Discrete Optimization

A multiclass, multicriteria logit-based traffic equilibrium assignment model under ATIS

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Abstract

This paper presents a multiclass, multicriteria (cost versus time) logit-based traffic equilibrium assignment model in road networks served by advanced traveler information systems (ATIS). All users are differentiated by their own value of time (VOT) that follows some probability distribution. Users of each class, having their own VOT, are further divided into two groups, equipped and unequipped with ATIS respectively. The travel disutility received by each user is defined as a linear bi-criteria combination of travel time and monetary travel cost. It is assumed that all users make their route choices in a logit-based stochastic manner, but the equipped users have lower perception variation on the travel disutility than the unequipped due to the ATIS service. The model is formulated as a fixed-point problem and solved by the method of successive averages in conjunction with logit assignment. Numerical results show that the traditional single-class and/or single-criterion models may overestimate or underestimate the benefit from ATIS services.
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1. Introduction

In recent years, there has been considerable interest in developing and evaluating advanced traveler information systems (ATIS) from both transportation practical and academic circles. Through provision of traffic information, ATIS can help commuters compensate for their limited knowledge and thus make more reasonable travel choice decisions. So far, a number of studies have been conducted for modeling the effects of ATIS on commuting behaviors and assessing the relevant benefits and risks. These studies can in general be classified into such three categories as field and laboratory experiments (Tsuji et al.,

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1985; Mahmassani and Jayakrishnan, 1991; Adler and Blue, 1998), computer simulation (Ben-Akiva et al., 1991; Al-Deek and Kanafani, 1993; Hall, 1993; Yang et al., 1993; Adler et al., 1999) and analytical modeling (Kanafani and Al-Deek, 1991; Yang, 1998; Lo et al., 1999; Yang and Meng, 2001; Lo and Szeto, 2002; Yin and Yang, 2003; Li et al., 2003).

Obviously, travelers equipped and unequipped with ATIS will behave differently in their route choices due to the fact that they obtain different quality of traffic information and then have different perception variances on travel disutility. In analytical modeling approaches, three criteria, namely user equilibrium (UE), system optimum (SO) and stochastic user equilibrium (SUE), are frequently used to model the commuters' route choice behaviors. Kanafani and Al-Deek (1991) estimated the ATIS benefit in a network with recurrent congestion by comparing the total network travel times generated under UE and SO respectively. In the studies by Harker (1988) and Bennett (1993), the equipped and unequipped drivers are modeled to follow the SO and UE criteria, respectively. Ben-Akiva et al. (1991), Yang (1998), and Yang and Meng (2001) developed mixed UE and SUE equilibrium formulae to model the different route choice behaviors of equipped and unequipped drivers. Recently, Lo and Szeto (2002), Yin and Yang (2003) and Li et al. (2003) adopted the logit-based SUE principle to describe the route choice behaviors of both classes of drivers. Their approaches are more close to the reality since either equipped or unequipped drivers cannot accurately compute the route travel disutility, considering the existence of various random factors in real world.

However, the aforementioned studies postulated all users have an identical and single value of time (VOT), i.e., all commuters are aggregated into one single user class. As we know, travelers are heterogeneous in many aspects due to their different socio-economic characteristics. They have different VOTs and often exhibit different decision-making behaviors in travel choices. In general, professional and self-employed workers have higher VOTs than assembly-line workers and clerks. Therefore, it is necessary to relax the single VOT assumption and then develop multiclass models that can consider the heterogeneity of commuters. In literature, these models are formulated either on a discrete set of VOTs for several user classes or by a continuously distributed VOT across the whole population (Leurent, 1993, 1996, 1998; Marcotte and Zhu, 2000; Nagurney, 2000; Yang and Huang, 2004).

On the other hand, users have their own personal understanding and preference on traffic condition. So, it is necessary to consider the travelers' trade-offs between travel time and out-of-pocket money when making route choices. The preference can be generally represented by the multicriteria modeling approaches. Adler et al. (1999) utilized simulation method to assess the impacts of the bi-objective route guidance systems on user and system performance. In their study, the travel disutility (or trip quality cost) is formulated as the linear weighted additive sum of travel time and cost. Nagurney and Dong (2002) proposed a multicriteria model in which commuters are allowed to perceive their travel disutility or generalized cost as the linear weighting of travel time and travel cost, with each of which being flow-dependent. Recently, Yang and Huang (2004) examined the multiclass, multicriteria traffic network equilibrium and system optimum problem.

In this paper, we propose a multiclass, multicriteria logit-based traffic equilibrium model for investigating the responses of different classes of users to ATIS. The user classes are differentiated by their own VOT that follows some probability distribution. Users of each class are further divided into two groups equipped with and without ATIS respectively. It is assumed that all users select the routes with minimum perceived travel disutility, which is a linear bi-criteria combination of travel time and monetary travel cost. The equipped users, served by ATIS, are better aware of the extent of disutility uncertainties than the unequipped, so they can find the routes with lower disutilities. The proposed model is formulated as a fixed-point problem, and solved by the method of successive averages in conjunction with logit assignment. Numerical results show that the proposed model can provide some new insights for assessing the impacts of ATIS on travel behaviors.

The paper is organized as follows. In Section 2, we describe some basic assumptions for the proposed modeling approach. In Section 3, we present a fixed-point formulation to state the multiclass, multicriteria equilibrium conditions under ATIS. Section 4 provides a solution algorithm for solving the proposed

model. In Section 5, a numerical experiment is used to explore the impacts by ATIS on travel behaviors. Section 6 concludes the paper and provides recommendations for future researches.

2. Assumptions

To facilitate the presentation of the essential ideas, the following assumptions are made in our study.

A1. Drivers are heterogeneous in terms of their VOTs that follow a certain probability density distribution. The distributions are further assumed to be identical for all origin–destination pairs. All drivers are categorized into finite number of classes according to their VOT distributions. In other words, each user class is associated with a specific VOT.

A2. Each driver evaluates a route according to that route's travel disutility which is defined as the linear bi-criteria combination of travel time and travel cost of the route. The travel cost is converted into equivalent time by the VOT multiplier.

A3. Both the equipped and unequipped drivers with ATIS make their route choices in a stochastic manner but with different degree of uncertainty of the travel disutility. The equipped drivers have lower perception variations on disutility than the unequipped due to the aid of information device.

A4. Each user class is associated with a market penetration that is the proportion of the drivers equipped with ATIS among the total number of that class of users. The higher VOT class is associated with a larger market penetration, implying that commuters of higher VOT class are more willing to purchase and use the ATIS. This modeling approach that simultaneously considers the VOT distribution of users and the attitudes of using ATIS for different user classes is never reported in literature.

Note that in this paper, the terms user, driver, commuter and traveler are used equally without difference, since we assume that there is only one person in each vehicle.

3. Model formulation

Consider a network $G = (N, A)$, where N is the set of nodes and A the set of links. Let W denote the set of origin–destination (O–D) pairs and R_w the set of routes between O–D pair w. Let M denote the number of user classes in the network and m a typical user class, $m = 1, 2, \ldots, M$. Let q_w^m and $\hat{q}_w^m$ be the demands of the equipped and unequipped drivers in class m respectively, and Q_w^m the given total demand of the user class m, then we have

$$q_w^m + \hat{q}_w^m = Q_w^m \quad \forall w \in W, \ m = 1, 2, \ldots, M. \tag{1}$$

Let v_a^m be the flow of user class m on link a and v_a the total flow on link a, which are respectively defined as

$$v_a^m = \sum_{w \in W} \sum_{r \in R_w} (f_{rw}^m + \hat{f}_{rw}^m) \delta_{ar} \quad \forall a \in A, \ m = 1, 2, \ldots, M, \tag{2}$$

$$v_a = \sum_m v_a^m \quad \forall a \in A, \tag{3}$$

where f_{rw}^m and $\hat{f}_{rw}^m$ are the flows on route r between O–D pair w for the equipped and unequipped drivers in user class m, respectively, δ_{ar} equals 1 if route r traverses link a and 0 otherwise. These route flows satisfy the following conservation conditions:

$$\sum_{r \in R_w} f_{rw}^m = q_w^m \quad \forall w \in W, \ m = 1, 2, \ldots, M, \tag{4}$$

$$\sum_{r \in R_w} \hat{f}_{rw}^m = \hat{q}_w^m \quad \forall w \in W, \ m = 1, 2, \ldots, M. \tag{5}$$

Let $\mathbf{v}$ denote the vector of link flows in the network, i.e., $\mathbf{v} = (\ldots, v_a, \ldots,)$. It is assumed that each link a associates with two flow-dependent attributes, namely travel time $t_a(\mathbf{v})$ and (monetary) travel cost $c_a(\mathbf{v})$, being measured by hours and dollars respectively. Each of these two attributes is the continuous function of all link flows. The travel cost may cover congestion toll for using the link, fuel and vehicle maintenance expenses shared onto the link. Let $u_a^m(\mathbf{v})$ be the measured disutility for user class m traveling on link a. Following Leurent (1993, 1996, 1998) and Yang and Huang (2004), this disutility can be formulated as the linear combination of travel time and monetary travel cost via the class-specified VOT, i.e.,

$$u_a^m(\mathbf{v}) = t_a(\mathbf{v}) + c_a(\mathbf{v})/\alpha_m \quad \forall a \in A, \ m = 1, 2, \ldots, M, \tag{6}$$

where the parameter α_m represents the average VOT of user class m. In Eq. (6), this parameter α_m converts monetary travel cost into equivalent travel time for user class m. The values of this parameter can be yielded by discretizing the probability distribution of the continuous VOT.

Let u_{rw}^m be the actual travel disutility of class m on route r between O–D pair w. It is formulated as the sum of all link travel disutilities along this route, i.e.,

$$u_{rw}^m = \sum_{a \in A} u_a^m(\mathbf{v})\delta_{ar} \quad \forall r \in R_w, \ w \in W, \ m = 1, 2, \ldots, M. \tag{7}$$

Let U_{rw}^m and $\widehat{U}_{rw}^m$ denote the travel disutility perceived by equipped and unequipped drivers in class m on route r between O–D pair w, respectively. They consist of systematic (or measured) and random components, i.e.,

$$U_{rw}^m = u_{rw}^m + \xi_{rw}^m \quad \forall r \in R_w, \ w \in W, \ m = 1, 2, \ldots, M, \tag{8}$$

$$\widehat{U}_{rw}^m = u_{rw}^m + \hat{\xi}_{rw}^m \quad \forall r \in R_w, \ w \in W, \ m = 1, 2, \ldots, M, \tag{9}$$

where ξ_{rw}^m and $\hat{\xi}_{rw}^m$ are the random terms representing the perception errors of travel disutility by equipped and unequipped drivers in class m on route r between O–D pair w, respectively. It is assumed that each perception error covers the variance caused by both time and cost uncertainties. In this study, we do not explicitly distinguish the disutility variance into two parts associated with the time and cost uncertainties respectively. In reality, however, the user classes with different VOTs would correspond to these two uncertainties in different manners.

Suppose that all random terms, ξ_{rw}^m and $\hat{\xi}_{rw}^m$ in Eqs. (8) and (9), are independent and identical distributed Gumbel variables with zero mean. According to the utility maximization theory, at equilibrium the route choice probabilities for equipped and unequipped drivers are governed by the following logit formulae (Oppenheim, 1995):

$$P_{rw}^m = \frac{\exp(-\theta_m u_{rw}^m)}{\sum_{k \in R_w} \exp(-\theta_m u_{kw}^m)} \quad \forall r \in R_w, \ w \in W, \ m = 1, 2, \ldots, M, \tag{10}$$

$$\widehat{P}_{rw}^m = \frac{\exp(-\hat{\theta}_m u_{rw}^m)}{\sum_{k \in R_w} \exp(-\hat{\theta}_m u_{kw}^m)} \quad \forall r \in R_w, \ w \in W, \ m = 1, 2, \ldots, M, \tag{11}$$

where θ_m and $\hat{\theta}_m$ are the parameters representing the travel disutility perception variations of equipped and unequipped drivers in class m, respectively. A higher θ_m-value means a smaller perception variation for the equipped drivers in class m. Hence, the θ_m-value can be used to represent the quality of the provided traffic information. The parameter $\hat{\theta}_m$ reflects the familiarity degree to traffic conditions by unequipped drivers. The relation $\theta_m > \hat{\theta}_m$ holds, which claims that the equipped drivers have lower perception variations on travel disutility than the unequipped.

From Eqs. (10) and (11), it can be noted that route choice probabilities P_{rw}^m and $\widehat{P}_{rw}^m$ are the functions of systematic components u_{rw}^m which are in turn the functions of f_{rw}^m and $\hat{f}_{rw}^m$ in terms of Eqs. (2)–(7). Hence, the

route choice probabilities P_{rw}^m and $\widehat{P}_{rw}^m$ are the functions of route flows f_{rw}^m and $\hat{f}_{rw}^m$. Then, the induced equilibrium route flows for the equipped and unequipped drivers can respectively be stated as

$$f_{rw}^m = q_w^m P_{rw}^m(\mathbf{f}, \hat{\mathbf{f}}) \quad \forall r \in R_w, \ w \in W, \ m = 1, 2, \ldots, M, \tag{12}$$

$$\hat{f}_{rw}^m = \hat{q}_w^m \widehat{P}_{rw}^m(\mathbf{f}, \hat{\mathbf{f}}) \quad \forall r \in R_w, \ w \in W, \ m = 1, 2, \ldots, M, \tag{13}$$

where $\mathbf{f}$ and $\hat{\mathbf{f}}$ are the vectors of route flows associated with the equipped and unequipped drivers, respectively.

Define the set of feasible route flows as $\Omega = \{(\mathbf{f}, \hat{\mathbf{f}}),$ where $\mathbf{f} \geqslant 0, \ \hat{\mathbf{f}} \geqslant 0$ and satisfy Eqs. (1), (4) and (5)}. Let $\mathbf{x}$ denote an element in this set, i.e., $\mathbf{x} = (\mathbf{f}, \hat{\mathbf{f}}) \in \Omega$, then the multiclass, multicriteria logit-based equilibrium conditions (12) and (13) can be further formulated as the following fixed-point problem:

$$\mathbf{x}^* - \mathbf{q}\mathbf{P}(\mathbf{x}^*) = \mathbf{0}, \quad \mathbf{x}^* \in \Omega, \tag{14}$$

where $\mathbf{q}$ and $\mathbf{P}$ are the vectors of travel demand and route choice probability, respectively.

Remark 1. Since the set Ω is compact and the link travel time and travel cost functions are assumed to be continuous, according to the Brouwer's fixed-point theory, there exists at least one solution to the fixed-point problem (14). However, the uniqueness of the model solution cannot be guaranteed because we cannot expect the travel disutility vector appearing in the fixed-point problem (14) to be strictly monotone even though all the link travel time and travel cost functions are strictly monotone (Nagurney, 2000).

Remark 2. The fixed-point formulation is the most general expression to various network equilibrium problems, including the ones with asymmetric link travel time and cost functions, but there still exist some shortcomings in the proposed modeling approach. For instance, the model confines our investigations to static or time-stationary equilibrium analyses, and thus cannot reveal the drivers' time-dependent responses to ATIS within a day or day-to-day. Moreover, the queue spillback which sometimes happens in real world is also not considered in our model. Daganzo (1998) explored the queue spillback phenomenon in a network with one O–D pair and two parallel links. This provides an excellent entry into the relevant researches. It is anticipated that the ATIS can help dissipating the physical queues through guiding the drivers' route choices. Hence, the extension of our model to inclusion of time dependence and queue spillback is planed to conduct in our future research.

4. Solution algorithm

The method of successive averages (MSA) proposed by Powell and Sheffi (1982) has successfully been used in solving various stochastic user equilibrium problems. Yang (1998) and Yin and Yang (2003) recently employed this method to solve the single-class and/or single-criterion behavior modeling problems. In this paper, a solution scheme which combines the MSA with the logit assignment (10) and (11) is developed for solving our model. The step-by-step procedure of this scheme is given below.

Step 0. For given O–D demands q_w^m and $\hat{q}_w^m$, use (10) and (11) to perform the logit assignment and get the route flows f_{rw}^m and $\hat{f}_{rw}^m$, $\forall r \in R_w, \ w \in W, \ m = 1, 2, \ldots, M$. Yield the initial link flows $v_a^{(1)}$, $\forall a \in A$, and set the iteration counter $n = 1$.

Step 1. According to the current link flows $v_a^{(n)}$, $\forall a \in A$, update the link travel disutility

$$u_a^{m(n)}(\mathbf{v}^{(n)}) = t_a(\mathbf{v}^{(n)}) + c_a(\mathbf{v}^{(n)})/\alpha_m \quad \forall a \in A, \ m = 1, 2, \ldots, M.$$

Step 2. Perform the logit assignment in terms of the current link travel disutility $u_a^{m(n)}(\mathbf{v}^{(n)})$, $\forall a \in A$, $m = 1, 2, \ldots, M$, then get route flows and the auxiliary link flows $x_a^{(n)}$, $\forall a \in A$.

Step 3. Update link flows by the following equation:

$$v_a^{(n+1)} = v_a^{(n)} + (x_a^{(n)} - v_a^{(n)})/(n+1) \quad \forall a \in A.$$

Step 4. If a certain equilibrium criterion, for instance, a relative gap (26) given later, is satisfied, stop and report the solution; otherwise, let $n = n + 1$ and go to Step 1.

Remark 3. In Steps 0 and 2, the logit assignment can be carried out by using the Dial (1971), Leurent (1997) or Bell (1995) approaches. The Dial approach just operates on the set of efficient routes which include only those links taking users away from their origins. When this approach is embedded in MSA algorithm for solving the SUE problems in congested networks, the set of efficient routes may change from one iteration to another since the link travel disutility are flow-dependent. Hence, the convergence cannot be guaranteed. Leurent (1997) made an improvement to Dial's logit assignment by introducing a new definition about efficient routes. His approach ensures the set of efficient routes unchangeable. The Bell approach obviates the explicit definition of efficient routes through considering all possible routes including those with loops. For eliminating the cyclic routes, Huang and Bell (1998) further improved this assignment technique.

Remark 4. The common advantage of the above three approaches is that the explicit route enumeration is not needed. Some authors, however, do not evade the route enumeration because the route flow information is particularly important in the content of route guidance and the advent of powerful processors makes the direct estimation of route flows be rapidly becoming practical. For this, various methods have been employed to achieve the route enumeration automatically, such as column generation method and preliminary phase scheme (Bell et al., 1993; Huang, 1995; Damberg et al., 1996). In this study, we adopt the route flow-based logit assignment by Huang and Bell (1998), which can guarantee the convergence of the solution algorithm.

5. Numerical experiments

5.1. Experiment settings

For making our experiments be more representative, i.e., covering more possible situations, we should carefully design the values of some parameters in the proposed model. In this study, all user classes are ranked according to their increasing VOTs. In the following, we first formulate the discrete set of VOTs and the corresponding O–D demands for all user classes from a given continuous probability density function of VOT. Let Q_w be the total demand between O–D pair w and $f(\tau)$ be the continuous probability density function of VOT for the whole population. It is assumed that $f(\tau)$ is identical for all O–D pairs. We divide the whole interval $[0, \tau_{\max}]$ into M intervals with equal length (where $\tau_{\max}$ is the maximum VOT), i.e.,

$$[\tau^{m-1}, \tau^m] = \left[\frac{m-1}{M} \tau^{\max}, \frac{m}{M} \tau^{\max} \right] \quad \forall m = 1, 2, \ldots, M, \tag{15}$$

where $\tau^0 = 0$ and $\tau^M = \tau^{\max}$. Let $F(\tau)$ be the cumulative distribution function of $f(\tau)$, then the demand of user class m between O–D pair w is given by

$$Q_w^m = Q_w[F(\tau^m) - F(\tau^{m-1})] \quad \forall w \in W, \ m = 1, 2, \ldots, M \tag{16}$$

and the VOT of user class m is computed by

$$\beta_m = \int_{\tau^{m-1}}^{\tau^m} \tau f(\tau) \mathrm{d}\tau \Big/ \int_{\tau^{m-1}}^{\tau^m} f(\tau) \mathrm{d}\tau \quad \forall m = 1, 2, \ldots, M. \tag{17}$$

Similar to Yang and Meng (2001), the following log-normal VOT distribution is adopted in our study:

$$f(\tau) = \frac{1}{\kappa\sqrt{2\pi}} \tau^{-1} \exp\left[-\frac{1}{2}\left(\frac{\ln\tau - \pi}{\kappa}\right)^2\right], \quad 0 < \tau < \infty, \ \kappa > 0, \tag{18}$$

where π and κ are the mean and the standard deviation of $\ln\tau$, respectively. These two parameters are related to the mean μ and the standard deviation σ of τ by the following equations:

$$\pi = \ln\mu - 0.5\kappa^2, \tag{19}$$

$$\kappa^2 = \ln\left(1 + \frac{\sigma^2}{\mu^2}\right). \tag{20}$$

To capture the willing behavior of purchasing and using ATIS devices of different user classes, the following simple market penetration rates are designed for the experiments:

$$\eta_w^m = \min\left\{\frac{i}{M - m + j}, \ 1.0\right\} \quad \forall w \in W, \ m = 1, 2, \ldots, M, \tag{21}$$

where the parameters i and j are integers and i takes values between 0 and $(M - 1 + j)$. These two parameters govern the market penetration level of each user class. In the later, we will investigate the ATIS impacts on system and user performance through changing the values of these two parameters. It should be noted that other forms of market penetration rates, different from Eq. (21), can also be used, but requiring that the market penetration rate of a user class is positively proportional to its VOT (Assumption A4). Table 1 shows all possible values of the market penetration rates when $M = 10$ and $j = 5$. For $i = 5$, all users of the class 10 are equipped with ATIS and users of other classes are partly equipped. For $i = 14$, all users in the network are equipped with ATIS.

Therefore, the demand of the equipped drivers in each user class can be computed by

$$q_w^m = Q_w^m \eta_w^m \quad \forall w \in W, \ m = 1, 2, \ldots, M. \tag{22}$$

In order to quantitatively analyze the impacts of ATIS on network and user performance, some performance measures should be specified. Average travel disutility saving, as an index of measuring user benefit,

Table 1
The market penetration rates given by Eq. (21) when $M = 10$ and $j = 5$

m	i													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1/14	2/14	3/14	4/14	5/14	6/14	7/14	8/14	9/14	10/14	11/14	12/14	13/14	1
2	1/13	2/13	3/13	4/13	5/13	6/13	7/13	8/13	9/13	10/13	11/13	12/13	1	1
3	1/12	2/12	3/12	4/12	5/12	6/12	7/12	8/12	9/12	10/12	11/12	1	1	1
4	1/11	2/11	3/11	4/11	5/11	6/11	7/11	8/11	9/11	10/11	1	1	1	1
5	1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10	1	1	1	1	1
6	1/9	2/9	3/9	4/9	5/9	6/9	7/9	8/9	1	1	1	1	1	1
7	1/8	2/8	3/8	4/8	5/8	6/8	7/8	1	1	1	1	1	1	1
8	1/7	2/7	3/7	4/7	5/7	6/7	1	1	1	1	1	1	1	1
9	1/6	2/6	3/6	4/6	5/6	1	1	1	1	1	1	1	1	1
10	1/5	2/5	3/5	4/5	1	1	1	1	1	1	1	1	1	1

is defined as the difference between the average travel disutility of unequipped drivers and that of equipped drivers. The average travel disutility generated from the logit-based SUE model is measured in log-sum formula (Oppenheim, 1995; Yang, 1999). In addition, for investigating the changes of total network travel disutility (TNTD) before and after providing ATIS services, the following relative reduction of TNTD is defined:

$$\text{Relative reduction of TNTD} = \frac{\text{TNTD}^b - \text{TNTD}^a}{\text{TNTD}^b} \times 100\%, \tag{23}$$

where the superscripts "b" and "a" refer to the cases of "before" and "after" providing ATIS services, respectively.

The numerical experiments are conducted in the Nguyen and Dupuis network (1984) as shown in Fig. 1. It consists of four O–D pairs, 19 links and 25 routes. The following link travel time and travel cost functions are adopted:

$$t_a(\mathbf{v}) = \frac{L_a}{S_a} + A_a \left(\frac{\tilde{v}_a}{C_a} \right)^m \quad \forall a \in A, \tag{24}$$

$$c_a(\mathbf{v}) = L_a K_a + B_a \left(\frac{\tilde{v}_a}{C_a} \right)^n \quad \forall a \in A, \tag{25}$$

where L_a and S_a are the length of link a and the free-flow speed of vehicles on link a, respectively, K_a is the vehicle operating cost per unit physical distance of link a, C_a is the link capacity, the parameters A_a and m, B_a and n reflect the congestion effects on time and cost, respectively. Note that in Eqs. (24) and (25), $\tilde{v}_a \neq v_a$ for considering the interaction among link flows. Table 2 gives the detailed expressions of $\tilde{v}_a$ for all links.

In the network, $L_a = \{24$ km for link 18, 16 km for links 4 and 13, and 8 km for all other links$\}$, $S_a = 40$ km/hour, $C_a = 3000$ veh/hour, and $K_a = 0.3$ ¥/km for all links. Here, the symbol ¥ (pronunciation 'yuan') stands for the monetary unit of Chinese currency (8.4 ¥ ≈ one US dollar). Other input data in the model are: $m = 4$, $n = 2$, $A_a = 0.00625$, $B_a = 0.15$, $\mu = 50$, $\sigma = 30$, $\tau^{\max} = 200$ ¥/hour, $\theta_m = 0.1$ for all equipped user classes and $\hat{\theta}_m = 0.01$ for all unequipped user classes. Given these μ, σ and $\tau^{\max}$ values, the demands with VOT larger than 200 ¥/hour, occupying the total 0.274%, is truncated off. Table 3 gives the O–D demand pattern.
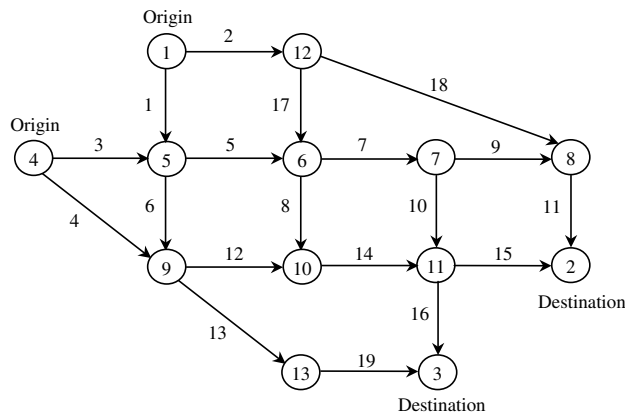

Fig. 1. The Nguyen and Dupuis network with node and link numbers.

Table 2
The interaction patterns among link flows

Link no.	$\tilde{v}_a$
1	$v_1 + 0.25v_3 + 0.25v_5 + 0.25v_6$
2	$v_2 + 0.25v_{17} + 0.25v_{18}$
3	$v_3 + 0.25v_1 + 0.25v_5 + 0.25v_6$
4	$v_4 + 0.25v_6 + 0.25v_{12} + 0.25v_{13}$
5	$v_5 + 0.25v_7 + 0.25v_8 + 0.25v_{17}$
6	$v_6 + 0.25v_4 + 0.25v_{12} + 0.25v_{13}$
7	$v_7 + 0.25v_9 + 0.25v_{10}$
8	$v_8 + 0.25v_{12} + 0.25v_{14}$
9	$v_9 + 0.25v_{11} + 0.25v_{18}$
10	$v_{10} + 0.25v_{14} + 0.25v_{15} + 0.25v_{16}$
11	$v_{11} + 0.25v_{15}$
12	$v_{12} + 0.25v_8 + 0.25v_{14}$
13	$v_{13} + 0.25v_{19}$
14	$v_{14} + 0.25v_{10} + 0.25v_{15} + 0.25v_{16}$
15	$v_{15} + 0.25v_{11}$
16	$v_{16} + 0.25v_{19}$
17	$v_{17} + 0.25v_5 + 0.25v_7 + 0.25v_8$
18	$v_{18} + 0.25v_9 + 0.25v_{11}$
19	$v_{19} + 0.25v_{16}$

Table 3
The O–D demands

From	To	
	2	3
1	3000	2000
4	2000	3000

5.2. Experiment results and analyses

The proposed solution algorithm was coded in Language C and run on a DELL INSPIRON/510m computer. Fig. 2 depicts the total network travel disutility against the number of user classes when j and i are fixed as 5 and 1 respectively. This figure shows that the total network travel disutility increases with the M-value and tends to be stable when the M-value is larger than 10. This indicates such a fact that an enough large M-value can result in a good approximation of multiclass, multicriteria equilibrium assignment with continuous VOT distribution across users. With this observation, the numerical results presented below are generated with $M = 10$.

Fig. 3 plots the values of the relative gap which is denoted by G and defined in Eq. (26) below, against the iteration number. This index measures how closely the outputs generated at iteration n approach the equilibrium conditions (10) and (11).

$$G = \frac{\sum_{rw}\sum_m f_{rw}^{m(n)}\left|f_{rw}^{m(n)} - q_w^m P_w^{m(n)}(\mathbf{f}^{(n)}, \hat{\mathbf{f}}^{(n)})\right|}{\sum_{rw}\sum_m f_{rw}^{m(n)}} + \frac{\sum_{rw}\sum_m \hat{f}_{rw}^{m(n)}\left|\hat{f}_{rw}^{m(n)} - \hat{q}_w^m \widehat{P}_w^{m(n)}(\mathbf{f}^{(n)}, \hat{\mathbf{f}}^{(n)})\right|}{\sum_{rw}\sum_m \hat{f}_{rw}^{m(n)}}. \tag{26}$$

Fig. 3 verifies the convergence of the proposed solution algorithm for $j = 5$ and $i = 1$. As shown in this figure, the G-value becomes very small (<0.001) after 100 iterations, only 0.21 seconds of CPU time required for this. The same convergence has also been observed for other j and i values.
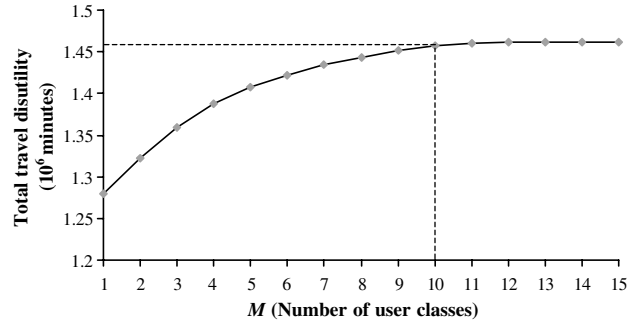
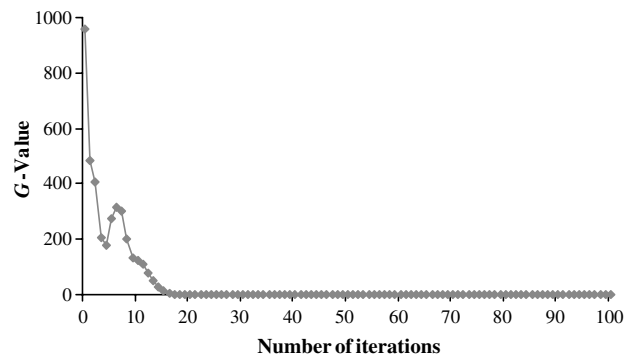Fig. 2. The total network travel disutility against number of user classes.

Fig. 3. Convergence of the proposed algorithm.

Figs. 4a and 4b show the average travel disutilities of different user classes against the ATIS market penetration level (reflected by the indicator i) for a fixed $j = 5$. It can be seen that as the ATIS market penetration first increases from zero, the average travel disutilities of all user classes decrease quickly. But, these decreases stop at $i = 5$, and the average travel disutilities for the first four lower VOT user classes (i.e., the classes 1–4) basically become constant when $i > 5$, whereas those for the remainder six higher VOT user classes (i.e., the classes 5–10) turn to ascend. This clearly demonstrates a so-called ATIS over-reaction (Ben-Akiva et al., 1991) that occurs for higher VOT user classes only. Fig. 4b also shows that the higher VOT users are slightly more sensitive to the ATIS market penetration than the lower VOT users. These findings, impossibly obtained by single user class models (Yang, 1998), can enrich our understanding on the ATIS impacts.

Fig. 5 displays the effects of information quality on average travel disutility saving for various O–D pairs in the network. It can be found that when $\theta_m < 0.1$, any information quality improvement can bring a significant average travel disutility saving. But, the saving does not continue to rapidly increase when $\theta_m > 0.1$; on the contrary, it declines a little for some O–D pairs like (4, 3) and (1, 2). When $\theta_m \geqslant 0.5$, the savings for all O–D pairs keep unchanged basically. In fact, that $\theta_m = 0.5$ in this example has already represented a very high information quality; so any further increasing on θ_m-value is meaningless. In addition, Fig. 5 shows that the inequity of average travel disutility savings from information quality improvement exists for different O–D pairs. This inequity may arise from the network structure specified in our study and the demand levels for different O–D pairs.
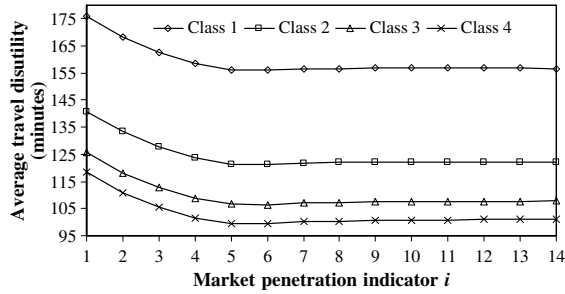
Fig. 4a. The average travel disutilities of lower VOT user classes (classes 1–4) against ATIS market penetration level.
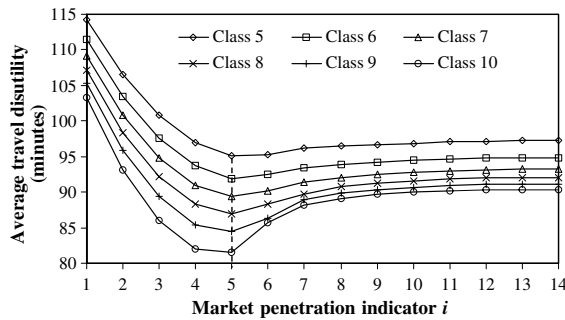

Fig. 4b. The average travel disutilities of higher VOT user classes (classes 5–10) against ATIS market penetration level.
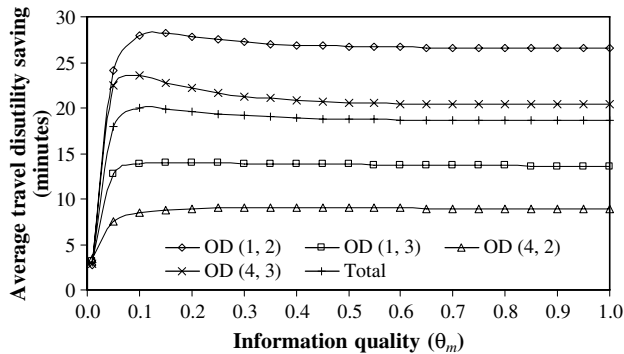

Fig. 5. The average travel disutility saving for various O–D pairs against information quality.

Finally, we compare the network performances generated by different modeling approaches under various ATIS market penetration rates. The following four models are considered: the multiclass, multicriteria model proposed in this paper (Model-I), the multiclass, travel time-based single-criterion model (Model-II), the multiclass, travel cost-based single-criterion model (Model-III), and the single-class, multicriteria model (Model-IV). In Model-IV, all users have an identical VOT equal to the mean of the VOT distribution. The relative reduction of total network travel disutility (TNTD) before and after providing ATIS services, computed by Eq. (23), is shown in Fig. 6 in which one curve corresponds to the results by one model. It can be seen that in comparison with Model-I, the Model-II and Model-IV overestimate the network benefits from
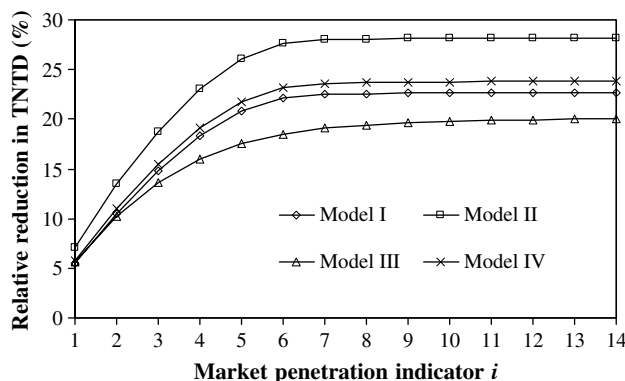
Fig. 6. The relative reduction in total network travel disutility by four models, against ATIS market penetration level.

ATIS, whereas Model-III underestimates. Fig. 6 also shows that the marginal effect of raising ATIS market penetration is decreasing, regardless of the models used.

6. Conclusions

In this paper, we present a multiclass, multicriteria logit-based traffic equilibrium assignment model under ATIS. The term 'multicriteria' refers to the trade-off between travel time and travel cost when making route choices. All users are discretized into finite number of classes according to their VOTs. Users of different classes are allowed to have different ATIS market penetration levels. This model can help us gain some insights into the different responses of heterogeneous users to ATIS, which may not be revealed by previously developed single-class and/or single-criterion models. The model is formulated as a fixed-point problem and solved by the method of successive averages in conjunction with logit assignment. Based on the numerical results, some initial but important findings are obtained: (i) ATIS over-reaction occurs more easily on higher VOT user classes; (ii) inequity of travel disutility savings from information quality improvement exists for different O–D pairs; (iii) traditional single-class and/or single-criterion models may overestimate or underestimate the network benefits from ATIS services.

The proposed modeling approach provides a useful tool for evaluating the effect of information service on travel behavior at a strategic level. One of future studies is to apply the proposed modeling approach into real networks. Aiming at the 2008 Olympic Games in Beijing, the local government has devoted a huge amount of resources to greatly improve the existing traffic system. Various advanced technologies including the intelligent transportation systems (ITS) will be adopted for enhancing the mobility to and from the Olympic central area. The ITS development strategy of the Beijing Olympic Games should be concentrated on not only how to satisfy the game-period traffic demand, but also how to bring the ITS facilities into full play after the Games. Hence, it is important and necessary to assess the long-term effects of various ITS facilities (including the ATIS devices) on system and user performances at a strategic level. The modeling approach proposed in this paper is expected to be applied in this research project.

Another interesting is to extend the proposed model in several aspects, for example, involving the non-additive cost functions (Gabriel and Bernstein, 1997) and the decreasing cost functions, treating the time and cost uncertainties separately, and considering the network uncertainty. In addition, a very valuable but challengeable work is to develop a dynamic modeling approach, thereby some dynamic phenomena of travel behaviors under ATIS, like the queue spillback (Daganzo, 1998), peak hour spreading (Lo and Szeto, 2004) and departure time choice (Huang and Lam, 2004), can be investigated.

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