\circ}\text{W}$ on two isopycnal surfaces: 100 on the same neutral density surface as the
545 tracer and the other 50 on the 27.2 kg m^{-3} neutral density surface (at a depth of approx.
546 500 m). These two density surfaces were chosen to see if eddy mixing is indeed suppressed
547 in the shallow ACC relative to eddy diffusivities at mid-depth, as found in our study (e.g.
548 Fig. 7).

549 To estimate the number of floats necessary to assess this vertical variation in the real
550 ocean, we advect synthetic floats using the same velocity field as above, with the only
551 difference that floats exiting the domain are not allowed to reenter (in the case of DIMES
552 these floats are lost past Drake passage). The velocity field for both surfaces is derived
553 by scaling the surface geostrophic velocity field according to the equivalent-barotropic scale
554 factors shown in Fig. 2 (i.e. $\mu(500 \text{ m}) \sim 0.9$ and $\mu(1500 \text{ m}) \sim 0.4$). The diffusivities are
555 computed for a bin in the ACC, which extends from 54° to 62°S , and a bin north of the ACC,
556 which extends from 42° to 50°S - both bins span the whole domain in longitude. These bins
557 are larger than the bins used above to increase the Lagrangian statistics per bin as needs to be
558 done in real experiments. We consider a bin to the north to the ACC, even though no floats
559 in DIMES were released in this region, to see if one can distinguish the eddy diffusivities
560 in the ACC and to the north using float numbers like those in DIMES. On both surfaces
561 floats are initialized every 0.2° as for previous calculations, giving approximately 5000 floats
562 per bin. The error bars are the 5th and 95th-percentiles calculated using a bootstrapping
563 technique with 100 subsamples. We use subsample sizes of (a) 50 floats, (b) 150 floats and
564 (c) 1000 floats. The error estimates are shown as gray shaded areas in Fig. 15.

565 The error bars in Fig. 15 show that with 50 or 150 floats, it is not possible to detect

566 differences in the eddy diffusivities between the two latitudes/depth levels with any signif-
567 icance. That is, we cannot resolve the lateral and vertical variations of the diffusivities in
568 the DIMES region. With 1000 floats however, one can resolve with statistical significance
569 the latitudinal and vertical variations in the diffusivity. This suggests that the 150 floats
570 deployed in DIMES are not sufficient to resolve the lateral and vertical variations, if they
571 are present. In addition note that the DIMES floats are not deployed homogeneously as in
572 our calculations. Hence the errors shown here are a lower limit for the errors expected from
573 float releases in DIMES.

574 We conclude that large errors in float-based estimates of diffusivities are endemic to
575 field campaigns, like DIMES, because it is not conceivable to deploy $O(1000)$ floats. Thus
576 float-derived estimates should be used in conjunction with other information to quantify
577 the absolute values and the vertical and lateral structure of eddy diffusivities. This could
578 come from the release of a tracer patch, such as in DIMES, satellite-derived geostrophic
579 velocity fields, high-resolution numerical models and from theoretical models. The challenge
580 for the future is to learn how to combine such diverse data sets to obtain statistically robust
581 estimates of diffusivities.

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APPENDIX

To advect a numerical tracer, we use the offline capabilities of the Massachusetts Institute of Technology general circulation model (MITgcm, Marshall et al. (1997)) with the non-divergent velocity field described above and solve the tracer advection-diffusion equation

$$\frac{\partial}{\partial t}C + \mathbf{u} \cdot \nabla C = \kappa \nabla^2 C, \quad (\text{A1})$$

where C is the tracer concentration and κ is the explicit numerical diffusivity. For the tracer advection we use an Adams-Bashforth time-stepping scheme and a centered 2^{nd} order advection scheme to advect the tracer for one year. The domain is zonally periodic with no-flux boundary conditions imposed at the northern and southern boundaries. The initial tracer concentration increases linearly with latitude, with values of zero at the southern boundary and values of one at the northern boundaries. As shown by Shuckburgh et al. (2009) the calculations of effective diffusivities are not very sensitive to the tracer initial condition.

The most problematic part in calculating effective diffusivities is the total numerical diffusivity. For an artificial tracer advected by a numerical model this is the sum of the explicit and implicit numerical diffusivities. Without knowing the total numerical diffusivity, but assuming that it is constant, one can determine the spatial structure of K_e , but to compute magnitudes of effective diffusivities one needs an estimate of the total numerical diffusivity (Shuckburgh et al. 2009). Marshall et al. (2006) show that the estimate of K_e becomes independent of numerical diffusivity only for large Péclet number, with $Pe = UL_{eddy}/\kappa_{total}$, where U is the characteristic scale of the velocity and L_{eddy} is an eddy length scale, and κ_{total} is the total numerical diffusivity. This would suggest to use a small κ for our calcula-

605 tions. On the other hand using a κ which is too small introduces implicit numerical diffusion
606 generated by noise on the grid scale which acts to increase κ_{total} . For every vertical level
607 we choose an explicit numerical diffusivity small enough to ensure that the estimate of K_e
608 is independent of the numerical diffusivity, but large enough to keep the implicit numerical
609 diffusivity small. To estimate the total numerical diffusivity inherent in the advection of the
610 artificial tracer we use the tracer variance equation,

$$\frac{1}{2} \frac{\partial \langle C^2 \rangle}{\partial t} = -\kappa \langle |\nabla C|^2 \rangle, \quad (\text{A2})$$

611 where $\langle \cdot \rangle$ indicates the average over the domain. κ_{total} is the total numerical diffusivity
612 necessary to explain the decay of the observed tracer variance $\langle C^2 \rangle$.

613 K_e is estimated by advecting the tracer with an explicit numerical diffusivity of $10 \text{ m}^2 \text{ s}^{-1}$.
614 Using Eq.(A2) gives us the numerical background diffusivity inherent in the numerical ad-
615 vection code, κ_{total} , which varies from $30 \text{ m}^2 \text{ s}^{-1}$ at the surface to $11 \text{ m}^2 \text{ s}^{-1}$ at depth. Note
616 that this variation of κ_{total} with depth can easily be accounted for in the two-dimensional
617 calculations performed in this study but might lead to serious issues in three-dimensional
618 calculations in which one value for κ_{total} is used everywhere.

619 We then use Eq.(5) to calculate K_e , where $L_0(y_{eq})$ is the width of the domain at the
620 equivalent latitude y_{eq} and L_{eq}^2 can be written as (see Shuckburgh and Haynes (2003) for a
621 derivation).

$$L_{eq}^2 = \left[\frac{1}{\left(\frac{\partial C}{\partial A}\right)} \frac{\partial}{\partial A} \int_A |\nabla C| dA \right]^2, \quad (\text{A3})$$

622 where C is the tracer concentration and A is the area between tracer contours.

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758 List of Figures

- 759 1 Snapshot of sea surface height [m] as estimated from the combined processing
760 of the TOPEX, ERS-1 and ERS-2 altimetries for an ACC sector upstream of
761 Drake Passage between 103° - $78^{\circ}W$ and 30° - $66^{\circ}S$. 45
- 762 2 The equivalent-barotropic structure function $\mu(z) = \langle |\mathbf{u}(x, y, z)| / |\mathbf{u}(x, y, 300)| \rangle$
763 as a function of depth calculated from SOSE. Note that the velocities are
764 scaled relative to 300 m to avoid Ekman fluxes in the mixed layer. 46
- 765 3 Zonal mean surface velocity for the domain shown in Fig. 1. 47
- 766 4 Numerical trajectories for two clusters of approximately 100 particles released
767 between $103^{\circ}W - 103.5^{\circ}W$ and $40^{\circ}S - 46^{\circ}S$ (upper cluster) and $103^{\circ}W -$
768 $103.5^{\circ}W$ and $56^{\circ}S - 62^{\circ}S$ (lower cluster). Particle trajectories are shown for
769 the first 150 days. When particles leave at the eastern or western boundary,
770 they are reintroduced from the opposite side for the eddy diffusivity calcula-
771 tions. Particles reentering the domain from the west are not shown here to
772 avoid cluttering the figure. 48
- 773 5 The effective diffusivity K_e defined in Eq. (5) at early times (dotted-dashed
774 lines) compared to the relative dispersion of pairs of particles released along
775 the same latitude line for two different sets of particles, one set released at
776 0.02° apart (dashed lines) and the other set released at 0.1° apart (solid lines).
777 The upper panel is for $45^{\circ}S$ and the lower for $59^{\circ}S$ computed as the equivalent
778 latitude for K_e (see Appendix) and as the latitude at which particles are released. 49

779 6 The effective diffusivity K_e (solid blue lines) defined in Eq. (5), the tracer-
780 based diffusivity K_{ty} defined in Eq. (25) from the spreading of tracer stripes
781 (solid black lines) and tracer patches (solid red lines). Also shown is the
782 effective diffusivity for a tracer advected with the eddy velocity only (dashed
783 blue lines). (a) Diffusivities computed with the surface geostrophic velocity
784 field. (b) Diffusivities calculated with the equivalent-barotropic velocity field
785 rescaled for ~ 1500 m. 50

786 7 Latitude-depth maps of (a) the effective diffusivity K_e (Eq. (5)) and (b) the
787 single-particle diffusivity K_{1y} (Eq. (22)) [m^2s^{-1}]. Eddy diffusivities were cal-
788 culated at 10 depth levels, using the equivalent-barotropic velocity field de-
789 scribed in Sec. 3. 51

790 8 The rectangular box sketches a bin sample used for calculating particle diffu-
791 sivities from two example trajectories. The orange dots are points at which
792 the trajectory positions are saved. Absolute/relative dispersion are calculated
793 using each of these points as a deployment location, i.e. each of these points
794 is the start of a pseudotrajectory. 52

795 9 The single-particle diffusivity, K_{1y} (Eq. (22)) for particles released in 2° bins
796 in (a) the subtropical gyre ($45^\circ S \pm 1^\circ$) and (b) in the ACC ($59^\circ S \pm 1^\circ$), both
797 for the full velocity field (solid lines) and for the eddy velocity field (dashed
798 lines). The shaded error bars show 2σ errors estimated by a bootstrapping
799 technique. 53

- 800 10 The Lagrangian velocity autocorrelations, R_{vv} , corresponding to the diffusiv-
801 ities shown in Fig.9, for (a) particles released in a 2° bin in the subtropical
802 gyre ($45^\circ S \pm 1^\circ$) and (b) particles released in a 2° bin in the ACC ($59^\circ S \pm 1^\circ$),
803 both for the full velocity field (solid lines) and the eddy velocity field (dashed
804 lines). 54
- 805 11 The effective diffusivity K_e (blue lines) as defined in Eq. (5), the single-particle
806 diffusivity K_{1y} (solid black line) as defined in Eq. (22) and the two-particle
807 diffusivity K_{2y} as defined in Eq. (24) for tracers and particles advected with
808 (a) the surface geostrophic velocity and (b) the velocity at ~ 1500 m, i.e.
809 the surface geostrophic velocity multiplied by 0.4 according to the equivalent-
810 barotropic model for that depth (see Fig. 2). The dashed blue lines are the
811 effective diffusivity K_e for a tracer advected with the eddy velocity only. 55
- 812 12 Relative error $2\sigma/K_{1y}$ of the single-particle diffusivity K_{1y} (Eq. (22)), where σ
813 is the standard deviation calculated from bootstrapping with 100 sub-samples
814 of n particles. The errors are shown for particles released in a 2° bin in the
815 subtropical gyre ($45^\circ S \pm 1^\circ$) (blue) and particles released in a 2° bin in the
816 ACC ($59^\circ S \pm 1^\circ$) (red) for the surface (solid lines) and deep surface (dashed
817 lines). The solid black line shows a line with slope of $n^{-1/2}$. 56
- 818 13 The time evolution of the single-particle diffusivities K_{1y} (Eq. (22), solid line)
819 and the relative diffusivities K_{2y} (Eq. (24), dashed line) for (a) particles re-
820 leased in a 2° bin in the subtropical gyre ($45^\circ S \pm 1^\circ$) and (b) particles released
821 in a 2° bin in the ACC ($59^\circ S \pm 1^\circ$). Shading shows uncertainties estimated
822 by a bootstrapping technique. 57

- 823 14 The time evolution of the two-particle diffusivities K_{2y} (Eq. (24)) for different
824 initial separations for (a) particles released in a 2° bin in the subtropical gyre
825 ($45^\circ S \pm 1^\circ$) and (b) particles released in a 2° bin in the ACC ($59^\circ S \pm 1^\circ$). 58
- 826 15 The single-particle diffusivity K_{1y} (Eq. (22)) for particles released in a bin
827 north of the ACC (a) at 500 m and (b) at 1500 m (the bin extends from
828 44° to $48^\circ S$) and for particles released in a bin in the ACC (c) at 500 m and
829 (d) at 1500 m (the bin extends from 56° to $60^\circ S$). Grey shading shows the
830 5th and 95th percentiles determined by a bootstrapping technique with 100
831 sub-samples using 50 floats (dark grey), 150 floats (medium grey) and 1000
832 floats (light grey). 59

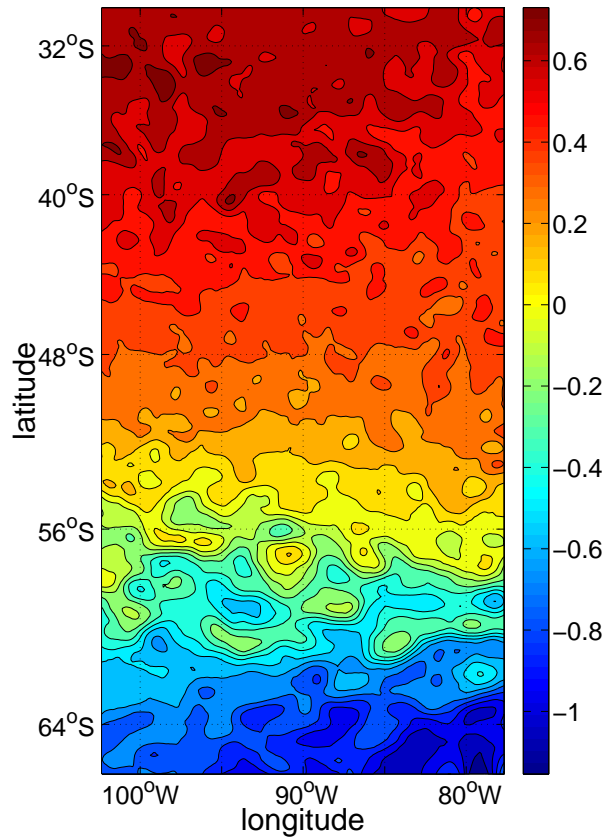


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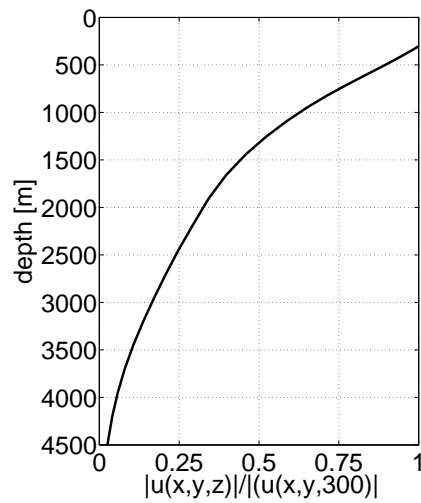


Figure 2: The equivalent-barotropic structure function $\mu(z) = \langle |u(x, y, z)| / |u(x, y, 300)| \rangle$ as a function of depth calculated from SOSE. Note that the velocities are scaled relative to 300 m to avoid Ekman fluxes in the mixed layer.

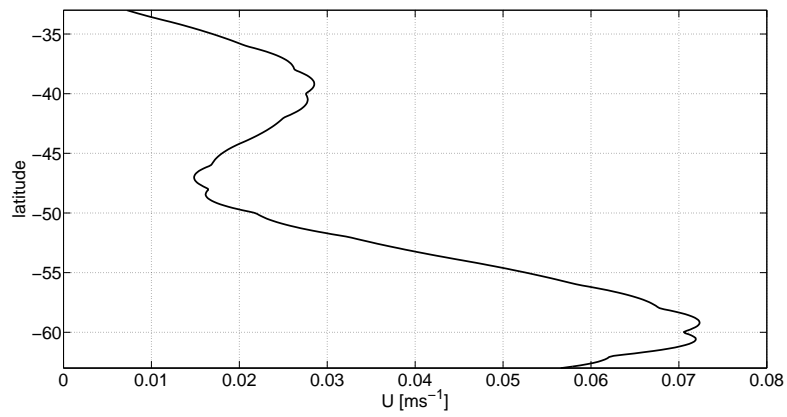


Figure 3: Zonal mean surface velocity for the domain shown in Fig. 1.

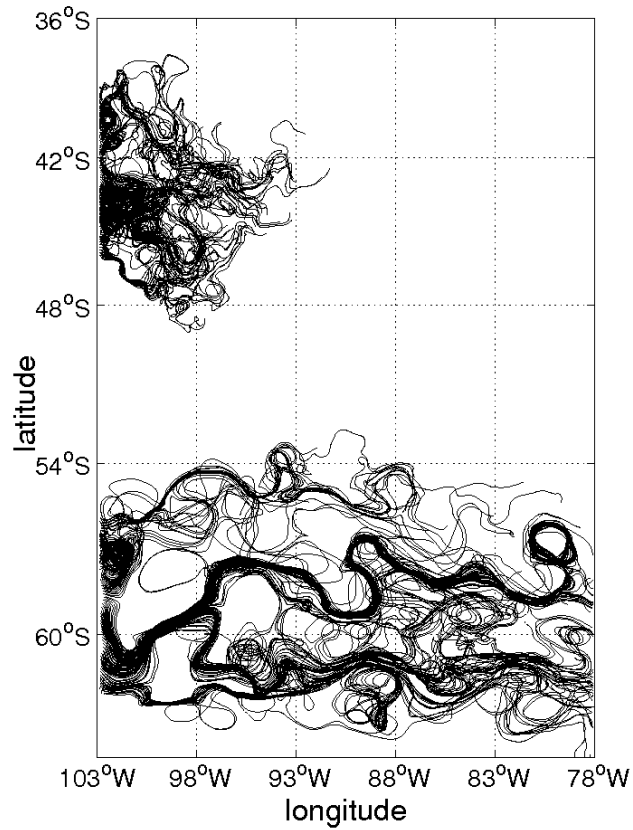


Figure 4: Numerical trajectories for two clusters of approximately 100 particles released between $103^{\circ}W$ - $103.5^{\circ}W$ and $40^{\circ}S$ - $46^{\circ}S$ (upper cluster) and $103^{\circ}W$ - $103.5^{\circ}W$ and $56^{\circ}S$ - $62^{\circ}S$ (lower cluster). Particle trajectories are shown for the first 150 days. When particles leave at the eastern or western boundary, they are reintroduced from the opposite side for the eddy diffusivity calculations. Particles reentering the domain from the west are not shown here to avoid cluttering the figure.

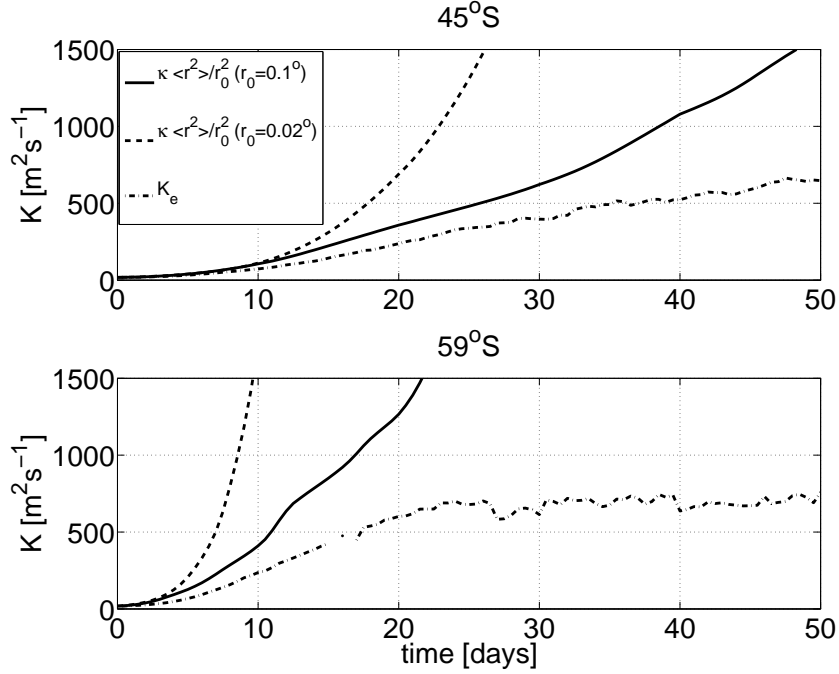


Figure 5: The effective diffusivity K_e defined in Eq. (5) at early times (dotted-dashed lines) compared to the relative dispersion of pairs of particles released along the same latitude line for two different sets of particles, one set released at 0.02° apart (dashed lines) and the other set released at 0.1° apart (solid lines). The upper panel is for 45°S and the lower for 59°S computed as the equivalent latitude for K_e (see Appendix) and as the latitude at which particles are released.

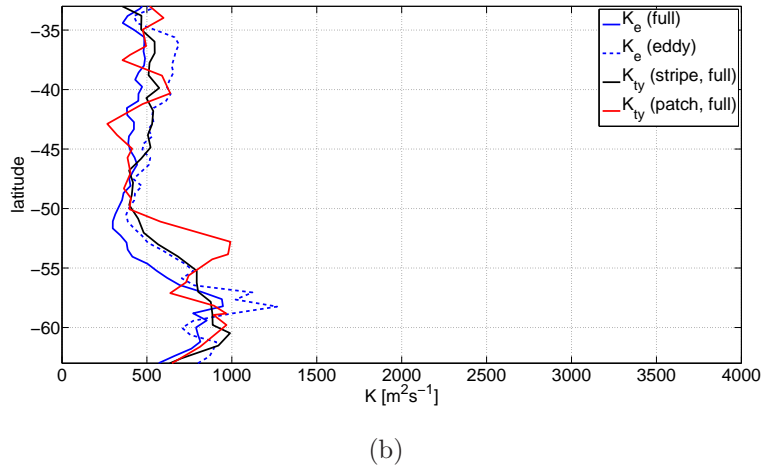
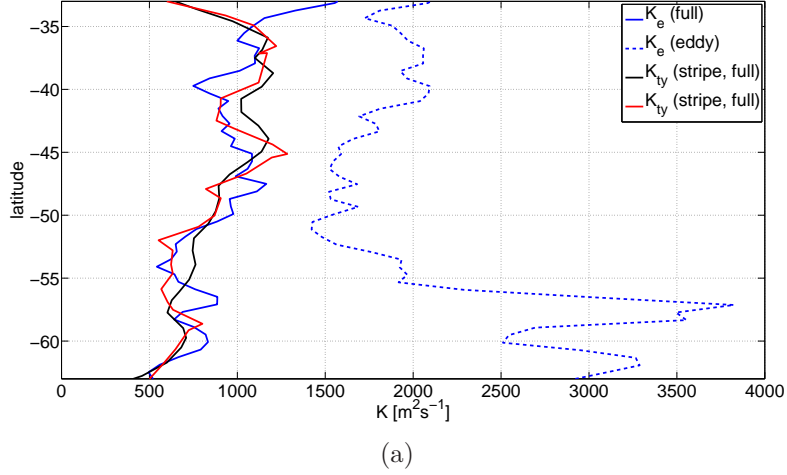
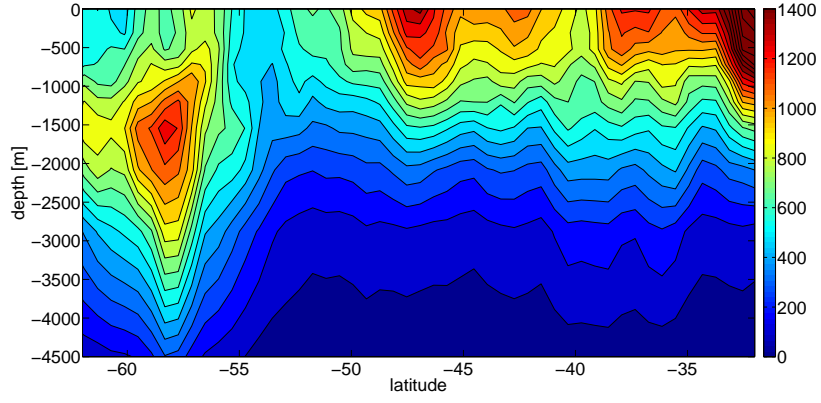
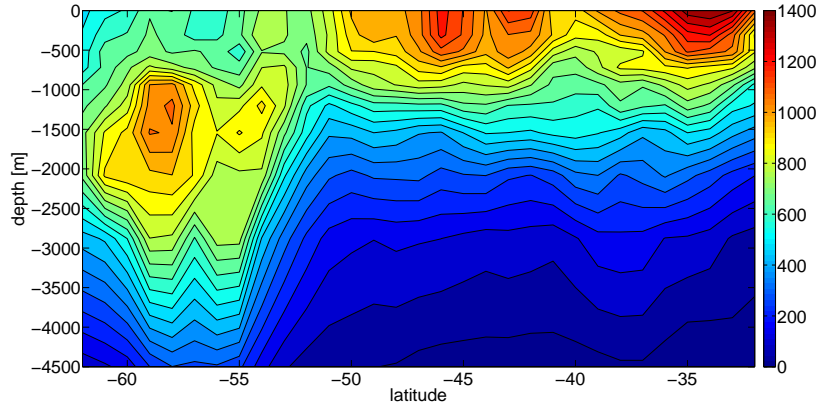


Figure 6: The effective diffusivity K_e (solid blue lines) defined in Eq. (5), the tracer-based diffusivity K_{ty} defined in Eq. (25) from the spreading of tracer stripes (solid black lines) and tracer patches (solid red lines). Also shown is the effective diffusivity for a tracer advected with the eddy velocity only (dashed blue lines). (a) Diffusivities computed with the surface geostrophic velocity field. (b) Diffusivities calculated with the equivalent-barotropic velocity field rescaled for $\sim 1500 m$.



(a)



(b)

Figure 7: Latitude-depth maps of (a) the effective diffusivity K_e (Eq. (5)) and (b) the single-particle diffusivity K_{1y} (Eq. (22)) [m^2s^{-1}]. Eddy diffusivities were calculated at 10 depth levels, using the equivalent-barotropic velocity field described in Sec. 3.

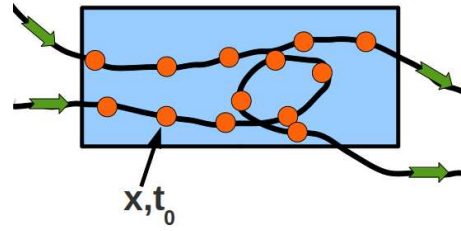
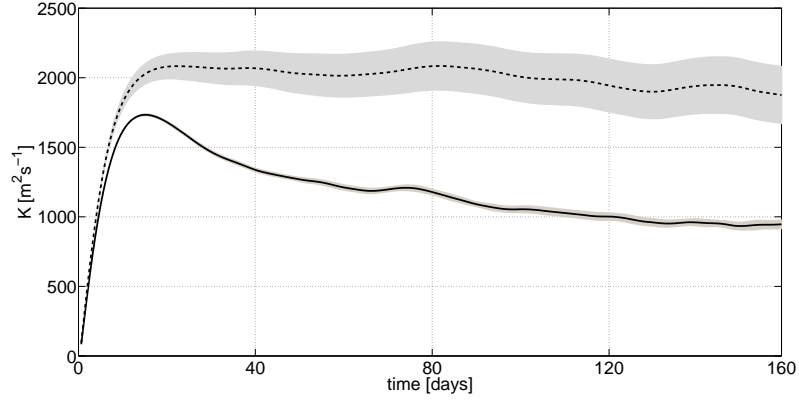
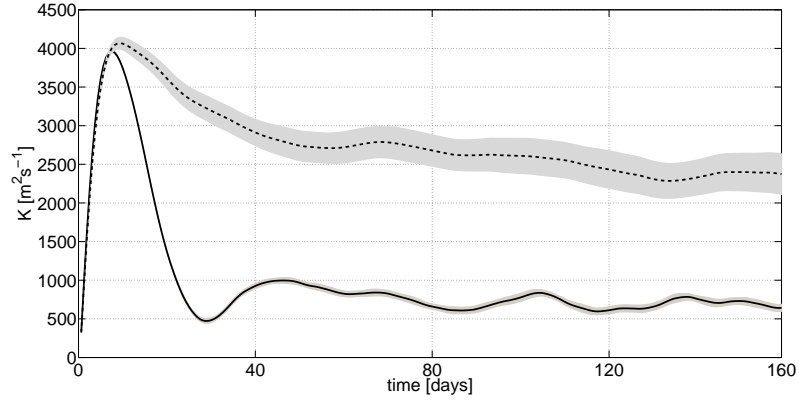


Figure 8: The rectangular box sketches a bin sample used for calculating particle diffusivities from two example trajectories. The orange dots are points at which the trajectory positions are saved. Absolute/relative dispersion are calculated using each of these points as a deployment location, i.e. each of these points is the start of a pseudotrajectory.

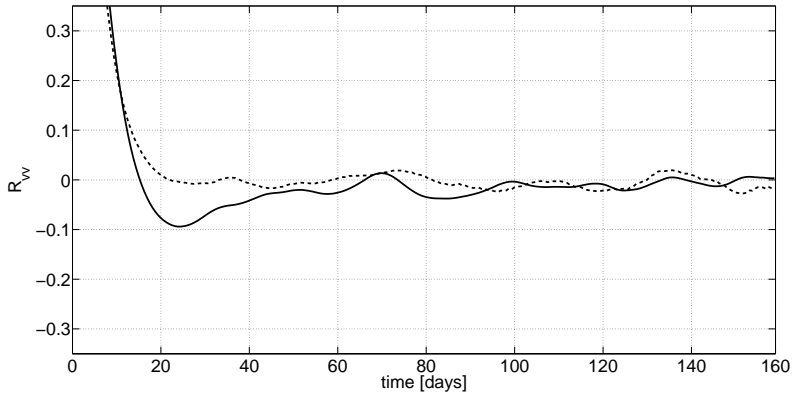


(a)

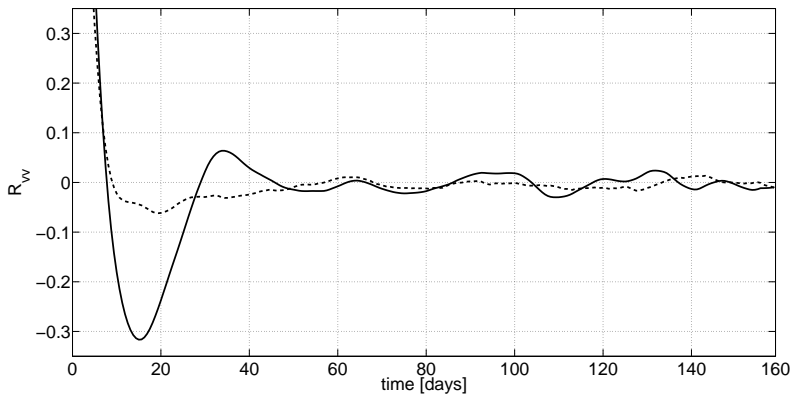


(b)

Figure 9: The single-particle diffusivity, K_{1y} (Eq. (22)) for particles released in 2° bins in (a) the subtropical gyre ($45^\circ S \pm 1^\circ$) and (b) in the ACC ($59^\circ S \pm 1^\circ$), both for the full velocity field (solid lines) and for the eddy velocity field (dashed lines). The shaded error bars show 2σ errors estimated by a bootstrapping technique.

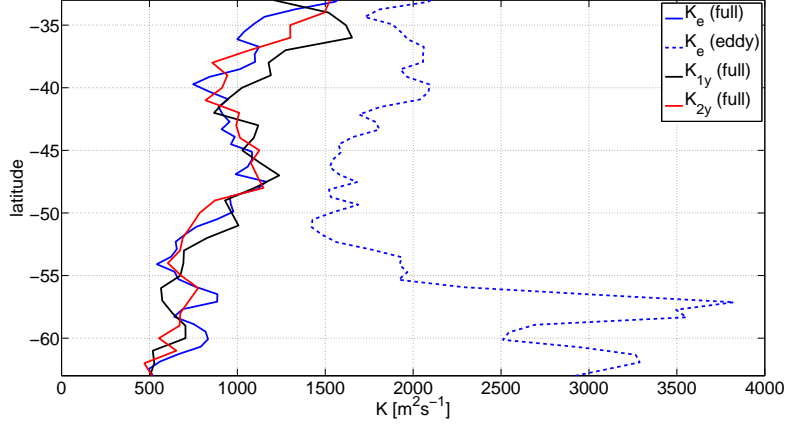


(a)

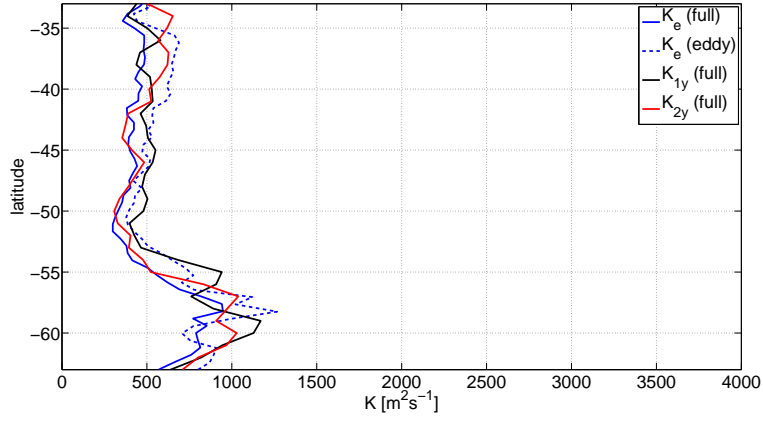


(b)

Figure 10: The Lagrangian velocity autocorrelations, R_{vv} , corresponding to the diffusivities shown in Fig.9, for (a) particles released in a 2° bin in the subtropical gyre ($45^\circ S \pm 1^\circ$) and (b) particles released in a 2° bin in the ACC ($59^\circ S \pm 1^\circ$), both for the full velocity field (solid lines) and the eddy velocity field (dashed lines).



(a)



(b)

Figure 11: The effective diffusivity K_e (blue lines) as defined in Eq. (5), the single-particle diffusivity K_{1y} (solid black line) as defined in Eq. (22) and the two-particle diffusivity K_{2y} as defined in Eq. (24) for tracers and particles advected with (a) the surface geostrophic velocity and (b) the velocity at ~ 1500 m, i.e. the surface geostrophic velocity multiplied by 0.4 according to the equivalent-barotropic model for that depth (see Fig. 2). The dashed blue lines are the effective diffusivity K_e for a tracer advected with the eddy velocity only.

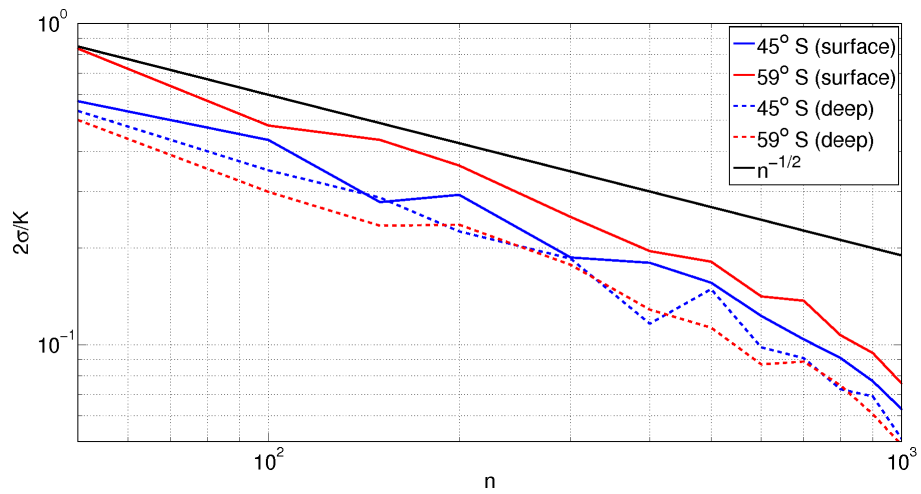
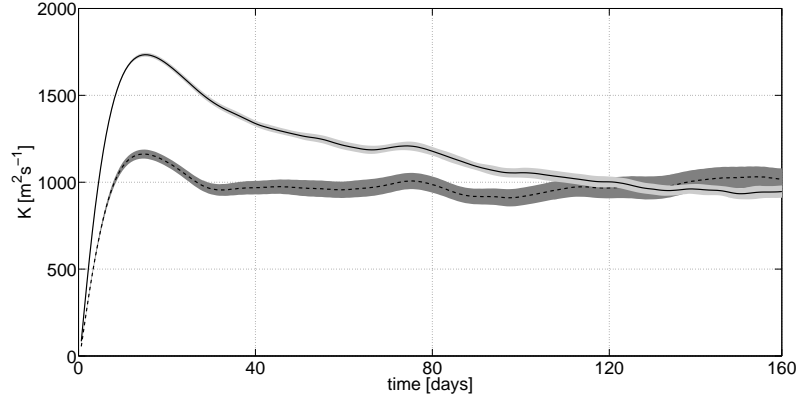
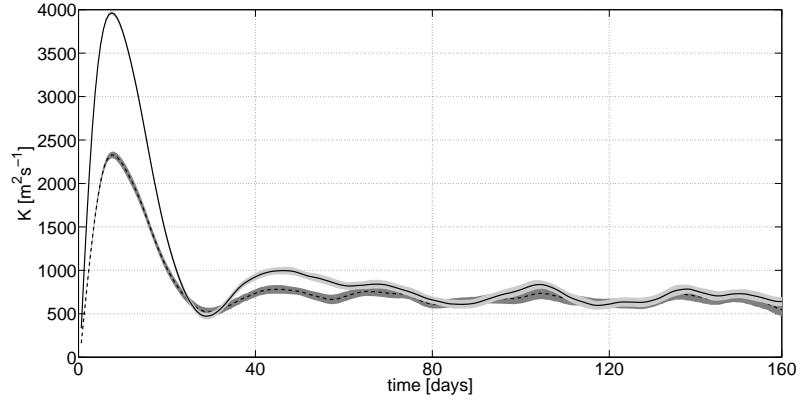


Figure 12: Relative error $2\sigma/K_{1y}$ of the single-particle diffusivity K_{1y} (Eq. (22)), where σ is the standard deviation calculated from bootstrapping with 100 sub-samples of n particles. The errors are shown for particles released in a 2° bin in the subtropical gyre ($45^\circ S \pm 1^\circ$) (blue) and particles released in a 2° bin in the ACC ($59^\circ S \pm 1^\circ$) (red) for the surface (solid lines) and deep surface (dashed lines). The solid black line shows a line with slope of $n^{-1/2}$.

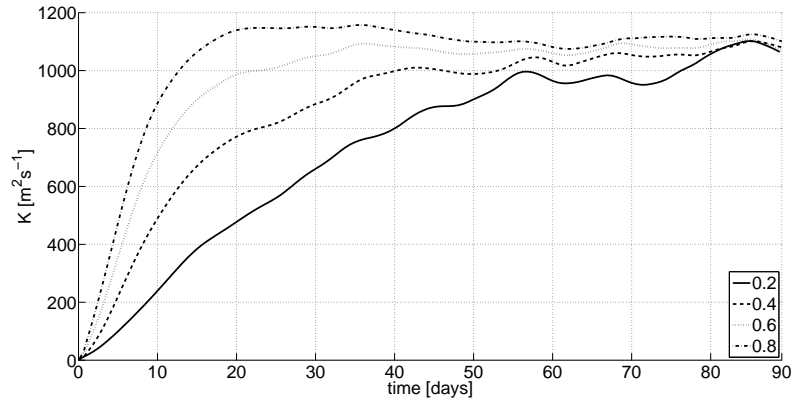


(a)

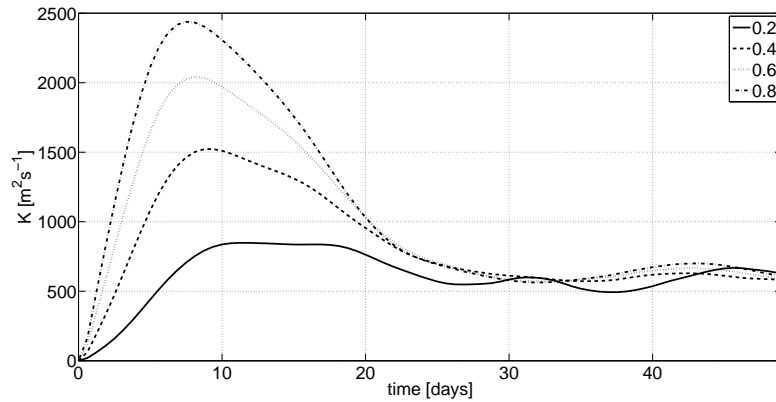


(b)

Figure 13: The time evolution of the single-particle diffusivities K_{1y} (Eq. (22), solid line) and the relative diffusivities K_{2y} (Eq. (24), dashed line) for (a) particles released in a 2° bin in the subtropical gyre ($45^\circ S \pm 1^\circ$) and (b) particles released in a 2° bin in the ACC ($59^\circ S \pm 1^\circ$). Shading shows uncertainties estimated by a bootstrapping technique.



(a)



(b)

Figure 14: The time evolution of the two-particle diffusivities K_{2y} (Eq. (24)) for different initial separations for (a) particles released in a 2° bin in the subtropical gyre ($45^\circ S \pm 1^\circ$) and (b) particles released in a 2° bin in the ACC ($59^\circ S \pm 1^\circ$).

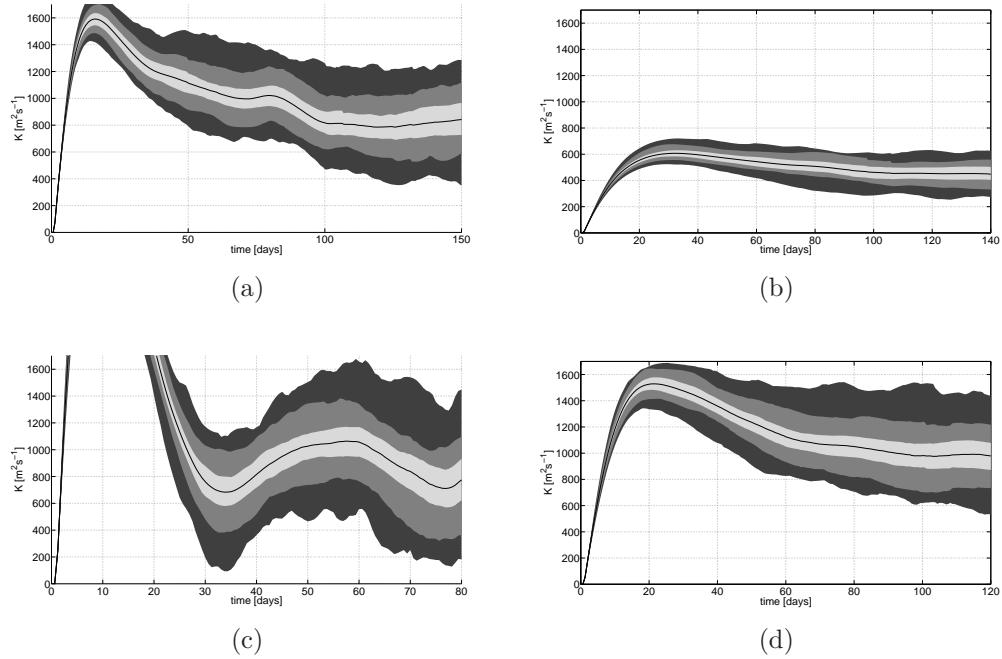


Figure 15: The single-particle diffusivity K_{1y} (Eq. (22)) for particles released in a bin north of the ACC (a) at 500 m and (b) at 1500 m (the bin extends from 44° to 48°S) and for particles released in a bin in the ACC (c) at 500 m and (d) at 1500 m (the bin extends from 56° to 60°S). Grey shading shows the 5th and 95th percentiles determined by a bootstrapping technique with 100 sub-samples using 50 floats (dark grey), 150 floats (medium grey) and 1000 floats (light grey).