

**Problem 1**

a) The 1-D heat equation is 
$$\frac{\partial(T - T_{init})}{\partial t} = \alpha \frac{\partial^2(T - T_{init})}{\partial x^2}$$

The first derivative of the complimentary error function is  $\frac{d}{du}[\text{erfc}(u)] = \frac{-2}{\sqrt{\pi}} \exp(-u^2)$

For the given temperature profile of 
$$T - T_{init} = \text{erfc}\left(\frac{x}{\sqrt{4\alpha t}}\right)(T_{surf} - T_{init}),$$

the first derivative with respect to time is obtained by carefully applying the chain rule

$$\begin{aligned}\frac{\partial(T - T_{init})}{\partial t} &= \frac{-2}{\sqrt{\pi}} \exp\left(\frac{-x^2}{4\alpha t}\right)(T_{surf} - T_{init}) \frac{x}{\sqrt{4\alpha}} \frac{-1}{2} t^{-\frac{3}{2}} \\ \frac{\partial(T - T_{init})}{\partial t} &= \frac{x}{\sqrt{4\alpha\pi}} \exp\left(\frac{-x^2}{4\alpha t}\right)(T_{surf} - T_{init}) t^{-\frac{3}{2}}\end{aligned}$$

The second derivative with respect to position x is obtained from...

$$\begin{aligned}\frac{\partial(T - T_{init})}{\partial x} &= \frac{-2}{\sqrt{\pi}} \exp\left(\frac{-x^2}{4\alpha t}\right)(T_{surf} - T_{init}) \frac{1}{\sqrt{4\alpha t}} \\ \frac{\partial^2(T - T_{init})}{\partial x^2} &= \frac{-2}{\sqrt{\pi}} \exp\left(\frac{-x^2}{4\alpha t}\right)(T_{surf} - T_{init}) \frac{1}{\sqrt{4\alpha t}} \frac{-2x}{4\alpha t} \\ \alpha \frac{\partial^2(T - T_{init})}{\partial x^2} &= \frac{x}{\sqrt{4\alpha\pi}} \exp\left(\frac{-x^2}{4\alpha t}\right)(T_{surf} - T_{init}) t^{-\frac{3}{2}}\end{aligned}$$

Therefore

$$\begin{aligned}\frac{\partial(T - T_{init})}{\partial t} &= \alpha \frac{\partial^2(T - T_{init})}{\partial x^2} \\ \frac{x}{\sqrt{4\alpha\pi}} \exp\left(\frac{-x^2}{4\alpha t}\right)(T_{surf} - T_{init}) t^{-\frac{3}{2}} &= \frac{x}{\sqrt{4\alpha\pi}} \exp\left(\frac{-x^2}{4\alpha t}\right)(T_{surf} - T_{init}) t^{-\frac{3}{2}}\end{aligned}$$

which checks as a solution to the heat equation.

b) In part (a) we have shown that the given temperature profile satisfies the heat equation. In order that this temperature profile apply specifically to a semi-infinite slab it must also satisfy the two boundary conditions and the initial condition.

Initial Condition:

**I.C.**  $T(x, 0) - T_{init} = 0$

$$T(x, 0) - T_{init} = \text{erfc}\left(\frac{x}{\sqrt{4\alpha \cdot 0}}\right)(T_{surf} - T_{init}) = \text{erfc}(\infty)(T_{surf} - T_{init})$$

**note**  $\text{erfc}(\infty) = 0$

$T(x, 0) - T_{init} = 0$

First Boundary Condition:

**B.C.**  $T(0, t) - T_{init} = T_{surf} - T_{init}$

$$T(0, t) - T_{init} = \text{erfc}(0)(T_{surf} - T_{init}) = (T_{surf} - T_{init})$$

Second Boundary Condition:

$$\text{B.C. } T(\infty, t) - T_{init} = 0$$

$$T(\infty, t) - T_{init} = \text{erfc}(\infty)(T_{surf} - T_{init}) = 0$$

c) The heat flux at the surface of the slab,  $x=0$ , is obtain by using Fourier's Law.

$$q(x, t) = -k \frac{\partial T}{\partial x} = -k \frac{\partial (T - T_{init})}{\partial x} = \frac{2}{\sqrt{\pi}} \exp\left(\frac{-x^2}{4\alpha t}\right) (T_{surf} - T_{init}) \frac{1}{\sqrt{4\alpha t}}$$

$$q(x, t) = \frac{k(T_{surf} - T_{init})}{\sqrt{\pi\alpha t}} \exp\left(\frac{-x^2}{4\alpha t}\right)$$

$$q(0, t) = \frac{k(T_{surf} - T_{init})}{\sqrt{\pi\alpha t}}$$

d) When two semi-infinite solids are brought into thermal contact, the interface temperature must be the same and the heat flux leaving one solid must be equal to the heat flux going into the second solid. In the problem statement we are told to assume that there is no interfacial thermal resistance between the solid. For this reason, there cannot be a temperature difference between the two interfaces. Secondly, since there is no work transfer at the interface, the First Law applied to a differential piece of the interface tells us that the flux leaving one solid must be equal to the flux going into the other solid.

$$T_{surf1} = T_{surf2}$$

$$k_1 \frac{\partial (T - T_{init})_1}{\partial x_1} = -k_2 \frac{\partial (T - T_{init})_2}{\partial x_2}$$

The negative sign is needed in the flux equation since the coordinate systems of the two slabs are different.

e) Find the expression for the interface temperature.

$$T_{surf1} = T_{surf2} = T_{surf}$$

$$k_1 \frac{\partial (T - T_{init})_1}{\partial x_1} \bigg|_{x_1=0} = -k_2 \frac{\partial (T - T_{init})_2}{\partial x_2} \bigg|_{x_2=0}$$

$$\frac{-k_1(T_{surf} - T_{init1})}{\sqrt{\pi\alpha_1 t}} = \frac{k_2(T_{surf} - T_{init2})}{\sqrt{\pi\alpha_2 t}}$$

$$T_{surf} = \frac{T_{init1}k_1\sqrt{\alpha_2} + T_{init2}k_2\sqrt{\alpha_1}}{k_1\sqrt{\alpha_2} + k_2\sqrt{\alpha_1}}$$

f) Using the thermophysical properties given in the problem, we can calculate  $T_{surf}$  for the three different blocks.

Copper Block:  $T_{surf} = 20.71^\circ\text{C}$

Stainless Steel Block:  $T_{surf} = 22.98^\circ\text{C}$

PVC Block:  $T_{surf} = 30.92^\circ\text{C}$

The copper block feels coldest to the touch.

g) Recall from part c that the expression for the heat flux at the surface is  $q(0, t) = \frac{k(T_{surf} - T_{init})}{\sqrt{\pi\alpha t}}$

The expression  $\sqrt{\pi\alpha t}$  has units of length. This expression can be thought of as the penetration depth, i.e., the depth into the solid that has experienced a change in temperature from its initial condition. This expression is often used in heat transfer analysis to obtain a scale for the time it takes an entire solid (or a part of a solid) to respond to a temperature change at its interface.

In the case of cooking a burger, we can use this length scale to obtain an approximate value for the time it takes to cook the burger. If we assume that the thickness of the burger is 15 mm and the thermal diffusivity of the burger is that of water, we can calculate a cooking time scale of

$$t = \frac{l^2}{\pi\alpha} = \frac{0.0075^2}{\pi(1.5 \times 10^{-7})} = 119 \text{ sec} = 2 \text{ min}$$

(We used half the thickness of the burger as our length scale, since heat diffuses in from both the top and bottom surfaces.)

### Problem 2

The rain keeps the surface of the asphalt at a constant 20 C. Let's model the asphalt as a semi-infinite solid, with an initial temperature of 50 C.

From equation 6.208 in the course reader (also derived in class), the heat flux into the asphalt at a given

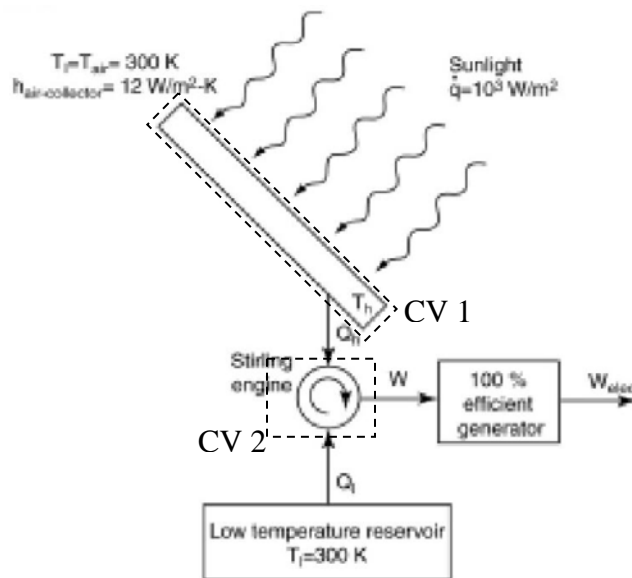
time is:  $\dot{q}_s(t) = \frac{k(T_s - T_i)}{\sqrt{\pi\alpha t}}$  where  $\alpha = \frac{k}{\rho c}$

We integrate with respect to  $t$  to get the total heat transfer per unit area *into* the asphalt, remembering that 30 minutes is 1800 seconds....:

$$\begin{aligned} Q'' &= \int_0^{30 \text{ min}} \dot{q}_s(t) \cdot dt \\ &= \frac{k(T_s - T_i)}{\sqrt{\pi\alpha}} \cdot 2\sqrt{t} \Big|_{t=30 \text{ min}} \\ &= -5.0 \times 10^5 \text{ J/m}^2 \end{aligned}$$

And of course, since we want the heat transferred *from* the asphalt, we take the negative of the value above:  $Q''_{out} = +5.0 \times 10^5 \text{ J/m}^2$

### Problem 3



a) Applying the First Law to CV 1 gives us an expression for  $\dot{Q}_h$   $\frac{dU}{dt} = \dot{Q} - \dot{W}$

Since the system is at steady state and there is no work transfer,  $\dot{Q} = 0$

$$\Rightarrow A\dot{q}_{sun} - (\dot{Q}_h + hA(T_h - T_{air})) = 0 \quad \Rightarrow \dot{Q}_h = A\dot{q}_{sun} - hA(T_h - T_{air})$$

The efficiency of a reversible engine operating between 2 thermal reservoirs is  $\eta_{max} = 1 - \frac{T_c}{T_h}$

Therefore the efficiency of this Stirling engine is  $\eta_{SE} = 0.2 \left( 1 - \frac{T_c}{T_h} \right) = \frac{\dot{W}}{\dot{Q}_h}$

Combining this result with the expression for  $\dot{Q}_h$  given above, we get an expression relating the power output of the Stirling engine to the temperature of the solar collector.

$$\dot{W} = 0.2 \left( 1 - \frac{T_c}{T_h} \right) \dot{Q}_h = 0.2 \left( 1 - \frac{T_c}{T_h} \right) (A\dot{q}_{sun} - hA(T_h - T_{air})) \quad (1)$$

To maximise the work, we take the derivative of the power with respect to  $T_h$  and set it equal to zero.

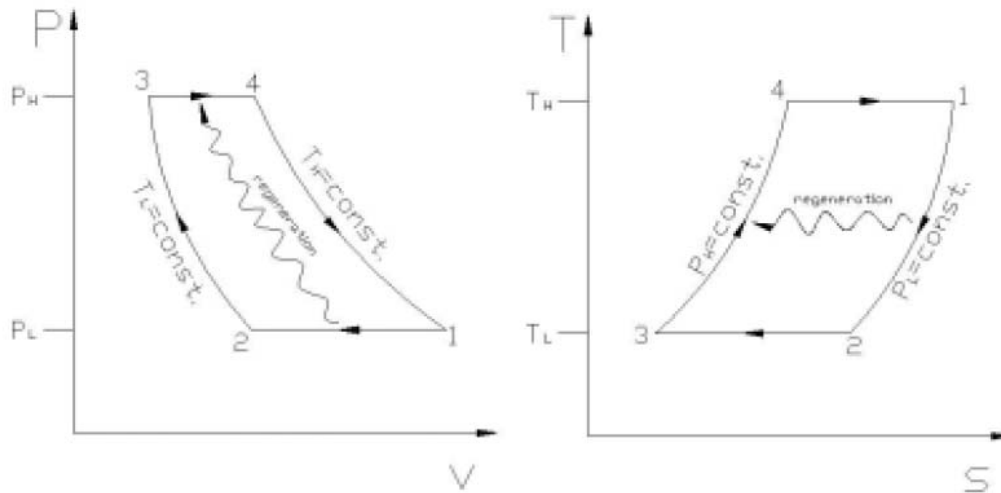
$$\frac{d\dot{W}}{dT_h} = 0.2 \left( \frac{T_c}{T_h^2} \right) (A\dot{q}_{sun} - hA(T_h - T_{air})) + 0.2 \left( 1 - \frac{T_c}{T_h} \right) (-hA) = 0$$

$$\Rightarrow T_h^2 = (T_c) \frac{\dot{q}_{sun}}{h} + T_c T_{air} \quad \Rightarrow T_h = \sqrt{T_c \left( \frac{\dot{q}_{sun}}{h} + T_{air} \right)} = 339.11 \text{ K}$$

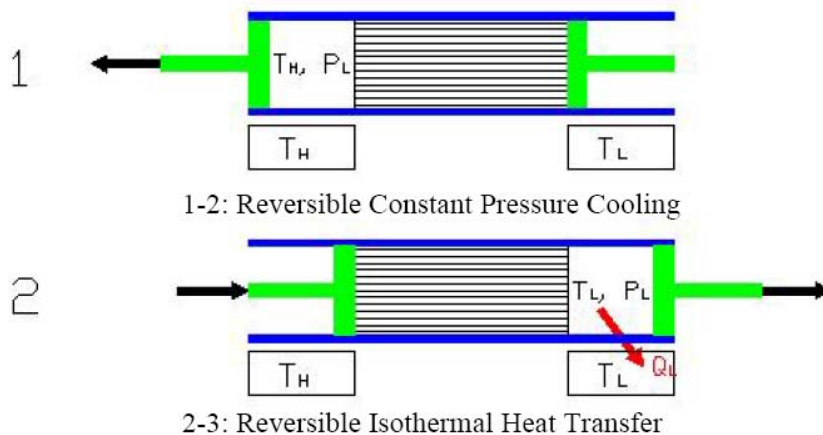
b) The power output of the generator at this operating condition is determined by plugging in the value of  $T_h$  into equation (1). We get  $\dot{W} = 122.41 \text{ W}$

c) The efficiency for solar to electric power conversion is  $\eta_{total} = \frac{\dot{W}}{A\dot{q}_{sun}} = \frac{122.41}{10 \times 10^3} = 0.0122$

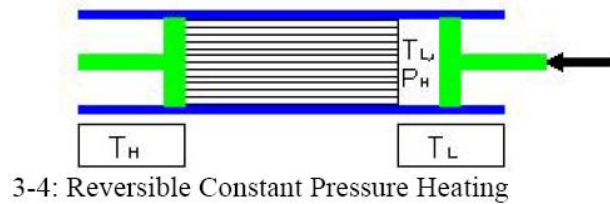
#### Problem 4



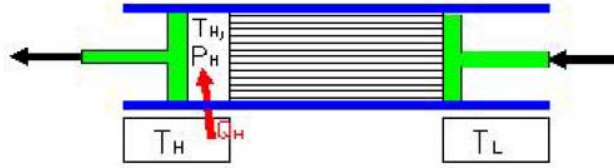
Below is one possible realisation of the Ericsson cycle. (You may have other designs that run the same thermodynamic cycle.)



3



4



For Process 1-2, the reversible constant pressure cooling at  $P_L$ ,

$$W_{1-2} = \int_1^2 P dV = P_L \int_1^2 dV = P_L (V_2 - V_1) = P_L \left( \frac{mRT_2}{P_2} - \frac{mRT_1}{P_1} \right) = mR(T_2 - T_1) = mR(T_L - T_H)$$

(In the above equation, we used the fact that the pressure stays constant over the process 1-2, and is equal to  $P_L$ )

From the 1<sup>st</sup> Law,  $Q_{1-2} = W_{1-2} + (U_2 - U_1) = W_{1-2} + mc_v(T_2 - T_1) = W_{1-2} + mc_v(T_L - T_H)$

$$\Rightarrow Q_{1-2} = mR(T_L - T_H) + mc_v(T_L - T_H) = mc_p(T_L - T_H)$$

This heat transfer is negative, which means that heat leaves the ideal gas and goes into the regenerator.

For Process 2-3, the reversible isothermal heat transfer at  $T_L$ ,

The analysis proceeds in the same way as for Process 4-1.

$$\text{In this case, } W_{2-3} = Q_{2-3} = mRT_L \ln \frac{P_L}{P_H}$$

Since  $P_L < P_H$ , the heat transfer and work transfer are negative – heat leaves the system and work enters the system.

For Process 3-4, the reversible constant pressure heating at  $P_H$ ,

The analysis proceeds in the same way as for Process 1-2.

$$\text{In this case, } W_{3-4} = mR(T_H - T_L) \text{ and } Q_{3-4} = mc_p(T_H - T_L)$$

Note that the heat and work transfers in this case are equal and opposite to the heat and work transfers in process 1-2. All the heat that was transferred to the regenerator in process 1-2 is transferred back to the gas in process 3-4.

To find the cycle efficiency, we need to find the net work output and the heat input.

The net work output is  $W_{net} = W_{4-1} + W_{1-2} + W_{2-3} + W_{3-4}$

$$= mRT_H \ln \frac{P_H}{P_L} + mR(T_L - T_H) + mRT_L \ln \frac{P_L}{P_H} + mR(T_H - T_L)$$

$$\Rightarrow W_{net} = mR(T_H - T_L) \ln \frac{P_H}{P_L}$$

The heat input occurs in process 4-1. (Heat is rejected to the low temperature reservoir in process 2-3. As shown previously, the heat transfers in processes 1-2 and 3-4 are equal and opposite: there is no net heat transfer to the regenerator.)

$$Q_{in} = Q_{4-1} = mRT_H \ln \frac{P_H}{P_L}$$

The efficiency of the cycle is defined as  $\eta = \frac{W_{net}}{Q_{in}}$

$$\text{In this case, it is calculated to be } \eta = \frac{mR(T_H - T_L) \ln \frac{P_H}{P_L}}{mRT_H \ln \frac{P_H}{P_L}} = \left(1 - \frac{T_L}{T_H}\right)$$

This is the same as that of a Carnot engine. This is not surprising, since any reversible engine operating between the same fixed temperature reservoirs  $T_H$  and  $T_L$  will have the same efficiency.

d) The heat absorbed at  $T_H$  is  $Q_{in} = 0.001 \times 287 \times 900 \times \ln 100 = 1189.5 \text{ J / cycle}$

The heat rejected at  $T_L$  is  $Q_{out} = Q_{2-3} = 0.001 \times 287 \times 300 \times \ln \frac{1}{100} = -396.5 \text{ J / cycle}$

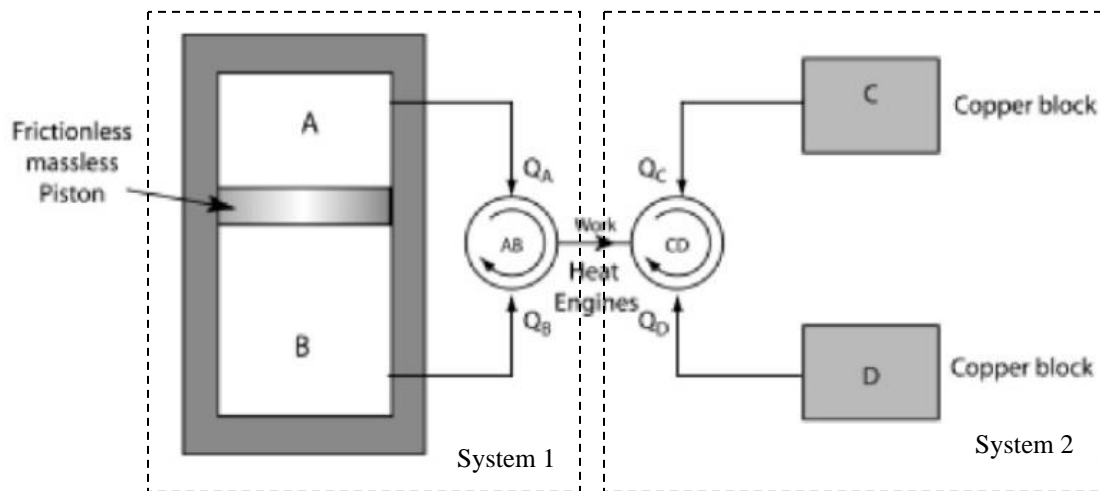
The net work transfer is  $W_{net} = 0.001 \times 287 \times (900 - 300) \times \ln 100 = 793.0 \text{ J / cycle}$

The thermal efficiency of the cycle is 0.667.

### Problem 5

a) Since System 1 (as shown in the figure below) is reversible and adiabatic, the entropy of the system does not change from the initial state to the final state (by the Second Law). Therefore, we have

$$\Delta S_A + \Delta S_B = m_A \left( c_V \ln \frac{T_{A,2}}{T_{A,1}} + R \ln \frac{V_{A,2}}{V_{A,1}} \right) + m_B \left( c_V \ln \frac{T_{B,2}}{T_{B,1}} + R \ln \frac{V_{B,2}}{V_{B,1}} \right) = 0$$



We are told that the final volumes of A and B are the same. Note that all times, the pressures in A and B must be same, because of the frictionless, massless piston. Since the masses of gases in A and B are also the same, the final temperatures of A and B must also be the same (this just follows from the ideal gas law). Let this final temperature be  $T_2$

We are not given the volumes of A and B, but we only require the volume ratios in the Second Law equation. Let the initial volume of A be  $V_{A,1}$ . Since the mass and initial pressure of A and B are the same,

$$\frac{m_{A,1} R}{P_{A,1}} = \frac{m_{B,1} R}{P_{B,1}} \Rightarrow \frac{V_{A,1}}{T_{A,1}} = \frac{V_{B,1}}{T_{B,1}} \Rightarrow \frac{V_{A,1}}{V_{B,1}} = \frac{250}{550}$$

$$\Rightarrow V_{B,1} = V_{A,1} \frac{T_{B,1}}{T_{A,1}} = 2.2 V_{A,1}$$

Therefore, the total initial volume is  $V_{A,1} + V_{B,1} = 3.2 V_{A,1}$ . Since the final volumes are equal, and the total volume remains the same, we have  $V_{A,2} = V_{B,2} = 1.6 V_{A,1}$ .

Therefore,  $\frac{V_{A,2}}{V_{A,1}} = 1.6$  and  $\frac{V_{B,2}}{V_{B,1}} = \frac{1.6}{2.2} = 0.727$

Plugging this back into the Second Law equation, we get

$$m_A \left( c_v \ln \frac{T_2}{250} + R \ln 1.6 \right) + m_B \left( c_v \ln \frac{T_2}{550} + R \ln 0.727 \right) = 0$$

Since the masses are equal, they drop out of the equation. Combining terms, we get

$$c_v \ln \left( \frac{T_2}{250} \frac{T_2}{550} \right) + R \ln(1.6 \times 0.727) = 0$$

$$T_2 = \sqrt{250 \times 550 \times (1.6 \times 0.727)^{-R/c_v}} = 352.75 \text{ K}$$

b) The total work extracted from the cylinder is found by applying the First Law to System 1:

$$\Delta U_A + \Delta U_B = Q - W$$

There is no heat transfer from the system ( $Q_A$  and  $Q_B$  are internal to the system). Therefore,

$$W = -(\Delta U_A + \Delta U_B) = -(m_A c_v (T_2 - T_{A,1}) + m_B c_v (T_2 - T_{B,1})) = 29.8 \text{ kJ}$$

c) If the heat engine is not allowed to communicate with any other thermal reservoir which is at a temperature different from the final temperature of gases A and B then the maximum amount of work has been extracted from the cylinder consisting of gas A and B. No more positive work can be extracted from the heat engine.

d) The process within System 2 (shown in the figure) is also reversible and adiabatic. If we apply the Second Law to this system, we get

$$\Delta S_C + \Delta S_D = 0$$

$$\Rightarrow C_{block} \ln \frac{T_{C,2}}{T_{C,1}} + C_{block} \ln \frac{T_{D,2}}{T_{D,1}} = 0 \quad \Rightarrow T_{D,2} = \frac{T_{C,1} T_{D,1}}{T_{C,2}}$$

Using this result and applying the First Law to System 2, we can solve for  $T_{C,2}$ :

$$\Delta U_C + \Delta U_D = Q - W$$

There is no heat transfer to this system; we are told that the work transfer is  $W = -80000 \text{ J}$

$$\Rightarrow C_{block} (T_{C,2} - T_{C,1}) + C_{block} (T_{D,2} - T_{D,1}) = -(-80000)$$

$$\Rightarrow (T_{C,2} - T_{C,1}) + (T_{D,2} - T_{D,1}) = \frac{80000}{C} = \frac{80000}{400} = 200$$

$$T_{C,1} = T_{D,1} = 300 \text{ K}$$

$$\Rightarrow T_{C,2} + \frac{T_{C,1} T_{D,1}}{T_{C,2}} = 200 + 300 + 300$$

$$\Rightarrow T_{C,2}^2 - 800 T_{C,2} + 90000 = 0$$

This equation has two solutions,  $T_{C,2} = 664.58 \text{ K}$  and  $T_{C,2} = 135.43 \text{ K}$

For  $T_{C,2} = 664.58 \text{ K}$ , we get  $T_{D,2} = 135.43 \text{ K}$

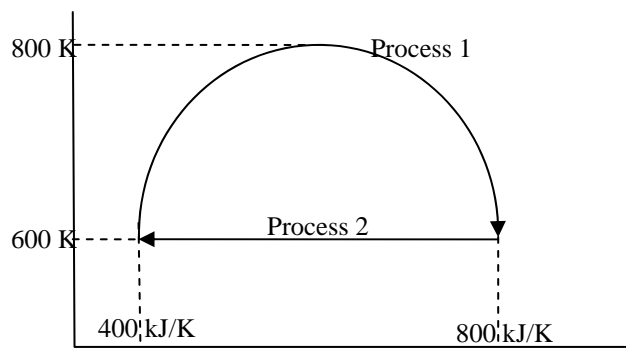
For  $T_{C,2} = 135.43 \text{ K}$ , we get  $T_{D,2} = 664.58 \text{ K}$

Since we are told that C is the colder block,  $T_{D,2} = 664.58 \text{ K}$  and  $T_{C,2} = 135.43 \text{ K}$

## Problem 6

A reversible cycle executes the cycle shown, between a maximum temperature of 800 K and a minimum temperature of 600 K. This cycle is different from the "two heat-reservoir" cycles we discussed in class

because the heat input takes place over a range of temperatures (600 K – 800 K). This engine interacts with several high-temperature reservoirs and one low-temperature reservoir (at 600 K).



(a) and (c) Determine the direction in which the engine executes the cycle, and indicate the path along which heat is rejected.

Since this cycle is a heat engine, i.e. a power-producing system, it must take heat (and entropy) from high temperature reservoirs and dump the entropy (along with some of the heat) to the low temperature reservoir, extracting some useful work in the process. So the heat transfer in must be at a higher temperature than the heat transfer out (in contrast to a refrigerator, where the heat transfer in is at a low temperature, and the heat transfer out is at a high temperature).

To figure out whether heat is entering or leaving a system during a reversible process, we look at the entropy. From the second law for a reversible process,  $dS = \int \frac{\delta Q}{T}$ , a positive heat transfer must increase the entropy of the system. A negative heat transfer decreases the entropy of the system.

We can break the cycle into two processes – the semi-circular arc (process 1) and the straight line (process 2). To produce power, the heat transfer must be positive during process 1. Therefore, the entropy of the system must increase during process 1. Similarly, the heat transfer must be out of the system during process 2, and therefore, the entropy of the system must decrease during process 2. So the engine executes a clockwise cycle and heat is rejected in process 2 (the straight-line part of the cycle).

(b) The net work

For a cycle, the 1<sup>st</sup> Law is  $\oint \delta Q = \oint \delta W$

The cyclic integral of heat for a reversible process is  $\oint \delta Q = \oint T dS$  (from the 2<sup>nd</sup> Law), which is just the area under the T-S graph. Since this region is semi-circular, its area can be calculated as  $\frac{1}{2} \pi r^2$ , where the radius is 200. Therefore, the cyclic integral of heat,  $\oint \delta Q$ , is 62832 kJ, which is also equal to the net work.

(d) The thermodynamic efficiency

The efficiency for an engine is defined as  $\eta = \frac{W}{Q_H}$

We found W in part (b); we now have to find  $Q_H$ . To do this, we apply the 2<sup>nd</sup> Law to process 1, the heat input process. The heat input is given by  $Q_H = \int_1 T dS$ , which is the area under the curve for process 1. This is the sum of the areas of the semi-circle, and the rectangle between (400 kJ/K to 800 kJ/K) and (0 K to 600 K). This total area is 62832 + 240000 = 302832 kJ. So the efficiency is  $W/Q_H = 62832/302832 = 20.75\%$

(e) The Carnot efficiency for a cycle running between 600 K and 800 K.

$$\eta_c = 1 - \frac{T_L}{T_H} = 1 - \frac{600}{800} = 25\%.$$

This is higher than the efficiency of the reversible cycle we calculated. This is not an inconsistency – the cycle we have analysed does not operate between two fixed temperature reservoirs like a Carnot cycle and thus does not have the same efficiency as the Carnot cycle, even though it is completely reversible.



### The irreversible engine

We now take a look at an irreversible cycle with  $W_{\text{irrev}} = 0.9W_{\text{rev}} = 56549 \text{ kJ}$ . The heat input is still 302832 kJ. The entropy transferred during the heat input is still the same as that in the reversible case.

We know that  $Q_{\text{L,rev}} = Q_{\text{H}} - W_{\text{rev}} = 240000 \text{ kJ}$ . This is transferred to the (fixed) low temperature reservoir at 600 K. The entropy transferred to the low temperature reservoir in the reversible cycle is  $240000/600 = 400 \text{ kJ/K}$ . Therefore, the entropy transferred in must also be 400 kJ/K.

A much easier way of seeing this is looking at process 1. The entropy change in process 1 (the heat input process) is 400 kJ/K (from the cycle plot). Since this process is reversible, all the entropy change must be due to entropy transfer.

(f) and (g) Calculate  $\oint \delta S_{\text{gen}}$ , and is  $Q_{\text{L,rev}}$  greater than, less than, or equal to  $Q_{\text{L,irrev}}$ ?

Applying the 1<sup>st</sup> Law to the reversible cycle,  $Q_{\text{H}} - Q_{\text{L,rev}} = W_{\text{rev}}$

$$Q_{\text{L,rev}} = Q_{\text{H}} - W_{\text{rev}} = 302832 - 60832 = 240000 \text{ kJ}$$

Applying the 1<sup>st</sup> Law to the irreversible cycle,  $Q_{\text{H}} - Q_{\text{L,irrev}} = W_{\text{irrev}}$

$$Q_{\text{L,irrev}} = Q_{\text{H}} - W_{\text{irrev}} = 302832 - 56549 = 246283 \text{ kJ}$$

$Q_{\text{L,rev}}$  is smaller than  $Q_{\text{L,irrev}}$ . This is expected – less work is extracted in the irreversible cycle. This is due to entropy generation in the irreversible engine. Because the entropy being transferred to the 600 K reservoir is the sum of the entropy transferred in during process 1 and the entropy generated, the  $Q_{\text{L}}$  required is greater in the irreversible case.

To find the entropy generation in one cycle, apply the 2<sup>nd</sup> Law to the irreversible engine over a complete cycle.

$$\oint \frac{\delta Q_{\text{H}}}{T} + \oint \frac{\delta Q_{\text{L,irrev}}}{T} + \oint S_{\text{gen}} = 0$$

$\oint \frac{\delta Q_{\text{H}}}{T}$  is the entropy transferred in during the heat input process, which we calculated to be 400 kJ/K.

$\oint \frac{\delta Q_{\text{L,irrev}}}{T}$  simplifies to  $\frac{Q_{\text{L,irrev}}}{T_{\text{L}}}$ , which is  $-246283/600 = -410.47 \text{ kJ/K}$  ( $Q_{\text{L}}$  leaves the system and is negative)

Plugging these values in,  $\oint S_{\text{gen}} = 10.47 \text{ kJ/K}$