

## 1 Rules of Inference

- (1) Rules of Inference
  - a. Modus Ponens  
Premises:  $P \rightarrow Q, P$   
Conclusion:  $Q$
  - b. Modus Tollens  
Premises:  $P \rightarrow Q, \neg Q$   
Conclusion:  $\neg P$
  - c. Hypothetical Syllogism  
Premises:  $P \rightarrow Q, Q \rightarrow R$   
Conclusion:  $P \rightarrow R$
  - d. Disjunctive Syllogism  
Premises:  $P \vee Q, \neg P$   
Conclusion:  $Q$
  - e. Simplification  
Premises:  $P \wedge Q$   
Conclusion:  $P$
  - f. Conjunction  
Premises:  $P, Q$   
Conclusion:  $P \wedge Q$
  - g. Addition  
Premises:  $P$   
Conclusion:  $P \vee Q$

## 2 Sample Inferences

Premises:  $p \rightarrow q, p \vee s, q \rightarrow r, s \rightarrow t, \neg r$

Conclusion:  $t$

Premises:  $p \rightarrow q, \neg p \rightarrow r$

Conclusion:  $q \vee r$

- (2) If the subject has not understood the instructions or has not finished reading the sentence, then he has pressed the wrong button or has failed to answer. If he has failed to answer, then the timer hasn't stopped. The subject has pressed the right button, and the timer has stopped. Therefore, the subject understood the instructions.

## 3 Conditional and Indirect Proofs

- (3) **Conditional Proofs:** Suppose we have  $P_1, \dots, P_n$  as premises and we want to prove  $Q \rightarrow R$ .

In a conditional proof, we add the antecedent  $Q$  of the conclusion as an additional *auxiliary* premise and then from the original premises together with the auxiliary premise, we derive  $Q$ .

If we can do this, it is equivalent to deriving  $Q \rightarrow R$ .

The validity of conditional proofs is based on the equivalence of

$((P_1 \wedge \dots \wedge P_n) \rightarrow (Q \rightarrow R))$

and

$((P_1 \wedge \dots \wedge P_n \wedge Q) \rightarrow R)$

Examples of Conditional Proofs:

(4) a. Premises:  $(p \rightarrow (q \vee r)), \neg r$

Conclusion:  $p \rightarrow q$

b. Premises:  $p \rightarrow (q \wedge r)$

Conclusion:  $((q \rightarrow s) \rightarrow (p \rightarrow s))$

(5) Indirect Proofs (*reductio ad absurdum*)

If we want to reach a certain conclusion, we assume its negation and show that its negation together with our premises leads to a contradiction, we have in effect shown that our conclusion follows from the premises.

Proving  $p$  from the premises  $(p \vee q), (q \rightarrow r), \neg r$

## 4 Several Types for or

(6) a. John is watching TV **or** he is eating dinner. [ $or^{tv}$ ]

b. John is watching TV **or** eating dinner. [ $or^{set}$ ]

c. John is boiling **or** poaching eggs.

With different types, go different semantic denotations.

The exact denotations depend upon what we take **or** to mean.

- Inclusive *or*

- 'Exclusive' *or*

(7) a. Either you eat your dinner or I'll spank you.

b. Coffee or tea comes with the meal.

c. Give me liberty or give me death!

Two options for  $or^{tv}$ :

Inclusive *or* ( $\vee$ )

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Exclusive *or* ( $\otimes$ )

p	q	$p \otimes q$
0	0	0
0	1	1
1	0	1
1	1	0

- Denotations for  $or^{set}$

(8) Mary jogs or swims or lifts weights.

The truth conditions of  $p \otimes q \otimes r$

## 5 Pelleter on *or*

- (9) a. *Or* in English is unambiguous, with the inclusive meaning as its only literal meaning.
- b. Complete utterance meaning is computed by drawing inferences from the literal meaning and assumptions about the speaker's beliefs and intentions.

### 5.1 Embedded *or*

When *or* appears embedded, it is read 'inclusively'.

- (10) a. I won't go fishing if it's raining or I have a headache.
- b. Anybody who is an American citizen or has already checked their bags can go to the front of the line.
- c. I wonder whether (or not) he wrote or called.

### 5.2 Interaction with Negation

- (11) He didn't write or call.

Possible evidence: if *neither .... nor = not + either .... or*

- (12) He neither wrote nor called.

Further support for the 'inclusive' proposal:

- (13) Putative instances of 'exclusive' *or*:
- a. Give me liberty or give me death!
- b. Arlene wants a marguerita or a cosmopolitan.
- c. Coffee or tea comes with the meal.
- (14) a. I demand neither liberty nor death.
- b. Arlene wants neither a marguerita nor a cosmopolitan.
- c. Neither Coffee nor tea comes with the meal.

Possible counterevidence:

- (15) He didn't write **OR** call - he did both!

### 5.3 Background Assumptions

Some non-arguments for 'exclusive' *or*:

- (16) a. Taxpayers must file exactly one return, but it may be a single or a joint return.
- b. Today is either Monday or Wednesday.
- c. Madonna was born in either Michigan or Minnesota.

When listeners spontaneously infer 'not both  $\phi$  and  $\psi$ ' from ' $\phi$  or  $\psi$ ', this is plausibly due to our background assumptions.

- (17) a. Salad or soup comes with this meal.  
 b. Either you shut up or I will throw you out.

(18) Plausible Explanations

- a. Restaurants have a commercial interest in fulfilling their commitment at minimal cost.  
 b. Speakers prefer to make good in their words with a minimum of antagonistic behavior.

- Assume you hear (17b), you shut up, and are still thrown out. Can you say the speaker of (17b) was lying?

#### 5.4 A sketch of Grice's proposal

The use of *and* would be more **informative** if we know that both disjuncts are true. Hence *or* is generally not used in such cases.

- Assumes that the basic meaning of *or* is inclusive.

## 6 Hurford's Counterproposal

Premise 1: ' $\phi$  or  $\psi$ ' is infelicitous whenever  $\phi$  entails  $\psi$  or vice versa.

- (19) a. ??John is an American or a Texan.  
 b. ??I will go to the Netherlands or to Amsterdam.

Premise 2: Statements of the form ' $[\phi$  or  $\psi]$  or both ( $\phi$  and  $\psi$ )' are generally felicitous.

- (20) a. I will apply to Stanford or Chicago or to both (Stanford and Chicago).  
 b. For breakfast, you can have eggs or pancakes or both.

Premise 3: ' $\phi$  and  $\psi$ ' entails ' $\phi$  or<sub>incl</sub>  $\psi$ ', but does not entail ' $\phi$  or<sub>excl</sub>  $\psi$ '.

Conclusion: The first *or* in (20) has to be or<sub>excl</sub>.

Pelletier's response: 1. The last disjunct is redundant.

2. This is reflected in its intonational contour.

3. It is an 'emphatic afterthought' calling attention to the willingness to the truth or both disjuncts.