
24.903
Language & Structure III: Semantics and Pragmatics
Spring 2003, 2-151, MW 1-2.30
February 18, 2003
Assignment 2, due in class on February 26

1. Determine whether each of the following is true or false (and say why).

(a) $\phi \subseteq \phi$

True. The empty set is a subset of every set, and so a subset of itself. A is subset of B iff every element of A is an element of B. Since an empty set has no elements, this condition is vacuously satisfied.

(b) $\phi \in \phi$

False. The empty set has no members.

(c) $\phi \in \{\phi\}$

True. Here ϕ is a member.

(d) $\phi \subseteq \{\phi\}$

True, see the reasoning in 1a.

(e) $\{a, b\} \in \{a, b, c, \{a, b\}\}$

True. $\{a, b\}$ is a member of $\{a, b, \{a, b\}\}$.

(f) $\{a, b\} \subseteq \{a, b, \{a, b\}\}$

True. Both a and b are members of $\{a, b, \{a, b\}\}$.

(g) $\{a, b\} \subseteq 2^{\{a, b, \{a, b\}\}}$

False. $2^{\{a, b, \{a, b\}\}} = \{\{\}, \{a\}, \{b\}, \{\{a, b\}\}, \{a, b\}, \{a, \{a, b\}\}, \{b, \{a, b\}\}, \{a, b, \{a, b\}\}\}$. a (or b) is not a member of $2^{\{a, b, \{a, b\}\}}$.

(h) $\{\{a, b\}\} \subseteq 2^{\{a, b, \{a, b\}\}}$

True. $\{a, b\}$ is a member of the power set, and hence $\{\{a, b\}\}$ is a subset of the powerset.

(i) $\{a, b, \{a, b\}\} - \{a, b\} = \{a, b\}$

False. $\{a, b, \{a, b\}\} - \{a, b\} = \{\{a, b\}\}$.

2. The set-theoretic laws concerning \cup and \cap seem to be related to the laws of arithmetics concerning $+$ (addition) and $*$ (multiplication). For example, $+$ is commutative, as we have $a + b = b + a$. Compare the set-theoretic laws with the basic arithmetic laws and note similarities and differences. (Do this with commutativity, associativity, distributivity and idempotency).

Addition and Multiplication like \cup and \cap are commutative.

$$a * b = b * a, a + b = b + a$$

Unlike \cup and \cap , addition and multiplication are not idempotent.

Unless $a = 0$, $a + a \neq a$, and unless $a = 0, 1$, $a * a \neq a$.

Addition and Multiplication like \cup and \cap are associative.

$$a * (b * c) = (a * b) * c, \text{ and } a + (b + c) = (a + b) + c$$

Multiplication distributes over addition like \cup distributes over \cap and \cap distributes over \cup .

$$a * (b + c) = (a * b) + (a * c)$$

However, addition does not distribute over multiplication.

$$a + (b * c) \neq (a * b) + (a * c) \text{ (in general)}$$

3. Which of the following statements are true (and why)?

(a) $\{x : x = a\} = \{a\}$

True. The LHS set consists all the things that are identical to a . Only a is identical to itself - therefore the LHS = RHS.

(b) $\{x : x \text{ is green}\} = \{y : y \text{ is green}\}$

True. Both LHS and RHS are the set of green things. Variables do not matter.

(c) $\{x : x \in A\} = A$

True. The set of all things that are members of A is nothing but A itself.

(d) $\{x : x \in \{y : y \in B\}\} = B$

True. Using (c), we simply the LHS to $\{x : x \in B\}$. Then we can use (c) again to simply the LHS to B .

(e) $\{x : \{y : y \text{ likes } x\} = \phi\} = \{x : \{x : x \text{ likes } x\} = \phi\}$

These two sets are not necessarily equal. But this does not mean they can never be equal. To determine the circumstances under which they are equal, let us see what they stand for.

The LHS is the set of people such that the set of people who like them is empty i.e. the LHS is the set of people who nobody likes.

The RHS is a curious set. The top-level x does not actually bind anything. Such sets can either denote the universal set U or ϕ depending upon whether the inner proposition is true or false. The inner proposition is $\{x :: x \text{ likes } x\} = \phi$. If this proposition is true, it means that the set $\{x :: x \text{ likes } x\}$ is empty i.e. there no people who like themselves. To rephrase: If the word contains no self-likers, the RHS = U . If the word contains self-likers, the RHS = ϕ .

Case 1: The LHS is ϕ . If LHS is ϕ , it means that everyone in this word is liked by someone or the other. If LHS = RHS, this means that RHS = ϕ i.e. the world contains self-likers. So the LHS and RHS can be ϕ when everyone is liked by someone and at least one person likes her/himself.

Case 2: The LHS is U . If LHS is U , then no one in this world is liked by anyone. If LHS = RHS, this means that RHS = U i.e. the world contains no self-likers. These two situations are compatible (in fact the LHS situation entails the RHS situation). So the LHS and RHS can be U when no one likes anyone.

To sum up, the two sets are equal if (i) everyone is liked by someone and at least one person likes her/himself, or (ii) no one likes anyone. In all other cases, the two sets are unequal.

4. Give the characteristic functions of the sets $\{ \}$, $\{1, 3\}$, $\{3, 4\}$ and $\{1, 2, 3, 4\}$, with respect to the universe $\{1,2,3,4\}$. Specify them as sets of pairs, or in a notation using arrows.

$$f_{\{ \}} = \{(1, 0), (2, 0), (3, 0), (4, 0)\}$$

$$f_{\{1,3\}} = \{(1, 1), (2, 0), (3, 1), (4, 0)\}$$

$$f_{\{3,4\}} = \{(1, 0), (2, 0), (3, 1), (4, 1)\}$$

$$f_{\{1,2,3,4\}} = \{(1, 1), (2, 1), (3, 1), (4, 1)\}$$

From Heim & Kratzer

1. Exercise on sentence connectives (pg. 23), and Exercise 2 on Currying (pg. 32). These exercises are closely related and should be answered as a unit.

a. Extend the rules.

We need to add the following rule to deal with V' 's:

$$\llbracket [V'\gamma \beta] \rrbracket = \llbracket \gamma \rrbracket \langle \llbracket \beta \rrbracket \rangle$$

b. Represented relationally, we can describe the denotation of *introduce* quite simply as: $\{(A, M, J), (M, J, A)\}$. From this set, the characteristic function of this set can easily be constructed. To convert this function which takes 3-tuples as arguments into a function that takes three arguments one by one, we need to schönfinkelize. Since the verb first combines with the second place argument, then the third place argument, and finally the first place argument, we need to do a 2-3-1 schönfinkelization

The 2-3-1 schönfinkelization (using set notation)

$$\begin{aligned} &\{(A, (A, (A, 0))), (A, (A, (J, 0))), (A, (A, (M, 0))), \\ &(A, (J, (A, 0))), (A, (J, (J, 0))), (A, (J, (M, 0))), \\ &(A, (M, (A, 0))), (A, (M, (J, 0))), (A, (M, (M, 0))), \\ &(J, (A, (A, 0))), (J, (A, (J, 0))), (J, (A, (M, 1))), \\ &(J, (J, (A, 0))), (J, (J, (J, 0))), (J, (J, (M, 0))), \\ &(J, (M, (A, 0))), (J, (M, (J, 0))), (J, (M, (M, 0))), \\ &(M, (A, (A, 0))), (M, (A, (J, 0))), (M, (A, (M, 0))), \\ &(M, (J, (A, 1))), (M, (J, (J, 0))), (M, (J, (M, 0))), \\ &(M, (M, (A, 0))), (M, (M, (J, 0))), (M, (M, (M, 0)))\} \end{aligned}$$

I hope it is transparent how the above information can be represented as a table. If it is not, please ask me.

c. We have already seen the denotation of *introduce* above. Applying the above function to the denotation of *Maria*, which I will assume is **M** yields the following function (again represented in set notation):

$$\begin{aligned} &\{(A, (A, 0)), (A, (J, 0)), (A, (M, 0)), \\ &(J, (A, 1)), (J, (J, 0)), (J, (M, 0)), \\ &(M, (A, 0)), (M, (J, 0)), (M, (M, 0))\} \end{aligned}$$

The next argument we get is the denotation of the PP *to Jacob*, which reduces to the denotation of *Jacob*, which I assume is **J**. Applying the above function to **J** yields the following function (again represented in set notation):

$$\{(A, 1), (J, 0), (M, 0)\}$$

The final argument is the denotation of *Ann*, which I assume is **A**. Applying the above function to **A** yields 1. This is good because we know that the sentence Ann introduces Maria to Jacob is true.

d. For any $x, y, z \in D$, $f(x)(y)(z) = 1$ iff $(z, x, y) \in X$

This question is just testing that you have understood that in order to get the correct denotation for *introduce*, you need to do a 2-3-1 schönfinkelization.

2. Exercise on functions as sets (pg. 24)

$$\llbracket [\beta \gamma] \rrbracket = \llbracket \beta \rrbracket \in \llbracket \gamma \rrbracket$$

3. Exercise 1 on Currying (pg. 31) ¹

Characteristic function for *Radores*:

(using set notation)

$$\{((J, J), 0), ((J, M), 1), ((M, J), 0), ((M, M), 1)\}$$

Characteristic function for *Rassignsto*:

¹In my solutions here, I am using parentheses to mark ordered pairs, while Heim & Kratzer use angled brackets.

(using set notation)

$\{((J, J, J), 0), ((J, J, M), 1), ((J, M, J), 0), ((J, M, M), 0),$
 $((M, J, J), 0), ((M, J, M), 1), ((M, M, J), 0), ((M, M, M), 0)\}$

Right-to-left Schönfinkelization of the characteristic function of R_{adore} :

(using set notation)

$\{(J, (J, 0)), (J, (M, 0)), (M, (J, 1)), (M, (M, 1))\}$

Right-to-left Schönfinkelization of the characteristic function of $R_{assignsto}$:

(using set notation)

$\{(J, (J, (J, 0))), (J, (J, (M, 0))), (J, (M, (J, 0))), (J, (M, (M, 0))),$
 $(M, (J, (J, 1))), (M, (J, (M, 1))), (M, (M, (J, 0))), (M, (M, (M, 0)))\}$

The right-to-left schönfinkeled characteristic function of R_{adore} is a suitable denotation for *adore* because the direct object of *adore*, which is the sister of the verb, is the outermost argument.

The right-to-left schönfinkeled characteristic function of $R_{assignsto}$ is not a suitable denotation for *assigns to* if we assume a syntactic structure like the kind Heim & Kratzer assume for *introduce* where the verb first combines with the direct object and then with the indirect object. With the right-to-left schönfinkelization, the third argument (the *to c*) becomes the outermost argument. However, under the indicated syntactic assumptions, this argument is not the one that is structurally closest to the verb. We need a different schönfinkelization order to get the correct denotation. However, if we assume a syntactic structure where the verb combines with the indirect object first and then with the direct object, a right-to-left schönfinkelization may be tenable.