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This paper uses recent results from incentive theory to study heretofore informal critiques of piece-rate compensation schemes. The informal critiques are based on the history of failed attempts to install piece-rate compensation schemes at the turn of the century. The formal analysis emphasizes the importance of information and commitment in contracting. The main result is as follows. In a work environment characterized by hidden information and a hidden action, if neither the firm nor the worker can commit to future behavior, then no compensation scheme, piece-rate or otherwise, can induce the worker not to restrict output.

I. Introduction

The incentive properties of piece-rate compensation schemes seem very attractive: workers are paid for the work they do, not the work they could have done, and this seems likely to solve problems associated with both hidden information (adverse selection) and hidden actions

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(moral hazard). On the other hand, modern accounts of the history of failed attempts to install piece-rate compensation schemes at the turn of the century emphasize exactly these information asymmetries. Edwards (1979), for instance, argues that piece-rate compensation schemes were ineffective because management did not know how fast a job could be done and therefore could not set the correct piece rate. Clawson (1980) argues that management could not use a worker's performance to determine the correct rate because the worker responded by restricting output.

Although informal versions of these critiques of piece rates have existed for some time, a formal analysis of the importance of information and commitment in compensation contracts has not been performed. This paper borrows freely from the recent developments in incentive theory in order to do so. The contribution of the paper, then, is not the development of new theory but rather the use of existing theory to study heretofore informal critiques.

The paper is organized as follows. Section II presents Edwards's and Clawson's arguments in more detail. Section III then develops a static model intended to capture Edwards's ideas. This model involves both hidden information and a hidden action. Also, the model assumes that the firm has at least a little (but perhaps far from complete) monopsony power. The results in this section are closely related to those in Sappington (1983) and Laffont and Tirole (1986). If one views a piece rate as a single price per unit of output, so that compensation is a linear function of output, then this model supports Edwards's view. The worker's private information makes a linear contract suboptimal; in fact, the optimal contract is nowhere linear. If one distinguishes only between salaries (functions of time) and piece rates (functions of output), however, then the optimal contract is a piece rate.

Section IV develops a dynamic model that formalizes Clawson's perspective. The model has 2 periods, each of which mimics the static model of Section III. The dynamic model emphasizes the importance of the firm's and the worker's commitment opportunities, in combination with the hidden information and hidden action of the static model. Loosely put, the results are as follows. First, if the firm can commit in advance to the second-period contract, then the optimal 2-period contract simply repeats the optimal static contract (see Baron and Besanko 1984*a*). Second, if the firm cannot commit to the second-period contract but the worker is committed to staying with the firm for the second period, then the firm revises the piece rate on the basis of first-period performance, as Clawson suggests, but the optimal contracts do not induce the worker to restrict output (see Lazear 1986). Third, if the firm cannot commit to the second-period contract and the worker cannot commit to stay with the firm for the second period, then no compensation scheme of any form can induce the worker not to restrict output, in the sense that workers in less difficult jobs often will produce no more than workers in more difficult jobs. This last result is based on the pathbreaking work of Laffont and Tirole (in press).

The stark contrast between the second and the third results deserves discussion. As the example in Section IV will make clear, everything hinges on whether the worker is committed to staying with the firm for the second period. Three (related) devices for achieving this commitment are as follows. First, a contract could impose a large financial penalty on the worker following a quit. Second, all or part of first-period compensation could be paid in the second period, to be forfeited following a quit. Third, the worker could post a bond before the first period, also to be forfeited following a quit.

These attempts to bind the worker to the firm face various obstacles. First, they create an incentive for the employer to make a layoff appear to be a quit as well as an incentive simply to harass the worker into quitting. Second, they may be incompatible with any or all of the following: a prohibition of involuntary servitude, a strong preference for smooth consumption streams, and an imperfect capital market in which loans cannot be secured by human capital. While these considerations do not imply that bonding is impossible, they suggest that complete bonding may be difficult. The analysis of incomplete bonding seems a worthwhile next step.

II. Two Critiques of Piece Rates

This section argues that piece rates have two serious shortcomings. The first arises because workers have private information about the difficulty of their jobs. Edwards (1979, pp. 98–99) summarizes the historical record as follows:

> Managers' ability to control soldiering resulted from their inadequate knowledge of the actual techniques of production. Most of the specific expertise—for example, knowledge of how quickly production tasks could be done—resided in workers. . . .
>
> Piece-rates always carried the allure of payment for actual labor done (rather than labor power), thus promising an automatic solution to the problem of translating labor power into labor. . . . [But] as long as management depended on its workers for information about how fast the job could be done . . . there was no way to make the piece-rate method deliver its promise.

In the language of information economics, management faces both adverse-selection and moral-hazard problems: only workers know the difficulty of their jobs, and they can shirk so as to obscure this information from management. For risk-averse workers, of course, agency theory proves that piece rates typically are an inferior solution to the problem of moral hazard and risk sharing and so presumably are an inferior solution to this more complicated problem as well.[1] Many jobs, however, simply do not involve a great deal of risk, which suggests that risk aversion is not entirely responsible for the unpopularity of piece rates. In order to focus on different culprits, this paper ignores the risk that piece rates impose on workers by assuming that workers are neutral to income risk.

The second shortcoming of piece rates stems from the firm's opportunity to revise the rate over time. After discussing many case studies at length, Clawson (1980, pp. 169–70) concludes:

> In theory, piecework was simple. The company set a fair price for each unit of completed work . . . and workers were paid according to their output. If workers could increase output, either by extra exertion or by improved methods of their own devising, they would receive higher wages. . . . In practice, piecework never worked this way, since employers always cut the price they paid workers. . . . Almost all employers insisted that they would never cut a price once it was set, yet every employer did cut prices. . . . Unless workers collectively restricted output they were likely to find themselves working much harder, producing much more, and earning only slightly higher wages.

If complete contracts could be written, the firm could commit to a fixed piece rate, but in practice the relevant contract is much too complex to write (not to mention to enforce) because the obvious simple contract will not suffice. As Clawson (1980, p. 170) observes, "Employers could cut rates in dozens of ways other than changing the piece price for a worker who continued to perform the same operations. New workers could be assigned to the job at a lower rate while the old workers were transferred elsewhere, information about output on one job could be used to lower the initial price on new work, and any sort of minor change could be made the excuse for large price cuts." This paper captures these contractual difficulties in a dynamic model by allowing the firm no interperiod commitment opportunities and requiring it to be sequentially rational: in each period, the firm's action must be optimal from that point onward, as in a dynamic program.

III. The Static Model

This section uses a static model to formalize Edwards's critique: "As long as management depended on its workers for information about

[1] It is rare but possible for a linear contract to be optimal. This follows from the proposition that any monotone sharing rule is optimal for some special case of the agency problem.

how fast the job could be done . . . there was no way to make the piece-rate method deliver its promise."

To keep things simple, consider one firm employing one worker.[2] Output, y, is determined by the difficulty of the job, θ, and the effort the worker expends, a, according to

$$y = \theta + a,$$

where effort is chosen from $[0, \infty)$. Note that jobs with lower θ's are more difficult.

Before contracting and production occur, the worker knows the difficulty of the job, but the firm knows only that θ has distribution $F(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, where $\underline{\theta} > 0$. To simplify the exposition, the inverse of the hazard rate,

$$\frac{1 - F(\theta)}{f(\theta)},$$

is assumed to decrease strictly in θ. Assumptions of this form are standard in the literature.[3]

The worker chooses effort to maximize the expectation of the separable utility function $u(w, a) = w - g(a)$, subject to the wage schedule $\tilde{w}(y)$ chosen by the firm. The disutility of effort, g, is increasing and strictly convex. Also, the analysis is simplified by several stronger but not counterintuitive assumptions.

ASSUMPTION 1.

$$g(0) = g'(0) = g''(0) = 0, \quad g''' \geq 0,$$

which guarantee that the optimal compensation scheme induces positive effort no matter what the job's difficulty.

ASSUMPTION 2. The marginal disutility of effort, $g'(a)$, approaches infinity as effort approaches infinity, which guarantees that the relevant first-order conditions have solutions. In particular, the efficient (or first-best) effort level solves $g'(a) = 1$ and will be denoted by a_{fb} in what follows.

[2] Alternatively, there could be as many workers as there are jobs in the firm, provided the jobs have independent difficulties, and there could be many firms, subject to the same proviso. What is important is that no two workers share the same private information, for if they did, then interdependent compensation schemes might help extract it from them, and these are beyond the scope of the paper.

[3] See, e.g., Baron and Besanko (1984*b*), who list many familiar distributions that satisfy a related condition. The analysis can proceed without this assumption but at some technical expense (see Myerson 1981; and Baron and Myerson 1982).

Finally, it is important to assume that the firm has at least a little (but perhaps far from complete) monopsony power. This seems to be a reasonable assumption; it holds unless absolutely perfect competition prevents the firm from earning *any* rents in the labor market. In the model, the worker's next-best alternative is assumed to be unemployment, which is characterized by zero wage and zero effort and therefore zero utility, but an arbitrary positive reservation utility, $\bar{U}$, would cause no substantive changes in the analysis.[4]

The firm's only cost is its wage bill, so it chooses a wage schedule to maximize expected profit, $E[y - \tilde{w}(y)]$, subject to optimizing behavior by the worker.[5] In this 1-period problem, the Revelation Principle (Dasgupta, Hammond, and Maskin 1979; and Myerson 1979) states that the firm's choice of a wage schedule $\tilde{w}(y)$ is equivalent to the choice of a suitable pair of functions $y(\theta)$ and $w(\theta)$ in a direct-revelation game: the firm chooses $\{y(\theta), w(\theta)\}$ to maximize expected profit

$$\int_{\theta=\underline{\theta}}^{\bar{\theta}} [y(\theta) - w(\theta)] f(\theta) d\theta \tag{1}$$

subject to incentive compatibility, individual rationality, and the feasibility constraint that $y(\theta) \geq \theta$ (since $a \geq 0$).[6]

To express the incentive-compatibility and individual-rationality constraints in the direct-revelation game, define $U(\tilde{\theta}, \theta)$ to be the utility of a worker of type θ who reports type $\tilde{\theta}$:

[4] Next-best alternatives other than unemployment, such as self-employment, could generate the reservation utility $\bar{U}$. For $\bar{U} > 0$, the firm may choose not to operate the technology if the worker reports that θ is very low. In modeling self-employment, however, it would be important that the worker not have access to the firm's technology since this would vitiate the problem of private information.

[5] As it stands, this is a model of a firm in a competitive product market facing a price of one. The model fits an imperfectly competitive product market as well because the notion of output can be suppressed and y can be interpreted as revenue. The competitive interpretation may be more useful, however, because it emphasizes that the rents the firm earns in this paper are due to monopsony power.

[6] The Revelation Principle works as follows. If the firm chooses a compensation scheme $\tilde{w}(y)$, a worker in a job of difficulty θ will choose effort to maximize $\tilde{w}(y) - g(a)$ subject to $y = \theta + a$. Let the optimal effort choice be $\tilde{a}(\theta)$. Then output will be $y(\theta) = \theta + \tilde{a}(\theta)$, and wages will be $w(\theta) = \tilde{w}[y(\theta)]$. Suppose that, instead of implementing $\tilde{w}(y)$, the firm allows the worker to choose one output-wage pair from the menu $\{y(\theta'), w(\theta'); \theta' \in [\underline{\theta}, \bar{\theta}]\}$, where the functions $y(\theta)$ and $w(\theta)$ are as determined above. Then, because $\tilde{a}(\theta)$ was optimal under $\tilde{w}(y)$, $\theta' = \theta$ is the optimal choice from the menu. Thus, any compensation scheme $\tilde{w}(y)$ can be represented by the appropriate pair $\{y(\theta), w(\theta)\}$.

$$U(\tilde{\theta}, \theta) \equiv w(\tilde{\theta}) - g[y(\tilde{\theta}) - \theta].$$

Also, let $U(\theta)$ denote $U(\theta, \theta)$, the utility from truthful reporting. Then the incentive-compatibility constraint is

$$U(\theta) \geq U(\tilde{\theta}, \theta) \tag{2}$$

for all θ, $\tilde{\theta}$, and the individual-rationality constraint is

$$U(\theta) \geq 0 \tag{3}$$

for all θ. In these terms, the firm's problem is to choose $\{y(\theta), w(\theta)\}$ to maximize (1) subject to the incentive-compatibility constraint (2), the individual-rationality constraint (3), and the feasibility constraint $y(\theta) \geq \theta$.

Lemmas 1 and 2 and proposition 1 solve this problem. The techniques in the lemmas are due to Mirrlees (1971) and Myerson (1981). Results similar to proposition 1 have been derived by many; this particular result is given in Sappington (1983) and Laffont and Tirole (1986). Since the proofs of these results are not new, they are relegated to the Appendix. Corollary 1 then concludes that the solution is not a linear piece-rate compensation scheme. Finally, three remarks following corollary 1 interpret the results.

LEMMA 1. The output and wage functions $\{y(\theta), w(\theta)\}$ satisfy the incentive-compatibility constraint (2) and the individual-rationality constraint (3) if and only if (*a*)

$$U(\theta) = U(\underline{\theta}) + \int_{\tilde{\theta}=\underline{\theta}}^{\theta} g'[y(\tilde{\theta}) - \tilde{\theta}] d\tilde{\theta},$$

(*b*) $U(\underline{\theta}) \geq 0$, and (*c*) $y(\theta)$ is nondecreasing.

The most important part of the lemma is condition *a*. The intuition behind this result is akin to that behind a separating equilibrium in Spence's (1973) signaling model. Here a worker in a job of difficulty θ must be persuaded not to overstate the job's difficulty, $\tilde{\theta} < \theta$; there, the incentive is to overstate one's ability. By the envelope theorem, for very small lies (i.e., as $\tilde{\theta}$ approaches θ from below), the worker stands to gain $g'[y(\theta) - \theta]$ from lying; in the signaling model, the analogous gain is the marginal product of ability. Condition *a* dictates that the worker's equilibrium utility must include a bribe of this size to prevent such a lie; in the signaling model, the marginal gain is matched by the cost of the extra education necessary to persuade the market that one's ability is as claimed. Condition *b* is clearly necessary for individual rationality, and condition c is a second-order condition in the incentive-compatibility problem.

Given lemma 1, substituting condition a and the definition of $U(\theta)$ into (1) and changing the order of integration yields a convenient restatement of the firm's objective.

LEMMA 2. The firm's problem can be reduced to choosing $y(\theta)$ and $U(\underline{\theta})$ to maximize

$$-U(\underline{\theta})+\int_{\theta=\underline{\theta}}^{\bar{\theta}}\left\{y(\theta)-g[y(\theta)-\theta]-\left[\frac{1-F(\theta)}{f(\theta)}\right]g'[y(\theta)-\theta]\right\}f(\theta)d\theta, \quad (4)$$

subject to condition b, condition c, and $y(\theta) \geq \theta$.

PROPOSITION 1. At the firm's optimum, $U(\underline{\theta}) = 0$, and $y^*(\theta)$ solves

$$1-g'(y-\theta)-\left[\frac{1-F(\theta)}{f(\theta)}\right]g''(y-\theta)=0. \quad (5)$$

The resulting effort level, $a^*(\theta) \equiv y^*(\theta) - \theta$, is strictly positive and strictly increasing and equals a_{fb} only at $\bar{\theta}$.

The intuition behind proposition 1 is straightforward. In the standard agency problem, if the agent is risk neutral, then the principal sells the firm for price p by offering the contract $\tilde{w}(y) = y - p$, and this induces the efficient effort level, a_{fb}. Here the problem is that only the agent knows how much the firm (or, more intuitively, the job) is worth. For a fixed price p there exists a type $\theta(p)$ such that all types $\theta < \theta(p)$ do not take the contract, while all types $\theta > \theta(p)$ take the contract, put forth the efficient effort level, and earn rents. Keeping the cutoff type $\theta(p)$ constant, the envelope theorem dictates that the second-order loss incurred in moving away from efficient effort is more than covered by the accompanying first-order reduction in the rents earned by those who take the contract. At the same time, it is efficient to reduce $\theta(p)$.

Mathematically, the optimal contract given by (5) is simply the first-order condition for the pointwise maximization of (4). It trades off productive efficiency against lost rents and has the familiar property that only the top type, $\bar{\theta}$, puts forth the efficient level of effort.

COROLLARY 1. A linear piece rate is not the optimal compensation scheme. Indeed, the optimum is nowhere linear.

Proof. Recovering $w^*(\theta)$ from the definition of $U(\theta)$ and condition a yields

$$w^*(\theta)=U(\underline{\theta})+g[y^*(\theta)-\theta]+\int_{\theta'=\underline{\theta}}^{\theta}g'[y^*(\theta')-\theta']d\theta'. \quad (6)$$

In a linear piece rate, $dw/dy = (dw/d\theta)(d\theta/dy)$ must be constant. But

$$dw/d\theta = g'(y' - 1) + g' = g'y',$$

so $dw/dy = g'$; and proposition 1 shows that $y^*(\theta) - \theta$ is strictly increasing, so $dw^*/dy = g'[y^*(\theta) - \theta]$ is nowhere constant. Q.E.D.

REMARK 1. It is possible to interpret $\{y^*(\theta), w^*(\theta)\}$ as the upper envelope of a menu of linear compensation schemes among which workers select. (As with lemma 1, the intuition for this parallels that for a separating equilibrium in a signaling model or in any other self-selection model based on the familiar condition on the cross-partial derivative of the relevant utility function.) Notice that the best response of a worker of type θ to the linear compensation scheme $\tilde{w}(y) = by + c$ is the effort $a(b)$ that solves $g'(a) = b$. Since the effort induced by $\{y^*(\theta), w^*(\theta)\}$ is $y^*(\theta) - \theta$, the linear compensation scheme designed for worker θ has slope $b(\theta) = g'[y^*(\theta) - \theta]$ and intercept $c(\theta) = w^*(\theta) - b(\theta)y^*(\theta)$. Such a menu of linear compensation schemes induces the worker to reveal the job's difficulty; this will not be possible in the dynamic model analyzed in the next section. For more on such menus of linear schemes, see Laffont and Tirole (1986).

REMARK 2. Suppose the firm chooses a linear compensation scheme—that is, a single price per unit of output that applies to workers of all types. (This is analogous to choosing a two-part tariff when optimality requires a nonlinear price schedule) The qualitative properties associated with the contract $\tilde{w}(y) = y - p$ reappear if the firm offers $\tilde{w}(y) = by + c$. As noted above, every worker who chooses to work will supply the effort $a(b)$ that solves $g'(a) = b$, while workers satisfying

$$\{b[\theta + a(b)] + c\} - g[a(b)] < 0,$$

or

$$\theta < \tilde{\theta} \equiv \frac{g[a(b)] - c}{b} - a(b),$$

will choose not to work. Assuming $\tilde{\theta} \in (\underline{\theta}, \bar{\theta})$, effort under a piece-rate compensation scheme is a step function: it is zero for $\theta \in [\underline{\theta}, \tilde{\theta})$ and $a(b)$ for $\theta \in [\tilde{\theta}, \bar{\theta}]$. Under the optimal contract, in contrast, the effort $a^*(\theta) \equiv y^*(\theta) - \theta$ is strictly positive.

REMARK 3. Edwards (1979, p. 100) describes a system of "differential-rate piece-work" designed by Taylor to strengthen workers' incentives for effort. According to this system, a contract should be piecewise linear with kinks, or even jumps, so that productive workers benefit from a higher rate once their output surpasses certain standards. In this connection, it is interesting that the optimal contract is nowhere linear.

IV. The Dynamic Model

This section uses a dynamic model to formalize Clawson's critique: "Unless workers collectively restricted output they were likely to find themselves working much harder, producing much more, and earning only slightly higher wages." This is the "ratchet effect" analyzed by Freixas, Guesnerie, and Tirole (1985) for the case of linear incentive schemes when θ can take only two values.

The goal of the section is to study the role of commitment in a dynamic model. The central result is that, if the firm cannot commit to the second-period contract and the worker cannot commit to stay with the firm for the second period, then no compensation scheme of any form can induce the worker not to restrict output. Note that this result says not that piece rates are suboptimal but rather that piece rates will induce unavoidable inefficiencies of the form described in earlier informal accounts.

Let there be 2 periods of work, each of which is identical to that described in the previous section. In period t, the worker's output, y_t, is determined by the difficulty of the worker's job, θ, and the effort the worker expends in that period, a_t, according to

$$y_t = \theta + a_t.$$

Notice that θ is constant over time.

As before, the worker knows the difficulty of the job but the firm does not. Before period 1, the firm believes that θ has distribution $G(\theta)$ on $[\theta^-, \theta^+]$. Its posterior belief given first-period output may be more refined, however. Denote the posterior by $F(\theta)$ on the subinterval $[\underline{\theta}, \bar{\theta}]$ of $[\theta^-, \theta^+]$.

The static analysis of Section III applies to the second period of this dynamic game, provided two assumptions are met. First, the firm cannot commit in advance to behavior that is not sequentially rational. Second, the worker's reservation utility must be the same in each case. The second assumption is closely related to the commitment question discussed in the introduction: for example, the assumption does not hold if the worker posts a large bond in period 1 that is returned after period 2 only if the worker does not quit. This paper ignores this possibility by assuming that imperfect capital markets make it impossible for workers to raise bonds and that wages are close enough to subsistence that deferred wages cannot act as bonds. Relaxing this assumption to study partial bonding, perhaps through pensions, would be interesting.

The worker's von Neumann–Morgenstern preferences are represented by the discounted sum of each period's utility, which is itself a separable function of consumption, c_t, and effort:

$$\sum_{t=1}^{2} \beta^{t-1}[c_t - g(a_t)].$$

The subjective discount factor, β, is assumed to be determined by the market interest rate, r, according to $\beta = 1/(1 + r)$. Since the worker is neutral to income risk and discounts at the market rate, borrowing and saving can be ignored. The firm shares the discount factor β and maximizes the expected discount sum of each period's profit,

$$E \sum_{t=1}^{2} \beta^{t-1}[y_t - \tilde{w}_t(y_t)],$$

where $\tilde{w}_t(\cdot)$ is the wage schedule the firm offers the worker for period t.

The results in this dynamic model depend critically on the firm's and the worker's commitment opportunities. Following a long tradition in labor market models, one could assume that the worker is free to quit the job, at no penalty, at any time; this is frequently described as a prohibition of involuntary servitude. As before, the worker's next-best alternative is assumed to be unemployment, which yields zero utility. On the firm's side, the difficulties in writing, not to mention enforcing, the appropriate 2-period contract may force the firm to offer only single-period contracts. (Clawson's persuasive description of these difficulties is quoted in Sec. II above.) Sequential rationality in the absence of commitment forces the firm to choose $\tilde{w}_2(\cdot)$ to maximize $E[y_2 - \tilde{w}_2(y_2)]$ with respect to its beliefs about θ conditioned on the observed value of y_1.

It is intuitive (but, for a proof, see Baron and Besanko [1984*a*]) that, if the firm can commit in advance to any second-period behavior it chooses, then the 2-period optimum simply repeats the static optimal compensation scheme. The absence of commitment opportunities has a profound effect, however. To see this, suppose there is a separating equilibrium in the first period; that is, suppose that each y_1 the firm could observe results in degenerate posterior beliefs about θ. Then (after the obvious changes in notation) the analysis of Section III can be applied to the second-period subgame by letting $\bar{\theta} \downarrow \underline{\theta}$. This yields $g' = 1$ in (5), so effort is at the first-best level, a_{fb}. Output is therefore $y_2(\theta) = a_{\text{fb}} + \theta$, and (6) then indicates that $\tilde{w}_2(\theta) = g(a_{\text{fb}})$ independent of θ. On the equilibrium path, therefore, the worker gets zero utility in the second period following a separating equilibrium in the first period because the firm confiscates all the surplus. (The same result follows from the agency contract $\tilde{w}[y] = y - p$ discussed following proposition 1: when θ is known, the principal can calculate that p should be $\theta + a_{\text{fb}}$

$-$ $g[a_{\text{fb}}]$, leaving the worker exactly zero surplus.) This is in the spirit of Clawson's contention that the firm will cut its piece rate, but it applies more generally to any separating first-period compensation scheme.

Given this second-period behavior, optimal first-period behavior can be determined by modifying the static game to include the second-period payoffs. Following the definitions in Section III, redefine $U(\tilde{\theta}, \theta)$ to be $U_1(\tilde{\theta}, \theta) + \beta U_2(\tilde{\theta}, \theta)$, where

$$U_1(\tilde{\theta}, \theta) = w_1(\tilde{\theta}) - g[y_1(\tilde{\theta}) - \theta],$$

$$U_2(\tilde{\theta}, \theta) = \max[g(a_{\text{fb}}) - g(a_{\text{fb}} + \tilde{\theta} - \theta), 0].$$

The form of $U_2(\tilde{\theta}, \theta)$ reflects the combination of the worker's opportunity to quit after the first period and the firm's attempt to extract all the surplus from the worker: on the basis of the belief that the job difficulty is $\tilde{\theta}$, the firm will pay the minimum wage $g(a_{\text{fb}})$ and require the output $y_2(\tilde{\theta}) = \tilde{\theta} + a_{\text{fb}}$; the worker quits if the implied second-period utility is negative.

Continue to denote $U(\theta, \theta)$ by $U(\theta)$. Then the incentive-compatibility constraints are again given by (2). Following the proof of lemma 1, substitute the definition of $U(\tilde{\theta})$ into (2):

$$U(\theta) - U(\tilde{\theta}) \geq g[y_1(\tilde{\theta}) - \tilde{\theta}] - g[y_1(\tilde{\theta}) - \theta] + \beta U_2(\tilde{\theta}, \theta).$$

Reversing the roles of θ and $\tilde{\theta}$ then yields

$$\begin{aligned} g[y_1(\theta) - \tilde{\theta}] - g[y_1(\theta) - \theta] - \beta U_2(\theta, \tilde{\theta}) &\geq U(\theta) - U(\tilde{\theta}) \\ &\geq g[y_1(\tilde{\theta}) - \tilde{\theta}] - g[y_1(\tilde{\theta}) - \theta] + \beta U_2(\tilde{\theta}, \theta). \end{aligned}$$

Take $\theta > \tilde{\theta}$, divide by $\theta - \tilde{\theta}$, and let $\theta \downarrow \tilde{\theta}$. This yields

$$g'[y_1(\theta) - \theta] \geq U'(\theta) \geq g'[y_1(\theta) - \theta] + \beta g'(a_{\text{fb}}), \tag{7}$$

which is impossible. This proves proposition 2, which is due to Laffont and Tirole (in press).

PROPOSITION 2. If neither the firm nor the worker can commit in advance to second-period behavior, then there is no sequentially rational pair of contracts $\{\tilde{w}_1(y_1), \tilde{w}_2(y_1, y_2)\}$ that separates any interval of worker types in the first period.

The intuition behind propostion 2 mimics that behind lemma 1: a worker in a job of difficulty θ must be persuaded not to claim that the job is more difficult, $\tilde{\theta} < \theta$, and this requires a bribe. In this two-period model, the bribe must be bigger than was necessary in lemma 1 because the claim that the job has difficulty $\tilde{\theta}$ now stands to earn rents in both periods. The catch is that the necessary bribe is so large that, provided the worker can quit after the first period, it is now profitable for a worker in a job of difficulty $\tilde{\theta}$ to claim that the job is less difficult, $\theta > \tilde{\theta}$, pocket the bribe, and then quit. This incentive-compatibility problem is described by the inequalities in (7): the first inequality concerns the incentive to claim that the job is less difficult and then pocket the bribe and quit (where quitting makes the $U_2[\theta, \tilde{\theta}]$ term disappear), while the second inequality concerns the incentive to claim that the job is more difficult, thereby earning the second-period rent, $U_2(\tilde{\theta}, \theta)$.

Proposition 2 says that the firm cannot perfectly infer the job's difficulty from the observed first-period output. This does not imply that all workers will produce the same output (although such a pooling equilibrium is possible). Rather, proposition 2 imples that, if a worker in a job of difficulty θ produces y_1, then there exists another job difficulty $\theta' \neq \theta$ such that, when the job difficulty is θ', the worker produces y_1 with positive probability (but perhaps probability less than one). Denoting the required effort levels by a_1 and a'_1, respectively, yields $\theta + a_1 = \theta' + a'_1$. Without loss of generality, let $\theta > \theta'$. Then $a_1 < a'_1$. This proves the main result of the paper, corollary 2.

COROLLARY 2. Piece-rate compensation schemes will not "translate labor power into labor" because workers will restrict output in the sense that workers in less difficult jobs often will produce no more than workers in more difficult jobs.

Proposition 2 makes strong use of the assumption that the worker can quit after the first period. Other assumptions have been studied by Lazear (1986) and Baron and Besanko (1987). Baron and Besanko work in terms of a direct-revelation game and impose the constraint that the worker is forced to accept a second-period contract that would yield at least the reservation utility (here zero) if the true type were the type announced in the first period. Lazear works with indirect mechanisms and makes the related assumption that the worker is committed to staying with the firm in the second period. As described in the introduction, one way to motivate this commitment is to assume that the worker posts a large bond before period 1 and that the bond is forfeited if the worker quits before period 2.

An example shows what an important difference this kind of assumption makes. For simplicity, assume that $\beta = 1$. Consider the pair of contracts

$$\tilde{w}_1(y_1) = 2y_1 - g(a_{\text{fb}}), \quad \tilde{w}_2(y_1, y_2) = y_2 - y_1 + g(a_{\text{fb}}).$$

These contracts are sequentially rational for the firm and induce the first-best effort level in both periods, provided the worker is committed to staying with the firm in the second period, as assumed by Lazear.

(Similarly, if the firm assumes that the worker chooses $a_1 = a_{\text{fb}}$, then the observed first-period output, y_1, is equivalent to the announced type $\tilde{\theta} = y_1 - a_{\text{fb}}$. On the basis of this calculation of an announced type, these contracts also induce the first-best effort level in both periods if the worker is committed as described by Baron and Besanko.) If the worker can quit, however, then the optimal effort strategy is to choose a_1^* to solve $g'(a) = 2$ and then quit, yielding utility

$$U_1^* = 2(\theta + a_1^*) - g(a_{\text{fb}}) - g(a_1^*),$$

rather than the utility that follows from $a_1 = a_2 = a_{\text{fb}}$,

$$U_1 = 2(\theta + a_{\text{fb}}) - 2g(a_{\text{fb}}).$$

A little algebra shows that $U_1^* > U_1$ if and only if

$$2 > \frac{g(a_1^*) - g(a_{\text{fb}})}{a_1^* - a_{\text{fb}}},$$

which follows from the convexity of $g(\cdot)$ and the definitions of a_1^* and a_{fb}.

Returning to proposition 2, one should not conclude that (when the stated assumptions hold) piece rates will never be observed: the result says not that piece rates are suboptimal but rather that it is not feasible for piece rates to induce workers to reveal their private information through their performance. The choice of compensation schemes is therefore a choice among second-best alternatives.

Appendix

LEMMA 1. The output and wage functions $\{y(\theta), w(\theta)\}$ satisfy the incentive-compatibility constraint (2) and the individual-rationality constraint (3) if and only if (*a*)

$$U(\theta) = U(\underline{\theta}) + \int_{\tilde{\theta}=\underline{\theta}}^{\theta} g'[y(\tilde{\theta}) - \tilde{\theta}]d\tilde{\theta},$$

(*b*) $U(\underline{\theta}) \geq 0$, and (*c*) $y(\theta)$ is nondecreasing.

Proof—only if. Substituting the definition of $U(\tilde{\theta})$ in the incentive-compatibility constraint (2) yields

$$U(\theta) - U(\tilde{\theta}) \geq g[y(\tilde{\theta}) - \tilde{\theta}] - g[y(\tilde{\theta}) - \theta],$$

and reversing the roles of θ and $\tilde{\theta}$ yields

$$g[y(\theta) - \tilde{\theta}] - g[y(\theta) - \theta] \geq U(\theta) - U(\tilde{\theta}) \geq g[y(\tilde{\theta}) - \tilde{\theta}] - g[y(\tilde{\theta}) - \theta]. \quad \text{(A1)}$$

Take $\theta > \tilde{\theta}$, divide by $\theta - \tilde{\theta}$, and let $\theta \downarrow \tilde{\theta}$ in (A1). This yields

$$U'(\theta) = g'[y(\theta) - \theta],$$

which implies condition a. Clearly, the individual-rationality constraint implies condition b. Finally, take $\theta > \tilde{\theta}$, and suppose for contradiction that $y(\tilde{\theta}) > y(\theta)$. By the convexity of g, if $\Delta > 0$, then $g(\delta + \Delta) - g(\delta)$ increases in δ, so

$$g[y(\tilde{\theta}) - \tilde{\theta}] - g[y(\tilde{\theta}) - \theta] > g[y(\theta) - \tilde{\theta}] - g[y(\theta) - \theta],$$

which contradicts (A1).

Proof—if. Since $g' \geq 0$, conditions a and b imply the individual-rationality constraint (3). For the incentive-compatibility constraint (2), use condition a to substitute

$$U(\underline{\theta}) + \int_{\theta'=\underline{\theta}}^{\tilde{\theta}} g'[y(\theta') - \theta'] d\theta'$$

for $U(\tilde{\theta})$ in

$$U(\tilde{\theta}, \theta) = U(\tilde{\theta}) + g[y(\tilde{\theta}) - \tilde{\theta}] - g[y(\tilde{\theta}) - \theta].$$

This yields

$$U(\tilde{\theta}, \theta) = U(\theta) + \int_{\theta'=\theta}^{\tilde{\theta}} \{g'[y(\theta') - \theta'] - g'[y(\tilde{\theta}) - \theta']\} d\theta',$$

which implies the incentive-compatibility constraint (2) because condition a and the convexity of g guarantee that the intergrand is negative for $\tilde{\theta} > \theta$ and positive for $\theta > \tilde{\theta}$ (in which case the limits of integration must be reversed). Q.E.D.

LEMMA 2. The firm's problem can be reduced to choosing $y(\theta)$ and $U(\underline{\theta})$ to maximize

$$-U(\underline{\theta}) + \int_{\theta=\underline{\theta}}^{\bar{\theta}} \left\{ y(\theta) - g[y(\theta) - \theta] - \left[\frac{1 - F(\theta)}{f(\theta)}\right] g'[y(\theta) - \theta] \right\} f(\theta) d\theta, \quad (4)$$

subject to conditions b and c and to $y(\theta) \geq \theta$.

Proof. By the definition of $U(\theta)$,

$$w(\theta) = U(\theta) + g[y(\theta) - \theta],$$

where $U(\theta)$ is given by condition a in lemma 1. Therefore

$$\int_{\theta=\underline{\theta}}^{\bar{\theta}} w(\theta) f(\theta) d\theta = U(\underline{\theta}) + \int_{\theta=\underline{\theta}}^{\bar{\theta}} \left\{ g[y(\theta)-\theta] + \left[\frac{1-F(\theta)}{f(\theta)}\right] g'[y(\theta)-\theta] \right\} f(\theta) d\theta$$

after reversing the order of integration in the double integral.

PROPOSITION 1. At the firm's optimum, $U(\underline{\theta}) = 0$, and $y^*(\theta)$ solves

$$1 - g'(y-\theta) - \left[\frac{1-F(\theta)}{f(\theta)}\right] g''(y-\theta) = 0. \tag{5}$$

The resulting effort level, $a^*(\theta) \equiv y^*(\theta) - \theta$, is strictly positive and strictly increasing and equals a_{fb} only at $\bar{\theta}$.

Proof. It suffices to show that for each θ the solution to (5) maximizes the kernel in (4),

$$y - g(y-\theta) - \left[\frac{1-F(\theta)}{f(\theta)}\right] g'(y-\theta),$$

subject to $y(\theta)$ being nondecreasing and $y(\theta) \geq \theta$. Since this kernel is concave (because $g''' \geq 0$), the solution to (5) yields the unconstrained maximum, unless the maximized value of the kernel is negative, in which case the firm chooses not to operate the technology. By the envelope theorem, this maximized value is increasing in θ. And at $\theta = \underline{\theta}$ and $a = 0$ the (nonmaximized) value of the kernel is

$$\underline{\theta} - g(0) - [g'(0)/f(\theta)] = \underline{\theta},$$

which is positive. Therefore, the maximized value of the kernel is positive for all θ, and (5) yields the maximizing value of y.

As for the effort level, $a^*(\theta) = y^*(\theta) - \theta$ is strictly increasing (and hence $y^*[\theta]$ is nondecreasing, as required) because implicitly differentiating (5) yields

$$y' - 1 = -\frac{\left\{ \dfrac{d}{d\theta}\left[\dfrac{1-F(\theta)}{f(\theta)}\right] \right\} g''(y-\theta)}{g''(y-\theta) + \left[\dfrac{1-F(\theta)}{f(\theta)}\right] g'''(y-\theta)} > 0,$$

since $[1 - F(\theta)]/f(\theta)$ strictly decreases in θ. Also, $a^*(\theta)$ is strictly positive (and hence $y^*[\theta] > \theta$, as required) because the left-hand side of (5) is positive at $y = \theta$:

$$1 - g'(0) - \left[\frac{1-F(\theta)}{f(\theta)}\right] g''(0) > 0$$

because $g'(0) = g''(0) = 0$. Finally, substituting $\theta = \bar{\theta}$ into (5) yields $1 - F(\bar{\theta}) = 0$ and $g' = 1$, so $a^*(\bar{\theta}) = a_{\mathrm{fb}}$. Q.E.D.

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