Problem 0: Please read the course notes up to Section 4.4

Problem 1: Thermal Models for a Solid.
The figure to the right shows the measured behavior of the heat capacity as a function of temperature for three solid metals, lead, gold and aluminum (See table of heat capacity values for these metals below). The behavior is typical of most solid materials. The specific heat capacity approaches a value of zero at low temperature. As the temperature is raised the heat capacity quickly rises to a large value initially and then settles to a slow (basically linear) increase in value with temperature.

In the very simple thermal models for solids that we will use throughout this course we assume that the heat capacity of a solid has a constant value. Looking at the graph, we can say that lead has a heat capacity of 130 J/kg K in the range of 150 to 400 K and never be more than 10% off for calculations in this temperature range. At room temperature, most solid substances have heat capacities that do not change significantly with temperature (provided the substance is not...
undergoing a phase change). The area in gray shows a representative temperature range that you might see during your day.

**Fixed heat capacity model:**
We are interested in developing a thermal model for a 1kg lead block for processes that will occur between 150 and 400 K.

a) Please choose a fixed heat capacity that is representative of lead block for this temperature range (150 to 400 K). Please use this chosen value next in sub-questions b)-i).

b) Under the assumption that the heat capacity of the lead is constant throughout the process please determine the heat transfer required to heat the Pb block from 273 K (state 1) to 350 K (state 2).

c) Using the first law, please determine the internal energy change of the lead block in process 1->2.

d) Assuming the internal energy is (arbitrarily) zero at T= 273 K, please develop an equation for the internal energy of the lead block as a function of temperature.

The entropy change of a material can be determined by subjecting the material to a reversible process and applying the second law to that process. A heat transfer process that occurs at uniform temperature is reversible (we will discuss this at greater length in class.) This can be approached by placing the lead block in an insulated environment with a heater attached as shown in the figure to the right.

The heater has negligible heat capacity (it is very small). The lead block can be heated while remaining at nearly uniform temperature if only a very small current is passed through the heater. The electrical dissipation in the heater is inherently not reversible but all the heat transfer within the (uniform temperature) lead block is reversible; hence the entropy generated within the lead block is zero for this process. The process is irreversible if the control mass (system) consists of the lead block and the heater. The process is reversible if the control mass (system) consists of ONLY the lead block.

(As an aside: if very large currents were passed through the heater, there would be large heat fluxes from the heater into the lead block. Very large temperature gradients would appear within the lead block. With these large temperature gradients, the block would not look like it was in equilibrium during the heating process and hence we would conclude the large-current process generates entropy within the lead block. In this case, the process is irreversible even when the control mass consists of ONLY the lead block.)

e) Using a control volume around the lead block, please determine the entropy change of the block if the temperature is changed from 273 K to 350 K in the internally reversible manner described above.

f) Assuming the entropy is (arbitrarily) zero at T= 273 K, please develop an equation for the entropy of the lead block as a function of temperature.
g) The lead block initially at 273 K (state 1) is placed in a microwave oven where the electric and magnetic fields create eddy currents that dissipate energy into the lead block (Dissipate = irreversible). The microwaves acting on the electrons in the lead is an example of a work transfer into the block. The microwaves are shut off and the isolated block is allowed to come to internal thermal equilibrium. The temperature of the block in this final state is 375 K (state 3). What is the change in the energy and entropy of the Pb block in process 1->3?

h) What is the entropy transferred from the magnetron to the lead by the microwaves?

i) What is the entropy generated in the lead block in process 1->3?

**Linear heat capacity model**

j) Please fit the heat capacity of lead as a linear function of temperature for use in the temperature range 150 to 400 K. Please use this fit in parts k)-o).

k) Under the assumption that the heat capacity of the lead is a linear function of temperature throughout the process please determine the heat transfer required to heat the Pb block from 273 K (state 1) to 350 K (state 2).

l) Using the first law, please determine the internal energy change of the lead block in process 1->2.

m) Assuming the internal energy is (arbitrarily) zero at T= 273 K, please develop an equation for the internal energy of the lead block as a function of temperature.

n) Using a control volume around the lead block, please determine the entropy change of the block if the temperature of the block is changed from 273 K to 350 K.

o) Assuming the entropy is (arbitrarily) zero at T= 273 K, please develop an equation for the entropy of the lead block as a function of temperature.

Two lead blocks one with mass 1 kg and temperature 500 K and the other with mass 2 kg and temperature 273 K are put into thermal contact and allowed to come to thermal equilibrium.

p) What is the predicted final temperature of the blocks using your fixed heat capacity model above?

q) What is the predicted final temperature using your linear heat capacity model above? Do you expect this temperature to be above or below the result in part p? Why?

r) What is the predicted entropy generated using your fixed heat capacity model?

s) What is the predicted entropy generated using your linear heat capacity model?

**Problem 2.**

A system consists of an externally insulated vessel filled with mercury. Immersed in the mercury is a paddle wheel that is driven by an external electric motor at a constant rotational speed of 180 rpm. When the vessel contains 65 kg of mercury, it is found that the torque the motor puts on the paddle wheel is 1.78 N-m. If the motor is operated for 8815 seconds and then shut off, the system settles to a temperature 10.00 °C warmer than the initial state.

If the mass of the mercury in the vessel is increased to 120 kg, the torque on the paddle wheel is 20% larger than before for the same operating speed. Under these latter conditions, running the motor for 5092 seconds increases the system temperature by 5.50 °C.

Finally, a new paddle wheel made of the same material as the original but with half the mass is swapped into the apparatus. The vessel is refilled with 120 kg of mercury. The electric motor is
turned on and at a rotational speed of 180 rpm, the torque on the paddle is 20% less than the original paddle under the same operating conditions. If the motor is operated for 10337 seconds and then shut off, the system settles to a temperature 8.00 C warmer than the initial state.

a) Describe a model of the composite system including its sub-components and their interactions, so as to be able to determine the heat capacity of the vessel, the heat capacity of the paddle wheel and the specific heat of mercury.

b) What are the heat capacities of the vessel and the original paddle wheel?

c) What is the specific heat of the mercury? (Looking the value up does not work here, be sure to show your work.)

Make a table similar to the one below showing whether the heat transfer, work transfer and the change in stored energy, change in stored entropy, entropy transfer and entropy generated of the paddle wheel, the vessel and the mercury is positive, negative or zero for a given change of temperature. Do the same for the composite system consisting of the paddle wheel, vessel and mercury.

<table>
<thead>
<tr>
<th></th>
<th>$Q_{1\rightarrow 2}$</th>
<th>$W_{1\rightarrow 2}$</th>
<th>$E_2-E_1$</th>
<th>$S_2-S_1$</th>
<th>$S_{\text{transfer}}$</th>
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<tbody>
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<td>Paddle wheel</td>
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<td>Vessel</td>
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<td>Mercury</td>
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<tr>
<td>Composite system</td>
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**Problem 3. The Putty Ball**

A ball of putty at a temperature of $T=300K$ falls on the ground from a height of $h$ m.

a) Does the putty system experience any energy transfer interactions across its boundary?

b) What is the temperature of the putty after it has just fallen down (but prior to having started a heat transfer with the environment) if $h = 1m$? If $h = 1000m$? ($g=9.81m/s^2$, specific heat $c = 2.0 \times 10^3 \text{ J/kg/K}$).

Assume now that $h$ is 1000m.

c) What is the entropy change of the putty system after it has just fallen to the ground?

The putty then cools down to the temperature of the surrounding air at $T=300K$.

d) What is the entropy change of the putty when this cooling process is completed?

e) What is the entropy change of the air?

f) Is the decrease in entropy of the putty equal to the increase in entropy of the air? Explain why or why not.
Problem 4.
An adiabatic cylinder, sketched to the right, contains nitrogen gas and a block of copper separated by an adiabatic membrane. Initially (state 1), the pressure and temperature of the gas are $6 \times 10^5$ Pa and 270 K, respectively. The copper block temperature is 380 K. A heavy adiabatic frictionless piston (M=45 kg) is pinned at a height of 20 cm above the surface of the copper block. The copper block is 1 cm thick. The surface area of the piston is 50 cm$^2$. The density of copper is 8.96 g/cm$^3$. The heat capacity of copper per unit mass is 385 J/kg K. The gas constant for nitrogen is 297 J/kg K and its specific heat capacity at constant volume is 741 J/kgK.

Please assume ambient pressure is zero absolute outside of the cylinder.

a) What is the mass of the contained nitrogen gas?
b) What is the total heat capacity of this gas?
c) What is the mass of the copper?
d) What is the total heat capacity of the copper?
e) How do these masses and heat capacities compare?

The pin is removed and the piston settles to a new position (state 2).

f) What is the position of the piston in state 2?
g) How much work was done on the copper block? What is the temperature of the copper block?
h) What are the temperature and volume of the nitrogen gas in state 2?
i) Has the entropy of the system changed during its transition from state 1 to state 2? If yes, what is the change in entropy?

The thin adiabatic membrane is now removed (with no work or heat interactions with the system). After some time the system settles to state 3.

j) What is the temperature of the system in state 3?
k) What is the position of the piston in state 3?
l) What is the total entropy generated in the transition from state 2 to state 3? Which of the components of the system has gone through the largest change in entropy?