Note to 8.13 students:

Feel free to look at this paper for some suggestions about the lab, but please reference/acknowledge me as if you had read my report or spoken to me in person. Also note that this is only one way to do the lab and data analysis, and there are nearly an infinite number of other things to do that would be better.

I made some mistakes doing this lab. Here are a couple I found (and some more tips):

- For the Compton scattering section, I added 2 incompatible sets of data. I should have performed some kind of combined fit.
- In figure 3, I multiplied by a random amplitude to make the graphs display more nicely.

Compton Scattering and Attenuation

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Compton scattering was studied using scintillators to measure the energies of the photons and electrons at different scattering angles. Using the relationship from the Compton formula, the rest mass of the electron was calculated to be $m_e c^2 = 552 \pm 7$ keV. The differential cross section at each scattering angle was also calculated and compared to the Thompson and Klein-Nishina predictions. Except for the scattering angles between $30^\circ \leq \theta \leq 50^\circ$, the data was consistent with the Klein-Nishina prediction. In the other section of the lab, the linear attenuation coefficients of three types of plastic and lead were found by measuring the intensity of the non-interacting photons as the thickness of the material was varied. The linear attenuation coefficients from the plastic were used to calculate the Compton cross scattering term for the electron to be $\sigma_e = 2.58 \pm 0.026 \times 10^{-25}$ cm^2.

I. INTRODUCTION

In the early 1900s, physicists used scattering experiments to help build a model for the structure of the atom. Around 1920, Arthur H. Compton performed one of the most important experiments of the era using x-ray photons scattering from electrons in light materials. This experiment was the first to show that photons are not just waves but, in collisions with electrons, should be treated as particles. Compton discredited the classical model used by J. J. Thompson and confirmed the relativistic model derived by Oskar Klein and Yoshio Nishina with his experiment.[1][2]

II. THEORY

A. Compton Scattering

When a photon collides with an electron at rest, the photon loses energy, imparting some of its momentum to the electron. This change in energy is known as the Compton effect and can be described by the equation

$$\frac{1}{E'_\gamma} - \frac{1}{E_\gamma} = \frac{1}{m_e c^2}(1 - \cos(\theta)) \tag{1}$$

where E_γ is the photon's initial energy, E'_γ is the photon's energy after scattering, c is the speed of light, m_e is the mass of the electron, and θ is the angle between the incident and scattered photon.

If the energy of the photon is close to or greater than the energy of the rest mass of the electron (551keV), the electron can be treated as if it were not bound by the nucleus. This is the case for the photons emitted by Cs-137 which has an energy of 661.6 keV.

B. Attenuation

When photons enter a material, they have a certain probability of interacting with the atoms in the material by photoelectric, Compton, or pair-production processes. The probability of the photon traveling a distance x without a photoelectric interaction is $e^{-\tau x}$, without a Compton collision is $e^{-\sigma x}$, and without a pair-production collision is $e^{-\kappa x}$. The values of τ,σ, and κ will vary depending on the incoming photons energy and the materials atomic number (Figure 1).

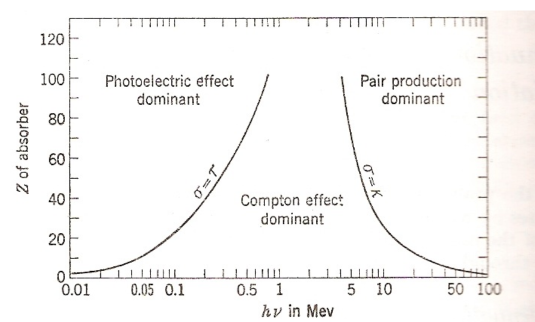

FIG. 1: As the energy of the incoming photon varies, the likelihood of a Compton, a photoelectric, or a pair-production interaction occurring will vary. Compton interaction are dominant when the incoming photon has an energy around 1 to 5 MeV. [3]

The three probabilities for interaction can be described by a single term called the total linear attenuation coefficient $\mu = \sigma + \gamma + \kappa$. The linear attenuation coefficient is used when calculating the intensity of the unaffected photons

$$I(x) = I_0 e^{-\mu x} \tag{2}$$

where $I(x)$ is the intensity, I_0 is the initial intensity, x is the thickness of the material (m), and μ is the total linear attenuation coefficient (m^{-1}).

If the majority of the interactions are due to the Compton effect, the scattering cross section can be found from

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the value of the total linear attenuation coefficient:

$$\sigma_{tot} = \frac{\mu}{n_e} \tag{3}$$

where $n_e = \rho Z N_a/A$ is the total number of electrons/cm^{-3}, ρ = density of the material, N_a = Avagdro's number, A is the atomic weight, and Z is the atomic number.

C. Thompson and Klein-Nishina

According to the Thompson model, the scattering cross section of the electron can be calculated by the formula:

$$\frac{dI}{dt} = \sigma_e I = \frac{8\pi e^2}{3m_e c^2} I \tag{4}$$

which predicts the scattering cross section of an electron to be $\sigma_e = 6.65 \times 10^{-25}$ cm^2. However, this formula does not account for the recoil of the electron, the relativistic effects, or the experimentally observed dependence of the power loss on the incoming photon's frequency. To account for these, Klein and Nishina derived a formula for the differential cross section,

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= r_0^2 \frac{1+cos^2(\theta)}{2} \frac{1}{[1+\gamma(1-cos(\theta)]^2} \\ &\times [1 + \frac{\gamma^2(1-cos(\theta))^2}{(1+cos(\theta)^2)[1+\gamma(1-cos(\theta)]}] \end{aligned} \tag{5}$$

where $r_0 = 2.82\times10^{-13}$cm is the classical electron radius, θ is the photon scattering angle and $\gamma = h\nu/mc^2$. The Klein-Nishina formula predicts that the scattering cross section of the electron is $\sigma_e = 2.53 \times 10^{-25}$ cm^2.

III. EXPERIMENTAL SETUP

A. Apparatus

To measure the energy spectrum of photons at different scattering angles, gamma rays emitted from Cs-137 were directed by a lead "howitzer", and sent towards the target detector (Figure 2). Some of these photons were Compton scattered off free-electrons in the target detector toward the scatter detector. Both of these detectors were scintillators with a sodium-iodide crystal, which sent signals to a pre-amplifier. The pre-amplifier sent the signals to an amplifier and an inverter, which prepared the signals to be measured by the discriminator.

To filter out signals due to background events, coincidence techniques were used. If the two signals from the target and scatter discriminators overlapped, the gate generator signaled the multi-channel analyzer in the computer to analyze the signal. If the signals did not overlap, the signal would not be analyzed. Most overlapping signals were caused by Compton scattering, but some were caused by a random overlap of background events. At small Compton angles, the target signal is small, so if the discriminator threshold is set too low some of the Compton events will be missed.

To measure the linear attenuation coefficients, the coincidence timing was turned off, the target detector was removed, and the scatter detector was placed at $\theta = 0°$. Blocks of different materials were placed at the exit of howitzer, and the intensity of the gamma rays at different energies were measured using the scattering detector.

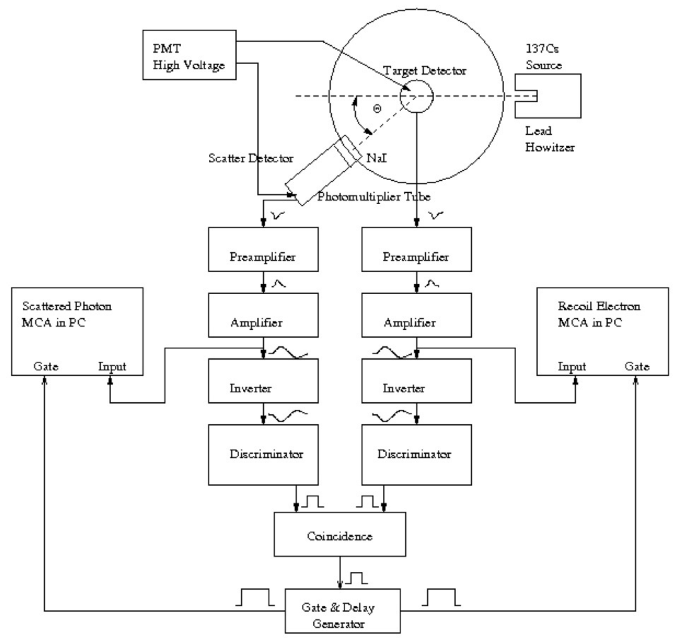

FIG. 2: Overall Setup to measure the relationship between the energy spectrum and changing the angle of the scatter detector. [1]

B. Calibration

A close relationship between the bin number outputted by the data interface program *Maestro* and the energy must be known to precisely measure the different energy values for the changing Compton angles. To find this relationship, the energy spectrum of three radioactive isotopes with known emission energies (Ba-133, Cs-137 and Na-22) were measured (Figure 3). Each peak intensity value corresponded to a measured bin number and a known energy. A weighted linear fit was created from the pairs of bin numbers and energies. The calibration used in the conversation of the bin numbers to energy for the scatter detector was $E = (0.571 \pm 0.001)x - 14.7 \pm 1.1$, where E is the energy and x is the bin number with a reduced chi square of 9.1. The calibration for the target detector was, $E = (0.567 \pm 0.001)x - 17.3 \pm 1.0$ with a reduced chi square of 5.0.

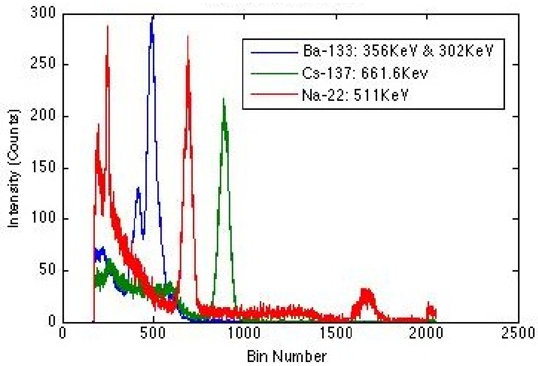

FIG. 3: Example Calibration Data. The energy value corresponding to each bin number was found by measuring the peaks from Ba-133, Cs-137 and Na-22.

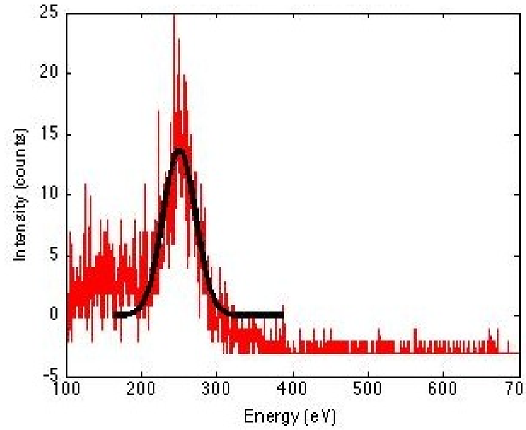

FIG. 4: Raw data with Gaussian Fit for Angle 110° in the Scatter Detector (Intensity vs Energy). The red line shows the raw data and the black line shows the fit used to extract an energy value (measured in keV). The same method was applied to extract energy values at other angles.

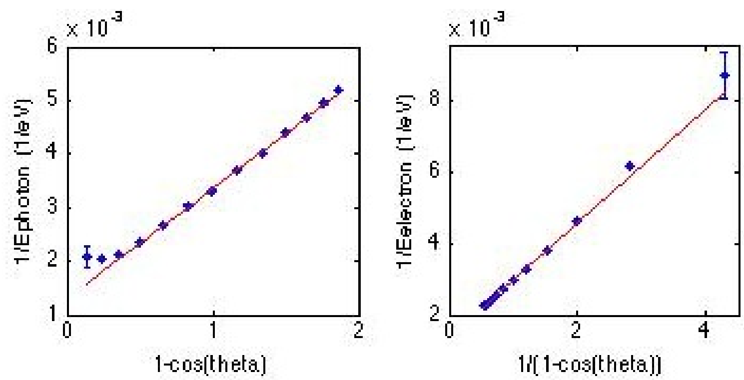

FIG. 5: (Left) Scatter Detector ($1/E_{photon}$ vs $1 - cos(\theta)$) (right) Target Detector ($1/E_{electron}$ vs $1/1 - cos(\theta)$) The blue points represent the measure data and the red line represents the line fitted by the method of least squares. The large error bar for the point at $\theta = 30°$ is due to an uncertainty in the position of the peak from the raw data.

IV. RESULTS AND DISCUSSION

A. Compton Scattering

The energy spectrum of the target and scattering detectors were measured at sixteen different Compton angles: -30° and from 0° to 150° in steps of 10°. For most angles, a Gaussian fit was made to extract an energy value for both detectors (Figure 4). In the target detector, energy values for the angles between and including 0° and 30° could not be determined because the energy was outside of the measurement range. In the scatter detector, the energy values for the Compton photons for angles between -30° and 20° could not be determined because they were not sufficiently resolved from unscattered photons.

A linearized version of the Compton formula (Equation 1) was graphed and fitted to determine a value for the rest energy of an electron to be $m_ec^2 = 488 \pm 5$ keV(left in Figure 5). The recoil electron data was graphed against, which was also linear and could be fitted to find the $m_ec^2 = 621 \pm 5$ keV (right in Figure 5). The weighted average of the two fits was $m_ec^2 = m_ec^2 = 552 \pm 7$ keV, which is within one standard deviation of the expected value of the energy of the rest mass of the electron $m_ec^2 = 511$keV.

The differential cross section was calculated using the formula

$$\frac{d\sigma}{d\omega} = \frac{I/e}{(d\omega)NI_0} \tag{6}$$

where I is the intensity, e is the efficiency, $d\omega$ is the solid angle and N is the number of electrons. The data was then graphed against the Thompson and the Klein-Nishina predictions (Figure 6). The data shows a close relationship with the Klein-Nishina prediction from the angles $60° \leq \theta \leq 150°$

From $30° \leq \theta \leq 50°$ the data does not fit either prediction. This anomaly is believed to be caused by a systematic error due to the method of coincidence timing. At sufficiently small angles, the energy of the recoil electron was below the threshold for the discriminator, so the event was not recorded. For these angles, it is likely that the discriminator was filtering some of the events so the measured intensity was only a fraction of what it should be. An attempt was made to correct for this systematic error by finding a predicted shape for the fitted Gaussian and overlaying it with the data. Unfortunately there was not a distinct trend for the width for the fitted Gaussians of non-affected angles, so a prediction for the Gaussians of the affected angles could not be obtained.

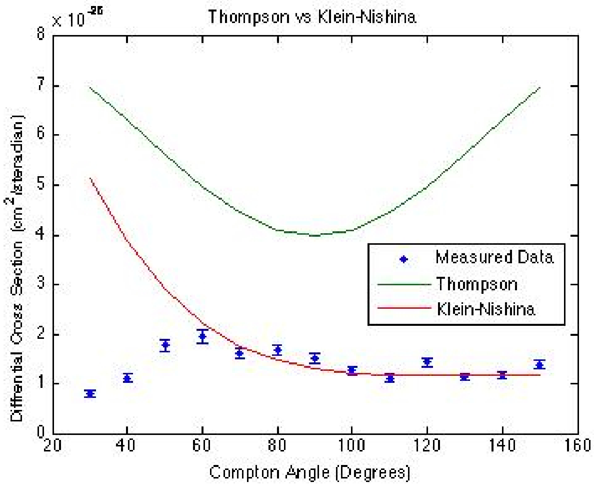

FIG. 6: Comparison of Thompson and Klein-Nishina with the Measured Data (Differential Cross Section vs Angle). The blue points represent the measured data with error, the dark green line is the Thompson prediction, and the red line is the Klein-Nishina prediction. Angle is measured in degrees and the differential cross section is measured in cm^2/steradian.

B. Attenuation

The energy spectra of four different materials, polycarbonate, polypropylene, polyvinyltoluene, and lead were measured using different thicknesses of each material. Five runs, for 20.0 second of live time, were performed at four thickness values for each material. From this count data, the intensity- the rate at which photons pass through the material without interacting - versus the thickness used of each material was found using the process described below.

First, each run's intensity value was extracted by fitting a Gaussian to the count versus energy data, and then taking the sum of the counts which fell between $\pm 2\sigma$ of the center. The average intensity of the five runs was found and was assigned as the intensity at that thickness. The error of the average intensity was found by calculating the standard deviation around the mean for the five runs.

Once the intensity was found at each thickness, the points were graphed and fitted using a linearized version of Equation 2, $-ln(I/I_0) = \mu x$, where I_0 is the measured intensity using no scattering material. The slope of the fitted line was equal to the linear attenuation coefficient (μ). The value for μ of each material is listed in Table 1.

Because the interactions in the three plastics were mostly due to Compton scattering, the linear attenuation values could be used to determine the Compton scattering cross section per electron (σ_e listed for each material in Table 1). The weighted average for the three plastics was $\sigma_e = 2.58 \pm 0.026 \times 10^{-25}$ cm^2 , which was within two standard deviations from the expected value for Klein-Nishina of $\sigma_e = 2.53 \times 10^{-25}$ cm^2. The value for μ_{lead} could not be used to calculate the Compton scattering cross section per electron because the number of Photoelectric interactions of an incoming photon of 661.6 keV is significant (see Figure 1).

Table 1: Linear Attenuation Coefficients and Compton Scattering Cross Section

Material	μ (1/m)*	μ data (1/m)	σ_e ($\mathrm{cm}^2 \times 10^{-25}$)
Polycarbonate	9.82	9.65 ± 0.23	2.49 ± 0.12
Polypropylene	7.80	7.92 ± 0.12	2.70 ± 0.08
Polyvinyltoluene	8.61	8.37 ± 0.14	2.49 ± 0.08
Lead	113.5	120.9 ± 1.7	n/a

*from [3]

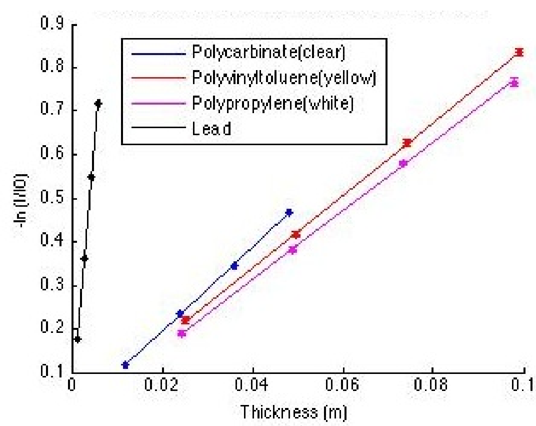

FIG. 7: Calculating Linear Attenuation Coefficient ($-ln(I/I_0)$ vs Thickness). The linear attenuation coefficient for each material was found from the slope of the fitted line. The thickness is in units of meters and the linear attenuation coefficient is in units of 1/meter.

V. SUMMARY

Compton scattering was studied by measuring the intensity versus energy data for the scattered photons and electrons at different angles. By applying the Compton formula, the rest mass of the electron was calculated to be $m_e c^2 = 552 \pm 7$ keV, which was within one standard deviation of the expected value of $m_e c^2 = 551$keV. The differential cross section at each angle was also calculated and compared to the Thompson and Klein-Nishina predictions. Except for angles $30^\circ \leq \theta \leq 50^\circ$, the data was consist with the Klein-Nishina prediction.

The linear attenuation coefficients of polycarbonate, polypropylene, polyvinyltoluene, and lead were calculated and the values from the three plastics were used to calculate the Compton cross scattering term for the electron to be $\sigma_e = 2.58 \pm 0.026 \times 10^{-25}$ cm^2. This value was within two standard deviations of the predicted value by Klein-Nishina ($\sigma_e = 2.53 \times 10^{-25}$ cm^2).

Acknowledgments

I would like to acknowledge Kathryn Decker French for taking data with me, and Sid Creutz for proof reading my paper.

[1] MIT Physics Department, *Junior lab written report notes* (2007).
[2] French and Taylor. *An Introduction to Quantum Physics* (Norton, 1978).
[3] Evans, Robley. *The Atomic Nucleus* (McGraw-Hill, 1955).