# Supermajority Voting Rules 

Richard T. Holden*

August[16, 2005


#### Abstract

The size of a supermajority required to change an existing contract varies widely in different settings. This paper analyzes the optimal supermajority requirement, determined by multilateral bargaining behind the veil of ignorance, where there are a continuum of possible policies. The optimum is determined by a tradeoff between reducing blocking power of small groups and reducing expropriation of minorities. We solve for the optimal supermajority requirement as a function of the distribution of voter types, the number of voters and the degree of importance of the decision. The findings are consistent with observed heterogeneity of supermajority requirements in different settings and jurisdictions.


Keywords: Supermajority, majority rule, qualified majority, special majority, constitutions, social contract, incomplete contracts.

JEL Classification Codes: D63, D72, D74, F34, G34, H40

[^0]
## 1 Introduction

Almost all agreements contain provisions governing the process by which the terms of the agreement can be changed. Often these clauses require a supermajority (more than $50 \%$ ) of the parties to agree in order to make a change. Constitutions of democratic countries are perhaps the most prominent example of this. Yet the phenomenon is far more widespread. Majority creditor clauses in corporate and sovereign debt contracts provide that a supermajority of the creditors can bind other creditors in renegotiations with the borrower. The size of the required supermajority varies. Filibuster rules mean that a $60 \%$ majority of the United States Senate is required to appoint a federal judge. Corporations laws in different countries specify different supermajorities, sometimes as high as $95 \%$, required to compulsorily acquire (or "squeeze-out") equity securities ${ }^{1}$. To change the International Monetary Fund Articles of Agreement requires an $85 \%$ vote of member nations.

The variation in supermajority requirements ${ }^{2}$ in different settings and jurisdictions raises the obvious question of why they differ. This paper presents a model of collective decision making in order to analyze supermajority rules. Individuals engage in multilateral bargaining, over a contract, behind the veil of ignorance. One of the elements of the contract specifies the rules governing changes to the contract if a state of nature arises which was not specified in the contract. The optimal supermajority requirement is determined by a tradeoff between two factors. On the one hand a high supermajority is attractive because it reduces the problem of minorities being expropriated by the majority. On the other hand, a high supermajority is detrimental because it provides a small group of voters with blocking power in the sense that they can prevent the efficient action from being taken. It is the tradeoff between minority protection and blocking minimization which determines the optimal supermajority requirement.

This paper is related to important literatures in economics and political science. Interest in rules chosen behind the veil of ignorance can be traced to Rousseau. Early works by economists using this notion include Vickrey (1945) and Harsanyi (1953), the classic papers of Harsanyi (1955) and Mirrlees (1971) and, of course,

[^1]Rawls (1971). The formal analysis of the construction of constitutions began with the seminal contribution of Buchanan and Tullock (1962). The literature on majority voting is known to have distant origins, dating at least to Condorcet. Arrow's extraordinary work (Arrow (1951)) ignited a vast literature attempting to overcome his Impossibility Proposition. Particularly pertinent to this paper, Arrow himself conjectured (Arrow (1951)) that a sufficient degree of social consensus could overcome his Impossibility Proposition ${ }^{3}$. This conjecture was formalized by Caplin and Nalebuff (1988) and with greater generality by Caplin and Nalebuff (1991). In fact, formal interest in voting under supermajority rules can be traced to Black (1948a). Despite large literatures on related issues there is, to our knowledge, no canonical exposition of the optimal supermajority rule.

Focusing on the role of contractual incompleteness, Aghion and Bolton (1992) show that some form of majority voting dominates a unanimity requirement in a world of incomplete social contracts. This paper highlights the fact that if a contract could be complete then the issue of supermajority requirements is moot if rules are chosen behind the veil of ignorance. In a similar framework, Erlenmaier and Gersbach (2001) consider "flexible" majority rules whereby the size of required supermajority depends on the proposal made by the agenda setter. Babera and Jackson (2004) consider "self-stable" majority rules, in the sense that the required supermajority does not wish to change the supermajority rule itself ex post. A related paper is Maggi and Morelli (2003), which finds that unanimity, in certain settings, is usually optimal if there is imperfect enforcement

Aghion and Bolton (1992) capture the key tradeoff which determines the optimal supermajority rule. Their model is somewhat special, however. They have only two types of voters, the voters are risk-neutral, and the fraction of voters who are harmed by a given proposal is constant across states of nature. These restrictive assumptions generate three particular problems. Firstly, they mean that asymmetric information essentially plays no role. Secondly, the two-point distribution of types of voters masks important issues in determining the optimal supermajority rule, related to the distribution of voters. Thirdly, risk-neutrality further masks a critical determinant of the optimal supermajority in a general setting. The first and second of these difficulties are overcome by Aghion et al. (2004). However, even this model is rather

[^2]specific. It is also one of public good provision, very much in the spirit of Romer and Rosenthal (1983). It is unclear that a very specific model such as this can be used to analyze situations as disparate as, for instance, the design of constitutions of countries and takeover provisions in stock markets.

We undertake a more general formulation where the policy set is a continuum. This allows us to study the effect of risk and risk-aversion on the voting rule. As discussed in section 2 , we consider a particularly strong form of incompleteness of the social contract. The social contract is not permitted to specify a state-contingent supermajority rule. In the context of the model this means that the supermajority requirement cannot differ based on realized draws from the distribution of types.

The distribution of voter types is particularly important in understanding the optimal tradeoff between minority protection and hold-up minimization. We begin with the benchmark case where there are a continuum of voters and the social decision space is unidimensional. Here we show that with risk-neutral or risk-averse voters the optimal supermajority rule is just majority rule (ie. $50 \%)^{4}$. This is essentially because, due to the law of large numbers, there is no new information revealed once voters' draws from the distribution are realized. We then turn attention to the more interesting case of a finite number of voters. Here there is a very real distinction between the ex ante and ex post stages. We show that as the importance of the decision increases the optimal supermajority requirement increases monotonically. Indeed, for a sufficient degree of importance unanimity is always optimal. We go on to show that the optimal supermajority rule is monotonically decreasing in the number of voters. Finally, our fourth main result is that as the spread of the distribution of voter types increases, so the optimal supermajority rule increases monotonically.

In establishing these latter three results, where the number of voters is finite, we solve for the expected utility of a given voter for any arbitrary absolutely continuous distribution of voter types as a function of the distribution, the number of voters, a measure of the importance of the decision, and the required supermajority. We are able to provide this expected utility characterization for any supermajority requirement.

[^3]This paper is organized as follows. Section 2 contains our model and main results. We begin with a benchmark case in the simplest possible setting: a unidimensional social choice set, risk-neutrality and a continuum of voters. These restrictions are gradually relaxed. Section 3 discusses the empirical content of various supermajority rules. Section 4 highlights areas of future research and contains some concluding remarks. All proofs of results are relegated to the appendix, as is a complete characterization of the expected utility calculations.

## 2 The Model

### 2.1 Statement of the Problem

Let $M$ be the set of voters, which may be finite or infinite. When it is finite we shall denote it's cardinality as $m$. The policy space is assumed to be the unit interval $[0,1]$. Voters preferences over this policy space are drawn from the distribution function $F(x)$.

Definition 1. A Social Decision is $\theta \in[0,1]$.
Assumption A1. Each voter $i$ has a utility function of the form:

$$
u_{i}=-\exp \left\{\beta\left\|\theta-x_{i}\right\|\right\}
$$

where $\beta$ is the coefficient of importance.
This is clearly related to the notion of risk-aversion. It is innocuous to think of the voters in this specification as being risk-averse about the social decision. However, since risk-aversion is usually thought of in the context of lotteries over money, we shall use the term "importance" throughout the paper.

Definition 2. A Supermajority Rule is a scalar $\alpha \in\left[\frac{1}{2}, 1\right]$ which determines the proportion of voters required to modify the social decision.

There are two time periods in the model. In period 1 voters know the distribution of types, $F(x)$, but they do not know their draw from the distribution. In this period they bargain, behind the veil of ignorance, over a Social Choice and a Supermajority Rule. In period 2 the social decision can be changed if a coalition
of voters $\geq \alpha m$ (for $M$ finite, or measure $\alpha$ of voters for $M$ infinite) prefer a new social decision.

As mentioned before, we restrict the (social) contracting space. State contingent supermajority rules are not permitted. An example of such a rule would be any kind of utilitarian calculus which would vary the supermajority requirement to change the status quo according to the aggregate utility to be gained ex post.

### 2.2 A Continuum of Voters

In this section we analyze the simple benchmark case where there is a continuum of voters. Let $F(x)$ be a continuous distribution with associated density function $f(x)$.

Proposition 1. Assume $A 1$ and $\beta=0$. Then the optimal social decision is to set $\theta$ equal to the median of $f(x)$.

This result is quite intuitive. The optimal social decision clearly must maximize the aggregate surplus of the voters. This is achieved by selecting the social decision which minimizes the sum of the distances from ideal points. This is simply the median of the distribution. With this Proposition in hand we can now state the following result.

Proposition 2. Assume $A 1$ and $\beta=0$. Then the optimal Supermajority Rule is the interval $\alpha=\left[\frac{1}{2}, 1\right]$.

With the distribution of voter types known at the ex ante bargaining stage, nothing essentially changes upon discovery of voters' positions in the distribution. Consequently the social decision must be unchanged. This will be the case if the required supermajority is at least one-half. This is basically a consequence of the Median Voter Theorem due to Black (1948b). The result is also parallel to the finding of Aghion and Bolton (1992) who show that if a complete social contract can be written then the required supermajority should be such that there is no change to the ex ante decision. In their model this requires unanimity. Here anywhere between a simple plurality and unanimity achieves this end.

It is natural to assume that voters' utility decreases more than proportionally as the social decision moves further from their ideal point. Moreover, we would like
to capture the idea that some decisions are more important to voters than other decisions. This is captured by $\beta$ positive.

Proposition 3. Assume $A 1$ and $\beta>0$. Then the optimal social decision requires the following condition to hold:

$$
\int_{\theta}^{1} \beta \exp \{\beta(x-\theta)\} f(x) d x+\int_{0}^{\theta}-\beta \exp \{\beta(\theta-x)\} f(x) d x=0
$$

This implies that the optimal social decision moves from the median in the direction of the end of the distribution with the longest tail. Since larger deviations from the social decision receive proportionally greater "punishment", the ex post decision moves toward the longer tail of the distribution. Note that this specification nests Proposition 1 as the special case where $\beta=0$.

Proposition 4. Assume $A 1$ and $\beta>0$. Then the optimal supermajority rule lies in the interval $\alpha=\left[\frac{1}{2}, 1\right]$, and increases as $f(x)$ becomes more skewed to the right.

As before there is essentially no change between period 1 and period 2 . Therefore the optimal social decision is unchanged - and the way to ensure that it remains unchanged is by requiring at least a simple plurality.

### 2.3 A Finite Number of Voters

With the assumption of a continuum of voters the law of large numbers ensures that the ex post optimal social decision is identical to the ex ante optimal social decision. Where the number of voters is finite this is no longer the case.

Definition 3. Assume $A 1$ and $M$ finite. Then the ex ante optimal social decision is:

$$
\widehat{\theta}=\underset{\theta}{\arg \min } \sum_{i=1}^{m}-\exp \left\{\beta\left|\theta-x_{i}\right|\right\}
$$

Definition 4. Assume $A 1$ and $M$ finite. Then the ex post optimal social decision is:

$$
\widehat{\theta}=\underset{\theta}{\arg \min } \sum_{i=1}^{m}-\exp \left\{\beta\left|\theta-x_{i}^{*}\right|\right\}
$$

With a finite number of voters the ex post optimal decision may well differ from the ex ante optimal decision because of the realized draws from $F(x)$. It is this wedge
between ex ante and ex post optimality which creates complexity in the choice of the optimal supermajority rule.

We make the following technical assumption which enables us to avail ourselves of several useful results from the theory of order-statistics.

Assumption A2. The parent distribution of voter types $F(x)$ is absolutely continuous.

By using order-statistics we are able to fully characterize the expected utility of a given voter for an arbitrary distribution of the population, number of voters, degree of risk-aversion and supermajority rule. We are, therefore, able to determine which rule yields the highest expected utility, and is hence optimal. The derivation of these expected utilities is contained in the appendix.

There is an obvious issue of how the ex post social decision is determined is a coalition has a sufficient number of members relative to the required supermajority who would be made better-off by a change to the ex ante social decision. In principle, any ex post social decision within the interval spanned by their preferences improves each of their payoffs. For simplicity we make the following assumption about how the bargaining power amongst members of such a coalition.

Assumption A3. If a coalition has the required supermajority ex post then the social decision is that preferred by the "final" member of the coalition. That is, the member of the coalition whose preference is closest to the ex ante social decision.

This assumption simplifies the analysis, but is not determinitive as far as the three major results, Propositions 1-3, are concerned. We could distribute the bargaining power amongst the members of the coalition in any (exogenous) way and not alter the results, but complicate the analysis. In section 4 we discuss the possibility of determining the formation of coalitions and the allocation of surplus within coalitions simultaneously ${ }^{5}$.

Before providing two examples, in order to build intuition, we state the major results of the paper.

[^4]Proposition 5. Assume A1-A3 and $M$ finite. Then the optimal supermajority rule is increasing in the coefficient of importance, $\beta$.

As the coefficient of importance increases voters are progressively more concerned with being expropriated. They essentially purchase insurance against this by requiring that the size of the majority required to expropriate them be large, thereby reducing the probability of that event occurring. In fact, when the coefficient of importance is sufficiently high a unanimity requirement is always optimal. If there is the prospect of a sufficiently bad payoff ${ }^{6}$ then voters require a veto in order to insure themselves against this outcome.

Proposition 6. Assume A1-A3 and $M$ finite. Then the optimal supermajority rule is decreasing in the number of voters, $m$.

As the number of voters increases the probability of being part of an expropriated minority decreases. The benefit gained from avoiding hold-up, however, is unchanged. For reasons analogous to those at play in Proposition 5 less insurance is required and therefore the optimal supermajority rule decreases.

Before stating Proposition 7, the following definition, due to Rothschild and Stiglitz (1970), is useful.

Definition 5. A distribution $\widehat{F}(\cdot)$ is Rothschild-Stiglitz Riskier than another distribution $F(\cdot)$ if either (i) $F(\cdot)$ Second Order Stochastically Dominates $\widehat{F}(\cdot)$, (ii) $\widehat{F}(\cdot)$ is a Mean Preserving Spread of $F(\cdot)$, or (iii) $\widehat{F}(\cdot)$ is an Elementary Increase in Risk from $F(\cdot)$.

As is well known, Rothschild and Stiglitz (1970) showed that these three statements are equivalent.

Proposition 7. Assume $A 1-A 3, \beta>0$ and $M$ finite. Then the optimal supermajority rule is larger for a distribution of voter types, $\widehat{F}(x)$ than for the distribution $F(x)$ if $\widehat{F}(x)$ is Rothschild-Stiglitz Riskier than $F(x)$.

This result obtains for reasons closely related to those of the two previous Theorems. As the spread of voter types increases more insurance is desired, which is effected by requiring the supermajority rule to be higher. This is, however, only

[^5]the case if the voters' utility is more than proportionally decreasing as the social decision moves away from their ideal point (i.e. $\beta>0$ ).

We now provide two examples in order to build intuition for the results. The calculations are somewhat involved, despite the apparent simplicity of the example. These calculations do, however, highlight the main technical issues involved in the proof of the above theorems.

### 2.3.1 Example 1

Voters' types are drawn from the uniform distribution on $[0,1]$ and $m=5$.

First note that the ex ante optimal social decision is simply $\theta^{*}=\frac{1}{2}$. First we focus on the outcome under majority rule, which is simply that the ex post social decision is the median of the voters' draws. Consider voter $i$ and let the other voters' draws be:

$$
x_{1}^{*} \leq x_{2}^{*} \leq x_{3}^{*} \leq x_{4}^{*}
$$

where $x_{k}^{*}$ is the $k$ th order-statistic. Now note that the density of $\left(x_{2}^{*}, x_{3}^{*}\right)$ on $[0,1] \times[0,1]$ is $^{7}:$

$$
f\left(a_{2}, a_{3}\right)=24 a_{2}\left(1-a_{3}\right)
$$

Denote $T=\{[0,1] \times[0,1] \mid l \geq j \forall l, j \in[0,1]\}$. Note that in considering the median we need only be concerned with voter $i$ 's position relative to $x_{2}^{*}$ and $x_{3}^{*}$. If they are between $x_{2}^{*}$ and $x_{3}^{*}$ then they are the median and their loss is zero. If $x_{i}^{*} \leq x_{2}^{*}$ then the expected "loss" is $\int_{0}^{a_{2}}-\exp \left\{\beta\left|t-a_{2}\right|\right\} d t$ and if $x_{i}^{*} \geq x_{3}^{*}$ it is $\int_{a_{3}}^{1}-\exp \left\{\beta\left|t-a_{3}\right|\right\} d t$. If $x_{2}^{*} \geq x_{i}^{*} \geq x_{3}^{*}$ then the expected loss is $-\exp (0)=-1$.

[^6]The expected utility of voter $i$ is, making use of Fubini's Theorem, therefore:

$$
\begin{aligned}
\mathbb{E}\left[u_{i}^{M}\right] & =\iint_{T}\left(\begin{array}{c}
\int_{0}^{a_{2}}-\exp \left\{\beta\left(a_{2}-t\right)\right\} d t \\
+\int_{a_{2}}^{a_{3}}(-1) d t \\
+\int_{a_{3}}^{1}-\exp \left\{\beta\left(t-a_{3}\right)\right\} d t
\end{array}\right) 24 a_{2}\left(1-a_{3}\right) d a_{2} d a_{3} \\
& =\int_{0}^{1} \int_{0}^{a_{3}}\binom{\frac{1-e^{\beta a_{2}}}{\beta}-\left(a_{3}-a_{2}\right)}{+\frac{1-e^{\beta\left(1-a_{3}\right)}}{\beta}} 24 a_{2}\left(1-a_{3}\right) d a_{2} d a_{3} \\
& =-\frac{\beta\left(\beta^{4}-10 \beta^{3}+120 \beta+480\right)+240 e^{\beta}(\beta-3)+720}{5 \beta^{5}}
\end{aligned}
$$

Now consider the expected utility of voter $i$ if we require unanimity in order to change the social decision ex post. Denote the ex post social decision as $t$. Let $B$ be the event where $0 \leq x_{1}^{*} \leq x_{4}^{*}<\frac{1}{2}$ and let $B^{\prime}$ be the event where $\frac{1}{2} \geq x_{1}^{*} \geq x_{4}^{*} \geq 1$. Let $A=\Omega \backslash\left(B+B^{\prime}\right)$. It is clear that $\operatorname{Pr}(A)=\frac{7}{8}$ and that $\operatorname{Pr}(B)=\operatorname{Pr}\left(B^{\prime}\right)=\frac{1}{16}$. The expected utility of voter $i$ conditional on event $A$ is:

$$
\begin{aligned}
\mathbb{E}\left[u_{i}^{U} \mid A\right] & =2 \int_{0}^{\frac{1}{2}}-\exp \left\{\beta\left(\frac{1}{2}-t\right)\right\} d t \\
& =\frac{2\left(1-e^{\beta / 2}\right)}{\beta}
\end{aligned}
$$

The density ${ }^{8}$ of $x_{4}^{*}$ is $f\left(a_{4}\right)=4\left(a_{4}\right)^{3}$. We now need the density of $x_{4}^{*}$ on $\left[0, \frac{1}{2}\right]$, which is found by applying the Change of Variables Theorem, yielding $g\left(a_{4}\right)=$ $2 \times 4\left(2 a_{4}\right)^{3}=64\left(a_{4}\right)^{3}$. Therefore:

$$
\begin{aligned}
\mathbb{E}\left[u_{i}^{U} \mid B\right]= & \int_{\frac{1}{2}}^{1}-\exp \left\{\beta\left(t-\frac{1}{2}\right)\right\} d t \\
& +\int_{0}^{\frac{1}{2}}\left(\int_{0}^{a_{4}}-\exp \left\{\beta\left(a_{4}-t\right)\right\} d t-\int_{a_{4}}^{\frac{1}{2}} 1 d t\right) 64\left(a_{4}\right)^{3} d a_{4} \\
= & \frac{2\left(1-e^{\beta / 2}\right)}{\beta}+\frac{1}{\beta}-\frac{8\left(48+e^{\beta / 2}\left(\beta^{3}-6 \beta^{2}+24 \beta-48\right)\right.}{\beta^{5}}-\frac{1}{10}
\end{aligned}
$$

${ }^{8}$ For the uniform distribution the density of the $i t h$ order statstic is:

$$
f_{i}(u)=\frac{n!}{(i-1)!(n-i)!} u^{i-1}(1-u)^{n-i}
$$

where $\int_{\frac{1}{2}}^{1}-\exp \left\{\beta\left(t-\frac{1}{2}\right)\right\} d t$ is the term associated with $x_{i} \geq \frac{1}{2}$ and the term associated with $x_{i} \leq \frac{1}{2}$ is $\int_{0}^{\frac{1}{2}}\left(\int_{0}^{a_{4}}-\exp \left\{\beta\left(a_{4}-t\right)\right\} d t-\int_{a_{4}}^{\frac{1}{2}} 1 d t\right) 64\left(a_{4}\right)^{3} d a_{4}$.

Under event $B^{\prime}$ the expected utility is given by:

$$
\begin{aligned}
\mathbb{E}\left[u_{i}^{U} \mid B^{\prime}\right]= & \int_{0}^{\frac{1}{2}}-\exp \left\{\beta\left(\frac{1}{2}-t\right)\right\} d t \\
& +\int_{\frac{1}{2}}^{1}\left(\int_{a_{1}}^{1}-\exp \left\{\beta\left(t-a_{1}\right)\right\} d t-\int_{\frac{1}{2}}^{a_{1}} 1 d t\right) 64\left(1-a_{1}\right)^{3} d a_{1} \\
= & \frac{2\left(1-e^{\beta / 2}\right)}{\beta}+\frac{1}{\beta}-\frac{8\left(48+e^{\beta / 2}\left(\beta^{3}-6 \beta^{2}+24 \beta-48\right)\right.}{\beta^{5}}-\frac{1}{10}
\end{aligned}
$$

Therefore the total expected utility under unanimity is:

$$
\begin{aligned}
\mathbb{E}\left[u_{i}^{U}\right] & =\frac{7}{8} \mathbb{E}\left[u_{i}^{U} \mid A\right]+\frac{1}{16} \mathbb{E}\left[u_{i}^{U} \mid B\right]+\frac{1}{16} \mathbb{E}\left[u_{i}^{U} \mid B^{\prime}\right] \\
& =\frac{-3840+\beta^{4}(\beta-160)+10 e^{\beta / 2}(-384+\beta(192+\beta(\beta(15 \beta+8) 48)))}{80 \beta^{5}}
\end{aligned}
$$

For majority rule to be preferable to unanimity therefore requires:

$$
\begin{gathered}
\frac{\beta\left(-\beta^{4}+10 \beta^{3}-120 \beta-480\right)+240 e^{\beta}(3-\beta)-720}{5 \beta^{5}}> \\
\frac{-3840+\beta^{4}(\beta-160)+10 e^{\beta / 2}(-384+\beta(192+\beta(\beta(15 \beta+8) 48)))}{80 \beta^{5}}
\end{gathered}
$$

Solving numerically shows that this is the case if and only if $0 \leq \beta \lesssim 3.9$. Therefore when the decision is relatively unimportant majority rule dominates, but with a sufficiently high enough degree of importance unanimity is preferred.

Now consider the case where the social decision can be altered ex post if four voters agree. In this example with five voters this reflects the only supermajority which is greater than simple majority but less than unanimity.

Now define events $B, B^{\prime}, C$ and $C^{\prime}$ as follows. $B$ is the event where $0 \leq x_{1}^{*} \leq$ $x_{2}^{*} \leq x_{3}^{*} \leq x_{4}^{*} \leq \frac{1}{2} . \quad B^{\prime}$ is the event where $\frac{1}{2} \leq x_{1}^{*} \leq x_{2}^{*} \leq x_{3}^{*} \leq x_{4}^{*} \leq 1 . \quad C$ is the event where $0 \leq x_{1}^{*} \leq \frac{1}{2} \leq x_{2}^{*} \leq x_{3}^{*} \leq x_{4}^{*} \leq 1 . C^{\prime}$ is the event where $0 \leq x_{1}^{*} \leq x_{2}^{*} \leq x_{3}^{*} \leq \frac{1}{2} \leq x_{4}^{*}$. Also, let $A=\Omega \backslash\left(B+B^{\prime}+C+C^{\prime}\right)$.


Event $B$


Event $B^{\prime}$


Event $C$


Event $C^{\prime}$

Note that $\operatorname{Pr}(B)=\operatorname{Pr}\left(x_{4}^{*} \leq \frac{1}{2}\right)=\frac{1}{16}=\operatorname{Pr}\left(B^{\prime}\right) . \operatorname{Pr}\left(C^{\prime}\right)=\operatorname{Pr}\left(x_{3}^{*} \leq \frac{1}{2} \wedge x_{4}^{*} \geq \frac{1}{2}\right)=$ $\frac{1}{4}=\operatorname{Pr}\left(C^{\prime}\right)$. Also note that $\operatorname{Pr}(A)=\frac{3}{8}$.

As before, if event $A$ occurs then there is no change to the ex ante social decision and hence the expected utility of voter $i$ is:

$$
\begin{aligned}
\mathbb{E}\left[u_{i}^{S} \mid A\right] & =2 \int_{0}^{\frac{1}{2}}-\exp \left\{\beta\left(\frac{1}{2}-t\right)\right\} d t \\
& =\frac{2\left(1-e^{\beta / 2}\right)}{\beta}
\end{aligned}
$$

Note ${ }^{9}$ that the density of $x_{2}^{*}$ conditional on event $C$ is simply the density of the first order-statistic of three on $\left[\frac{1}{2}, 1\right]$. In fact, order statistics from a continuous

[^7]parent form a Markov Chain. It follows that the density of the first-order statistic of three ${ }^{10}$ on $U[0,1]$ is $3\left(1-a_{2}\right)^{2}$. By a change of variables the density on $\left[\frac{1}{2}, 1\right]$ is therefore $24\left(1-a_{2}\right)^{2}$. Hence the expected utility conditional on event $C$ is:
\[

$$
\begin{aligned}
\mathbb{E}\left[u_{i}^{S} \mid C\right]= & \int_{\frac{1}{2}}^{1}\left(-1 \int_{1 / 2}^{a_{2}} d t+\int_{a_{2}}^{1}-\exp \left\{\beta\left(t-a_{2}\right)\right\} d t\right) 24\left(1-a_{2}\right)^{2} d a_{2} \\
& +\int_{0}^{1 / 2}-\exp \left\{\beta\left(\frac{1}{2}-t\right)\right\} d t \\
= & -\frac{1}{8}+\frac{1}{\beta}+\frac{1-e^{\beta / 2}}{\beta}-\frac{6\left(-8+e^{\beta / 2}\left(8-4+\beta^{2}\right)\right.}{\beta^{4}}
\end{aligned}
$$
\]

The density of $x_{3}^{*}$ conditional on event $C^{\prime}$ is the third of three uniformly distributed order-statistics on $\left[0, \frac{1}{2}\right]$, which is $g\left(a_{2} \mid C^{\prime}\right)=24\left(a_{3}\right)^{2}$. Hence the expected utility conditional on event $C^{\prime}$ is:

$$
\begin{aligned}
\mathbb{E}\left[u_{i}^{S} \mid C^{\prime}\right]= & \int_{0}^{\frac{1}{2}}\left(-1 \int_{a_{3}}^{\frac{1}{2}} d t+\int_{0}^{a_{3}}-\exp \left\{\beta\left(a_{3}-t\right)\right\} d t\right) 24\left(a_{3}\right)^{2} d a_{3} \\
& +\int_{1 / 2}^{1}-\exp \left\{\beta\left(t-\frac{1}{2}\right)\right\} d t \\
= & -\frac{1}{8}+\frac{1}{\beta}+\frac{1-e^{\beta / 2}}{\beta}-\frac{6\left(-8+e^{\beta / 2}\left(8-4+\beta^{2}\right)\right.}{\beta^{4}}
\end{aligned}
$$

Now note that the joint density of $\left(x_{3}^{*}, x_{4}^{*}\right)$ on $[0,1]$ is $f\left(x_{3}, x_{4}\right)=12\left(a_{3}\right)^{2}$ and so on $\left[0, \frac{1}{2}\right]$ it is $192\left(a_{3}\right)^{2}$.

The expected utility conditional on event $B$ is therefore:

$$
\begin{aligned}
\mathbb{E}\left[u_{i}^{S} \mid B\right] & =\int_{0}^{\frac{1}{2}} \int_{0}^{a_{4}}\left(\begin{array}{c}
-\int_{a_{3}}^{a_{4}} 1 d t \\
+\int_{0}^{a_{3}}-\exp \left\{\beta\left(a_{3}-t\right)\right\} d t \\
+\int_{a_{4}}^{1}-\exp \left\{\beta\left(t-a_{4}\right)\right\} d t
\end{array}\right) 192\left(a_{3}\right)^{2} d a_{3} d a_{4} \\
& =\frac{-3840 e^{\beta / 2}-\beta^{4}(\beta-20)+1920(\beta+6)+80 e^{\beta / 2}\left(\beta^{4}+72 \beta-96\right)}{10 \beta^{5}}
\end{aligned}
$$

The joint density of $\left(x_{1}^{*}, x_{2}^{*}\right)$ on $[0,1]$ is $f\left(x_{1}, x_{2}\right)=f\left(x_{1}, x_{2}\right)=12\left(1-a_{2}\right)^{2}$ and

[^8]so on $\left[\frac{1}{2}, 1\right]$ it is $192\left(1-a_{2}\right)^{2}$. The expected utility conditional on event $B^{\prime}$ is:
\[

$$
\begin{aligned}
\mathbb{E}\left[u_{i}^{S} \mid B^{\prime}\right] & =\int_{\frac{1}{2}}^{1} \int_{1}^{a_{1}}\left(\begin{array}{c}
-\int_{a_{1}}^{a_{2}} 1 d t \\
+\int_{0}^{a_{1}}-\exp \left\{\beta\left(a_{1}-t\right)\right\} d t \\
+\int_{a_{2}}^{1}-\exp \left\{\beta\left(t-a_{2}\right)\right\} d t
\end{array}\right)\left(-192\left(1-a_{2}\right)^{2}\right) d a_{2} d a_{1} \\
& =\frac{-3840 e^{\beta / 2}-\beta^{4}(\beta-20)+1920(\beta+6)+80 e^{\beta / 2}\left(\beta^{4}+72 \beta-96\right)}{10 \beta^{5}}
\end{aligned}
$$
\]

Therefore the total expected utility under a supermajority of four voters (ie. $80 \%$ supermajority) is:

$$
\begin{aligned}
\mathbb{E}\left[u_{i}^{S}\right]= & \frac{3}{8} \mathbb{E}\left[u_{i}^{S} \mid A\right]+\frac{1}{16} \mathbb{E}\left[u_{i}^{S} \mid B\right]+\frac{1}{16} \mathbb{E}\left[u_{i}^{S} \mid B^{\prime}\right] \\
& +\frac{1}{4} \mathbb{E}\left[u_{i}^{S} \mid C\right]+\frac{1}{4} \mathbb{E}\left[u_{i}^{S} \mid C^{\prime}\right]
\end{aligned}
$$

Which, upon simplification, is:

$$
\mathbb{E}\left[u_{i}^{S}\right]=\frac{\begin{array}{c}
-1920 e^{\beta}+\beta^{4}(80-3 \beta)+1920(\beta+3) \\
-10 e^{\beta / 2}(284+\beta(-192+\beta(\beta+4)(5 \beta-12)))
\end{array}}{40 \beta^{5}}
$$

For an $80 \%$ supermajority to be preferable to majority rule therefore requires:

$$
\begin{gathered}
-1920 e^{\beta}+\beta^{4}(80-3 \beta)+1920(\beta+3) \\
\frac{-10 e^{\beta / 2}(284+\beta(-192+\beta(\beta+4)(5 \beta-12)))}{40 \beta^{5}} \geq \\
\frac{11520+(60-\beta) \beta^{4}-30 e^{\beta / 2}(-384+\beta(192+\beta(\beta-4)(12+\beta)))}{30 \beta^{5}}
\end{gathered}
$$

Solving numerically shows that this is the case if and only if $\beta \gtrsim 2.69$.
For unanimity to be superior to an $80 \%$ supermajority rule requires:

$$
\begin{gathered}
\frac{-3840+\beta^{4}(\beta-160)+10 e^{\beta / 2}(-384+\beta(192+\beta(\beta(15 \beta+8) 48)))}{80 \beta^{5}} \geq \\
-1920 e^{\beta}+\beta^{4}(80-3 \beta)+1920(\beta+3) \\
\frac{-10 e^{\beta / 2}(284+\beta(-192+\beta(\beta+4)(5 \beta-12)))}{40 \beta^{5}}
\end{gathered}
$$

Solving numerically reveals that this the case for $\beta \gtrsim 9.02$. That is, the $80 \%$ supermajority rule dominates unanimity until the degree of importance becomes sufficiently large. For sufficiently large degrees of importance unanimity dominates because the fear of expropriation dominates and a veto provides them with insurance against this possibility. Therefore, in this example, for $0 \gtrsim \beta \gtrsim 2.69$ majority rule is optimal, for $2.69 \gtrsim \beta \gtrsim 9.02$ an $80 \%$ supermajority requirement is optimal, and for $\beta \gtrsim 9.02$ a unanimity requirement is optimal.

### 2.3.2 Example 2

Voters' types are drawn from the uniform distribution on $[0,1]$ and $m=3$.
This example illustrates that as the number of voters increases the optimal supermajority rule decreases. We again use the uniform distribution, but with 3 voters rather than 5 .

The expected utility under majority rule (here 2 out of three voters) is ${ }^{11}$ :

$$
\begin{aligned}
\mathbb{E}\left[U_{M}\right] & =\int_{0}^{1} \int_{0}^{a_{2}}\left[\begin{array}{c}
\int_{0}^{a_{1}}-\exp \left\{\beta\left(a_{1}-t\right)\right\} d t \\
+\int_{a_{2}}^{1}-\exp \left\{\beta\left(t-a_{2}\right)\right\} d t \\
-1 \int_{a_{1}}^{a_{2}} d t
\end{array}\right] 2 d a_{1} d a_{2} \\
& =\frac{12-12 e^{\beta} \beta(12-\beta(\beta-6))}{3 \beta^{3}}
\end{aligned}
$$

The expected utility under a unanimity requirement is:

$$
\begin{array}{rc}
\mathbb{E}\left[U_{U}\right]= & \int_{0}^{1 / 2}-\exp \{\beta(1 / 2-t)\} d t \\
= & \frac{\left.-48+\int_{0}^{1 / 2}\left(\int_{0}^{a_{2}}-\exp \left\{\beta\left(a_{2}-t\right)\right\} d t-1 \int_{a_{2}}^{1 / 2} d t\right) 8 a_{2} d a_{2}\right)}{6 \beta^{3 / 2}(-8+\beta(\beta+4))}
\end{array}
$$

Now consider $\beta=5$. In this case, where $m=3$, a unanimity requirement is optimal and yields expected utility of approximately -3.44 . Where $m=5$ (ie. example 1) and $\beta=5$ an $80 \%$ supermajority is optimal and the expected utility is approximately -4.12 . For majority rule under $m=3$ expected utility is -4.50 . This illustrates the general point made in Proposition 6, that the optimal supermajority

[^9]requirement decreases as the number of voters gets larger. In this example, for a coefficient of importance of 5, it falls from $100 \%$ to $80 \%$.

## 3 Applications and Empirics

### 3.1 Constitutional Design

In recent years a number of countries have written completely new constitutions. The list includes: the Republic of South Africa, Cambodia, Namibia and East Timor ${ }^{12}$. The process by which constitutions are drafted and adopted have been of major interest to scholars of constitutional law. The standard process for constitutional adoption is the popular election of a constituent assembly, usually by proportional representation, and then a vote of that assembly, under a supermajority requirement, to ratify a constitution ${ }^{13}$. The constitution to be voted on is the outcome of negotiations between members of the constituent assembly. Whilst a careful analysis of the optimal supermajority required to ratify these constitutions has been absent, there has been debate regarding the impact of a two-thirds supermajority rule. Indeed it appears as though the default supermajority rule for constitutional ratification is taken to be two-thirds. This was the case in all of the aforementioned nations constitutional changes. The potentially adverse implications of such a default has begun to be acknowledged by legal scholars and practitioners ${ }^{14}$ and serves to illustrate some of the main features of our model. It has become apparent that the heterogeneity of different countries means that a "one size fits all" constitutional supermajority requirement is unlikely to be appropriate. As Dixon (2002) puts it:
"However, the experience of most post-colonial, post-conflict societies is likely to be very different. In circumstances where a single party is associated with liberation of the nation from a colonial oppressor, that liberation party will often command overwhelming support, which, particularly in combination with historically weak notions of political opposition, will produce large super-majority support for the liberation party.

[^10]In this context, whether a two-thirds majority requirement promotes any deliberative consideration will be entirely fortuitous."

In East Timor the two-thirds supermajority rule was arguably too low. At the constituent assembly election stage ${ }^{15}$ FRETELIN, the leading East Timorese proindependence party, received $57.3 \%$ of the popular vote and gained 12 of 13 district seats and 43 of he 75 national seats. This gave them 55 of the 88 seats, representing $62.5 \%$ of the seats. The Partido Democratico ("PD") won 7 seats, the Partido Social Democrata ("PSD") won 6 seats, the Associacao Social Democrata Timorense ("ASDT") $7.8 \%$ of the vote or 6 seats, and the remaining seats were divided amongst 8 smaller parties. Thus FRETELIN had nearly enough seats to unilaterally adopt a constitution of their choice. Dixon (2002) reports that "Informal alliances between minority parties and FRETELIN meant that FRETELIN was able to adopt the final constitution with minimal "opposition" support." Given the distribution of voter preferences, a higher supermajority requirement may have been appropriate in order to protect minority interests.

In Namibia in 1989 the pro-independence SWAPO party received 41 of the 72 seats in the constituent assembly, representing $57 \%$ of the votes, while the Democratic Turnhalle Alliance ("DTA") (who favored integration) won 21 seats or $29 \%$. The fact that SWAPO had to negotiate with more than one minority party meant that they had significantly less bargaining power than FRETELIN in East Timor (who could choose among a variety of potential coalition partners). SWAPO eventually agreed to a form of government which contained significant structural differences from their initial preference ${ }^{16}$. The supermajority requirement may still have been too low, but the Namibian experience contrasted with the East Timorese highlights how the supermajority rule can effect negotiations and how it depends on the distribution of voter preferences.

In contrast, in Cambodia the two-thirds requirement appears to have been too high. In the 1993 constituent assembly election the royalist party FUNINPEC

[^11]won $45.5 \%$ of the vote and gained 58 of the 120 seats in the Constituent Assembly. The the CPP, who were the incumbent government, gained $38.2 \%$ of the vote or 51 seats. This meant that neither party could govern in a coalition without the support of the other major party. Giving an in-effect veto to each of the major parties in a situation with a significant degree of polarization has been problematic. The ensuing "power-sharing" arrangement between FUNINPEC and the CPP led to what Dixon (2002) describes as a "co-government" model which she argues "has proven an unworkable compromise". Again, the supermajority requirement appears to have been unrelated to the distribution of voter preferences, leading to important difficulties.

The proposed constitution for the European Union has also received significant attention - in particular the required supermajority for particular kinds of decisions. At the time of this writing the draft constitution includes a set of issues, considered to be the most critical, such as border protection, over which their will be a national veto. This is in accordance with Proposition 1, that when an issue has a sufficiently high degree of importance the optimal supermajority rule is $100 \%$.

### 3.2 Collective Action Clauses and Sovereign Debt

Attention has been drawn in recent years to the absence of collective action clauses in many sovereign bond contracts, particularly those governed by New York law. Collective actions clauses allow a supermajority of bondholders to renegotiate terms of the bond contract, in particular payment terms such as the amortization schedule. The difficulties associated with debt restructuring where a small number of bondholders could hold-out and prevent renegotation and cause inefficient liquidation. Although formal models of inefficient liquidation are relatively recent (Aghion and Bolton (1992), Hart and Moore (1998)), the notion that restructuring provision are needed to prevent minorities taking actions which reduce the value bonds held by the majority dates to such clauses in English law governed corporate bonds in the nineteenth century. The US Treasury, the IMF (who also proposed Chapter 11 style procedures) and the G-10 Working Group on Contractual Clauses have all been supportive of wider use of collective actions clauses. In February 2003 Mexico began issuing New York Law bonds with such clauses. Uruguay, Brazil, Costa Rica, Colombia, Hungary, Italy, Panama, Peru, South Africa, Turkey and Venezuela. and
others have followed (Galvis and Saad (2004)). In fact Mexico's 2003 issuance expanded the use of collective actions clauses to important non-payment terms such as: events of default, governing law, pari passu ranking and submission to jurisdiction ${ }^{17}$. Brazil's and Venezuela's issuance have contained an $85 \%$ supermajority provision while the remainder have stipulated $75 \%$.

The has been relatively little attention paid, however, to the appropriate supermajority rule. There has been speculation that investment grade issuers such as Mexico would use $75 \%$, while non-investment grade issuers such as Brazil and Venezuala would use the higher $85 \%$ rule (Salmon (2003)). Although a number of recent non-investment grade issuers such as Panama, Peru, Turkey and Uruguary have used the $75 \%$ rule, it is unclear at this point whether a two-tier system will prevail, or whether $75 \%$ will become standard. Our model suggests that it would be rather fortitous for a $75 \%$ rule to be optimal for all issuers. The rule should optimally depend on the distribution of lender preferences, for instance their risk-tolerance. As syndicates vary across bond issues, so may the distribution of preferences. Furthermore, the number of members in a syndicate is typically a function of the size of the issue.

### 3.3 Financial Contracting and Renegotiation

There is a substantial literature arguing that having a large number of parties can aid commitment not to renegotiate a contract because of free-riding at the renegotiation phase. In the context of debt contracts it has been noted that this could be either good or bad for the creditors (Bolton and Scharfstein (1996)). If the borrower is solvent then having a diffuse group of creditors, thereby hindering renegotiation, allows the creditors to extract more surplus from the borrower. Where the borrower is not solvent, however, hindering renegotiation can lead to inefficient liquidation decisions. Bolton and Scharfstein (1996)also point out that varying the optimal voting rule governing renegotiation is an alternative instrument to changing the number of creditors. Since the number of creditors may address other issues, such as the size of the loan, an appropriately chosen majority creditor seems a useful

[^12]instruemnt. Their model implies that the voting rule should always be a supermajority rule, and should be higher when default risk is low, and when the liquidation value (alternative use valuation) of the firms' assets is low.

Our model does not address such specific issues as the risk of default or liquidation value of assets. However, our model does address two issues which Bolton and Scharfstein (1996) does not. The first is heterogenous preferences amongst creditors. Our model implies that the supermajority rule should be higher, all else equal, the more diffuse are the preferences of the creditors. Such heterogeneity could arise, for instance, from different risk appetites among the group of creditors. Secondly, our model suggests that the supermajority rule should be decreasing in the size of the syndicate. This factor suggests that the size of the syndicate and the supermajority rule are directly related, whereas in Bolton and Scharfstein (1996) syndicate size and the supermajority rule are perfect substitutes. To gain a complete understand of these issues would require integrating the detail of Bolton and Scharfstein (1996) with the type of model presented in this paper.

Furthermore, both this paper and Bolton and Scharfstein (1996) imply that the optimal supermajority rule is very different in situations of default than situtations of solvency. A contract which could specify a low supermajority rule contingent on certain events of default (eg. monetary default), so that inefficient liquidation would be more easily avoided, but a higher supermajority rule in other states so that maximum surplus can be extracted from the borrower, would appear to be a superior contract to one where the supermajority rule is not state contingent. This seems a particular attraction of the majority creditor clause, since the number of creditors can clearly not be easily varied contingent on events of default ${ }^{18}$.

### 3.4 The United Nations Security Council

It is well known that the five permanent members of the United Nations ("UN") Security Council each hold a veto over resolutions of the Council. We argue here that this veto was generally regarded as a sound device during the period from the creation of the UN until the end of the Cold War. In the post Cold War era the presence of a veto has attracted significant criticism, however. Our model suggest

[^13]that in a highly polarized environment, such as during the Cold War, unanimity would be the optimal supermajority rule. In a setting where views are significantly closer together a lower supermajority requirement, perhaps four of the five members, might be preferable. This prediction of the model seems consistent with current views regarding the veto power on the Security Council.

## 4 Discussion and Conclusion

### 4.1 Extensions

An obvious direction for further research is to explore settings in which the social choice space is multidimensional. A key issue in analyzing such a model is that with a multidimensional social choice space Euclidean Preferences no longer imply single peakedness. Consequently voting cycles cannot be ruled out and agenda setting may be an important issue in determining the social decision. Caplin and Nalebuff (1991) have shown that if the distribution of voter types is log-concave then as the number of voters goes large a $64 \%$ supermajority requirement ensures no voting cycles. Considering supermajority rules in the interval $[0.64,1]$ and logconcave voter distributions, may be a useful starting point in the exploration of multidimensional social choices.

The veil of ignorance setup is a powerful device, but there is a real question as to its accuracy in describing decision making processes. In reality, decision makers have some understanding of their preferences, even if they are imperfect, when at the ex ante stage. In this sense they are not ex ante identical. One could capture this by assuming that each voter receives an imperfect signal of their subsequent draw from the distribution. This would be a non-trivial change to the mode of analysis employed in this paper, since once one departs from voters being ex ante identical one can no longer make use of the notion of a representative individual.

As previously discussed, enriching the contracting space to consider state-contingent supermajority rules would be another direction for future research. Although such rules are rarely observed in their pure form, many democratic institutions are a proxy for such rules. A non-trivial portion of democratic decision making is based on notions of interpersonal utility comparisons and aggregate welfare gain, despite the difficulties inherent in such comparisons.

In this paper we have abstracted from the possibility of monetary transfers between voters in order to affect the ex post social decision. There are circumstances where this implies significant inefficiencies in the sense that aggregate welfare could be improved by a different ex post decision. While a simple compensation principle can lead to obvious Pareto improvements in certain circumstances, this relies strongly on the verifiability of voters preferences. If each voter's draw from the distribution remains private information, which seems realistic, then compensation and the associated Pareto improvements are not so straightforward. An incentive compatible mechanism would be required so that voters reveal their type in equilibrium. Understanding the existence and properties of such a mechanism may prove useful in understanding the practicalities of designing supermajority rules in the presence of private information. We feel that the issues of monetary transfers and private information regarding voter's preferences are inextricably linked if one seeks a deeper understanding of actual voting systems.

Finally, we have imposed an exogenous allocation of bargaining power between parties in coalitions. An interesting question is how the analysis might change if the formation of coalitions and the distribution of the surplus within them was determined simultaneously. Ray and Vohra (1999) analyze this problem generally and consider stationary perfect equilibria of games where the negotiation process is conducted via alternating offers with costly delay, à la Rubinstein. A number of issues complicate the application of their approach to this setting, however. Firstly, they require binding agreements within coalitions, though there is non-cooperative play between coalitions. Though complete contracts between coalitions after the realization of the state of nature is not inconsistent with an ex ante incomplete social contract, the familiar questions regarding the completeness of the within coalition contract arise. More problematic, however, is that their approach requires transferable utility - that is, a linear frontier of coalition payoffs. This is not consistent with our specification. Finally, whilst with symmetric players Ray and Vohra derive an algorithm which generates a unique coalition structure, with asymmetric players this uniqueness necessarily breaks down. Each of these difficulties would need to be overcome in order to consider endogenously generated bargaining power and coalition formation.

### 4.2 Conclusion

We have sought to provide an understanding of the key effects at work in determining the optimal supermajority rule in a general setting. The expansion of this setting to capture important "real-world" factors such as multidimensionality of the choice space, private information, state-contingent rules, and transfers remains an enticing prospect for future research. In addition, this paper has been based in large part on a methodology utilizing certain results from order-statistics. Since legal and political decision making processes generally deal with individual choice among a finite number of agents, this methodology seems to have broader application. Perhaps certain problems which have failed to be theoretically analyzed in this field will be tractable when these order-statistic techniques are brought to bear.

## References

Aghion, Philippe, Alberto Alesina, and Francesco Trebbi, "Endogenous Political Institutions," Quarterly Journal of Economics, 2004, 119, 565-611.

- and Patrick Bolton, "An Incomplete Contracts Approach to Financial Contracting," Review of Economic Studies, 1992, 59, 473-494.

Arrow, Kenneth J, Social Choice and Individual Values, John Wiley, 1951.
Babera, Salvador and Matthew O Jackson, "Choosing How to Choose: SelfStable Majority Rules," Quarterly Journal of Economics, 2004, 119, 1011-1048.

Balakrishnan, N and C R Rao, Handbook of Statistics 16: Order Statistics Theory and Methods, North-Holland, 1998.

Black, Duncan, "The Decisions of a Committee Using a Special Majority," Econometrica, 1948, 16, 245-261.
_ , "On the Rationale of Group Decision Making," Journal of Political Economy, 1948, 56, 23-34.

Bolton, Patrick and David Scharfstein, "Optimal Debt Structure and the Number of Creditors," Journal of Political Economy, 1996, 104, 1-25.

Buchanan, James and Gordon Tullock, The Calculus of Consent: Logical Foundations of Constitutional Democracy, University of Michigan Press, 1962.

Caplin, Andew and Barry Nalebuff, "On 64-Percent Majority Rule," Econometrica, 1988, 56, 787-814.

Caplin, Andrew and Barry Nalebuff, "Aggregationa and Social Choice: A Mean Voter Theorem," Econometrica, 1991, 59, 1-23.

Dixon, Rosalind, "Constitutional Transitions: Lessons for East Timor from Namibia, Cambodia and South Africa," Constitutional Law and Policy Review, 2002, pp. 50-70.

Erlenmaier, Ulrich and Hans Gersbach, "Flexible Majority Rules," CESInfo Working paper No.464, 2001.

Galvis, S J and A L Saad, "Collective Actions Clauses: Recent Progress and Challenges Ahead," Mimeo, Sullivan and Cromwell LLP, 2004.

Harsanyi, John, "Cardinal Utility in Welfare Economics and the Theory of RiskTaking," Journal of Political Economy, 1953, 61, 434-435.
_ , "Cardinal Welfare, Individual Ethics and Interpersonal Comparability of Utility," Journal of Political Economy, 1955, 61, 309-321.

Hart, Oliver and John Moore, "Default and Renegotiation: A Dynamic Model of Debt," Quarterly Journal of Economics, 1998, 113, 1-41.

Maggi, Giovanni and Massimo Morelli, "Self Enforcing Voting in International Organizations," NBER Working Paper 10102, 2003.

Mirrlees, James A., "An Exploration in the Theory of Optimum Income Taxation," Review of Economic Studies, 1971, 38, 175-208.

Rawls, John, A Theory of Justice, Cambridge, MA: Belknap Press, 1971.
Ray, Debraj and Rajiv Vohra, "A Theory of Endogenous Coalition Structures," Games and Economic Behavior, 1999, 26, 286-336.

Romer, Thomas and Howard Rosenthal, "A Constitution for Solving Asymmetric Externality Games," American Journal of Political Science, 1983, 27, 1-26.

Rothschild, Michael and Joseph E Stiglitz, "Increasing Risk: 1. A Definition," Journal of Economic Theory, 1970, 2, 225-243.

Salmon, F, "Brazil Goes Off On a CAC Tangent," Euromoney, 2003, p. 156.
Vickrey, William, "Measuring Marginal Utility by Reactions to Risk," Econometrica, 1945, 13, 319-333.

## 5 Appendix A: Optimal supermajority rule Characterization

We provide the expected utility calculations for the case where the number of voters is odd. It is straightforward to calculate the $m$ even case, but no additional insights are generated.

### 5.1 Majority Rule

We now calculate expected utility of a given voter $i$ under majority rule. First note that the ex ante social decision is $\theta^{*}$. Also note that under majority rule, in a one dimensional social choice space, the median voter determines the ex post social decision. Consider voter $i$ and let the other voters' draws be:

$$
x_{1}^{*} \leq \ldots \leq x_{m-1}^{*}
$$

where $x_{k}^{*}$ is the $k t h$ order-statistic. Denote $T=[0,1] \times[0,1] \mid l \geq j \forall l, j \in$ $[0,1]$. Note that in considering the median we need only be concerned with voter $i$ 's position relative to $x_{(m-1) / 2}^{*}$ and $x_{(m+1) / 2}^{*}$. If they are between $x_{(m-1) / 2}^{*}$ and $x_{(m+1) / 2}^{*}$ then they are the median and their payoff is -1 . If $x_{i}^{*} \leq x_{(m+1) / 2}^{*}$ then the expected distance is $\int_{0}^{a_{(m-1) / 2}^{*}}-\exp \left\{\beta\left|x-a_{(m-1) / 2}^{*}\right|\right\} d x$ and if $x_{i}^{*} \geq x_{(m+1) / 2}^{*}$ it is $\int_{a_{(m+1) / 2}^{*}}^{1}-\exp \left\{\beta\left|x-a_{(m+1) / 2}^{*}\right|\right\} d x$.

In order to derive the joint density of the two order statistics $a_{(m-1) / 2}^{*}$ and $a_{(m+1) / 2}^{*}$ note that the following formula gives the joint density (Balakrishnan and Rao (1998)):

$$
\begin{aligned}
f\left(x_{i}, x_{j}\right)= & \left\{\frac{n!}{(i-1)!(j-i-1)!(n-j)!} F\left(x_{i}\right)^{i-1}\left(F\left(x_{j}\right)-F\left(x_{i}\right)\right)^{j-i-1}\right\} \\
& \cdot\left[\left(1-F\left(x_{j}\right)\right)^{n-j} f\left(x_{i}\right) f\left(x_{j}\right)\right]
\end{aligned}
$$

Denoting $a_{(m-1) / 2}^{*}$ as $a_{\underline{m}}$ and $a_{(m+1) / 2}^{*}$ as $a_{\bar{m}}$ we therefore have:

$$
f\left(a_{\underline{m}}, a_{\bar{m}}\right)=\frac{(m-1)!F\left(a_{\underline{m}}\right)^{\frac{m-3}{2}}\left(1-F\left(a_{\bar{m}}\right)\right) f\left(a_{\underline{m}}\right) f\left(a_{\bar{m}}\right)}{\left(\frac{1}{2}(m-3)!\right)^{2}}
$$

Making use of Fubini's Theorem the expected utility of voter $i$ is given by:

$$
\begin{aligned}
\mathbb{E}\left[u_{i}^{M}\right]= & \iint_{T}\left[\begin{array}{c}
\int_{0}^{a_{\underline{m}}}-\exp \left\{\beta\left|x-a_{\underline{m}}\right|\right\} f(x) d x \\
+\int_{a_{\bar{m}}}^{1}-\exp \left\{\beta\left|x-a_{\bar{m}}\right|\right\} f(x) d x \\
+\int_{a_{\underline{m}}}^{a_{\bar{m}}}-1 f(x) d x
\end{array}\right] f\left(a_{\underline{m}}, a_{\bar{m}}\right) d a_{\underline{m}} d a_{\bar{m}} \\
= & \int_{0}^{1} \int_{0}^{a_{\bar{m}}}\left[\begin{array}{c}
\int_{0}^{a_{\underline{m}}}-\exp \left\{\beta\left|x-a_{\underline{m}}\right|\right\} f(x) d x \\
+\int_{a_{\bar{m}}}^{1}-\exp \left\{\beta\left|x-a_{\bar{m}}\right|\right\} f(x) d x \\
+\int_{a_{\underline{m}}}^{a_{\bar{m}}}-1 f(x) d x
\end{array}\right] f\left(a_{\underline{m}}, a_{\bar{m}}\right) d a_{\underline{m}} d a_{\bar{m}}
\end{aligned}
$$

This becomes the following:
where $\Gamma(z)$ is the Euler Gamma function given by $\Gamma(z)=\int_{0}^{\infty} t^{z-1} \exp \{-t\} d t$.

### 5.2 Unanimity

Now consider the expected utility of voter $i$ if we require unanimity in order to change the social decision ex post. Let $B$ be the event where $0 \leq x_{1}^{*} \leq \ldots \leq x_{m-1}^{*}<\theta^{*}$ and let $B^{\prime}$ be the event where $\theta^{*} \geq x_{1}^{*} \geq \ldots \geq x_{m-1}^{*} \geq 1$. Let $A=\Omega \backslash\left(B+B^{\prime}\right)$. It is clear that $\operatorname{Pr}(B)=F\left(\theta^{*}\right)^{m-1}, \operatorname{Pr}\left(B^{\prime}\right)=1-\left(1-F\left(\theta^{*}\right)\right)^{m-1}$ and therefore that $\operatorname{Pr}(A)=\left(1-F\left(\theta^{*}\right)\right)^{m-1}-F\left(\theta^{*}\right)^{m-1}$. The expected utility of voter $i$ conditional on event $A$ is:

$$
\mathbb{E}\left[u_{i}^{U} \mid A\right]=\int_{0}^{\theta^{*}}-\exp \left\{\beta\left(\theta^{*}-x\right)\right\} f(x) d x+\int_{\theta^{*}}^{1}-\exp \left\{\beta\left(x-\theta^{*}\right)\right\} f(x) d x
$$

In order to evaluate the expected utility conditional on events $B$ and $B^{\prime}$ note that the density of the $i$ th order-statistic from a continuous distribution $F(x)$ is given by:

$$
f_{i}(x)=\frac{n!}{(i-1)!(n-i)!} f(x)[F(x)]^{i-1}(1-F(x))^{n-i}
$$

(Balakrishnan and Rao (1998).

The density of $x_{m-1}^{*}$ on $[0,1]$ is $g\left(a_{m-1}\right)=\frac{m!}{(m-2)!} f(x)[F(x)]^{m-2}(1-F(x))$. By the Change of Variables Theorem its density on $[0, \theta]$ is given by:

$$
\begin{aligned}
g\left(a_{m-1} \mid B\right) & =\int_{0}^{\theta^{*}}\left(\frac{m!}{(m-2)!} f(y)[F(y)]^{m-2}(1-F(y))\right) \frac{d y}{d x} d y \\
& =1
\end{aligned}
$$

where $y=x / \theta^{*}$.
Therefore the expected utility conditional on event $B$ is given by:

$$
\begin{aligned}
& \mathbb{E}\left[u_{i}^{U} \mid B\right]= \int_{\theta^{*}}^{1}-\exp \left\{\beta\left(x-\theta^{*}\right)\right\} f(x) d x \\
&\left.+\int_{0}^{\theta^{*}}\left(\begin{array}{c}
\int_{0}^{a_{m-1}}- \\
\\
\\
\\
+\int_{a_{m-1}}^{\theta^{*}}-1 f(x) d x
\end{array}\right) g\left(a_{m-1}-x\right)\right\} f(x) d x \\
&\left.a_{m-1} \mid B\right) d a_{m-1}
\end{aligned}
$$

where $\int_{\theta^{*}}^{1}-\exp \left\{\beta\left(x-\theta^{*}\right)\right\} f(x) d x$ is the term associated with $x_{i} \geq \theta^{*}$ and the term associated with $x_{i} \leq \theta^{*}$ is

$$
\int_{0}^{\theta^{*}}\binom{\int_{0}^{a_{m-1}}-\exp \left\{\beta\left(a_{m-1}-x\right)\right\} f(x) d x}{+\int_{a_{m-1}}^{\theta^{*}}-1 f(x) d x} g\left(a_{m-1} \mid B\right) d a_{m-1} \text {, since if voter } i \text { 's }
$$ draw is less than $x_{m-1}^{*}$ then their payoff is $\int_{0}^{a_{m-1}}-\exp \left\{\beta\left(a_{m-1}-x\right)\right\} f(x) d x$ and if it is between $x_{m-1}^{*}$ and $\theta^{*}$ then they determine the ex post social decision and therefore receive utility of -1 .

Similarly for event $B^{\prime}$ :

$$
\begin{aligned}
\mathbb{E}\left[u_{i}^{U} \mid B^{\prime}\right]= & \int_{0}^{\theta^{*}}-\exp \left\{\beta\left(\theta^{*}-x\right)\right\} f(x) d x \\
& +\int_{\theta^{*}}^{1}\binom{\int_{a_{m-1}}^{1}-\exp \left\{\beta\left(x-a_{m-1}\right)\right\} f(x) d x}{+\int_{0}^{a_{m-1}}-1 f(x) d x} g\left(a_{m-1} \mid B^{\prime}\right) d a_{m-1}
\end{aligned}
$$

Therefore the expected utility under a unanimity requirement is:

$$
\begin{aligned}
\mathbb{E}\left[u_{i}^{U}\right]= & \left(1-\left(1-F\left(\theta^{*}\right)\right)^{m-1}-F\left(\theta^{*}\right)^{m-1}\right) \mathbb{E}\left[u_{i}^{U} \mid A\right] \\
& +F\left(\theta^{*}\right)^{m-1} \mathbb{E}\left[u_{i}^{U} \mid B\right]+\left(1-\left(1-F\left(\theta^{*}\right)\right)^{m-1}\right) \mathbb{E}\left[u_{i}^{U} \mid B^{\prime}\right]
\end{aligned}
$$

which upon simplification becomes:

$$
\left.\begin{array}{c}
F\left(\theta^{*}\right)^{m-1}\left[+\int_{0}^{\theta^{*}}\left(\begin{array}{c}
-\int_{0}^{\theta^{*}}-e^{\beta\left(\theta^{*}-x\right)} f(x) d x \\
\int_{0}^{a_{m-1}}-\exp \left\{\beta\left(a_{m-1}-x\right)\right\} f(x) d x \\
+\int_{a_{m-1}}^{\theta^{*}}-1 f(x) d x
\end{array}\right)\right]+ \\
\cdot g\left(a_{m-1} \mid B\right) d a_{m-1}
\end{array}\right]+\begin{gathered}
\int_{\theta^{*}}^{1}-\exp \left\{\beta\left(x-\theta^{*}\right)\right\} f(x) d x \\
+2 \int_{0}^{\theta^{*}}-\exp \left\{\beta\left(\theta^{*}-x\right)\right\} f(x) d x \\
\left(1+\frac{\left(1-F\left(\theta^{*}\right)^{m}\right.}{F\left(\theta^{*}\right)-1}\right)\left[\begin{array}{c}
1
\end{array}\right] \\
+\int_{\theta^{*}}^{1}\left[\begin{array}{c}
\int_{a_{m-1}}^{1}-\exp \left\{\beta\left(x-a_{m-1}\right)\right\} f(x) d x \\
+\int_{0}^{a_{m-1}}-1 f(x) d x \\
\cdot g\left(a_{m-1} \mid B^{\prime}\right) d a_{m-1}
\end{array}\right]
\end{gathered}
$$

### 5.3 Interior Supermajority Rules

Now consider the case where a supermajority of $\frac{1}{2}<\alpha<m$ voters can determine the ex post social decision. Define $\gamma=m-k$ for $k \in \mathcal{K}$, where $\mathcal{K}=\left\{\mathbb{N} \cap\left[1, \frac{m-3}{2}\right]\right\}$. This implies that $\gamma \in\left\{\mathbb{N} \cap\left[\frac{m+3}{2}, m-1\right]\right\} . \gamma$ represents the number of voters required to change the social decision ex post. Define event 1 as $0 \leq x_{1}^{*} \leq \ldots \leq$ $x_{m-2}^{*} \leq \theta^{*} \leq x_{m-1}^{*}$, event 2 as $0 \leq x_{1}^{*} \leq \ldots \leq x_{m-3}^{*} \leq \theta^{*} \leq x_{m-2}^{*} \leq x_{m-1}^{*}$ and so on through event $m-2$, which is $0 \leq x_{1}^{*} \leq \theta^{*} \leq x_{2}^{*} \leq \ldots \leq x_{m-1}^{*}$. Let event $m-1$ be $0 \leq x_{1}^{*} \leq \ldots \leq x_{m-1}^{*} \leq \theta^{*}$ and event $m$ be $\theta^{*} \leq x_{1}^{*} \leq \ldots \leq x_{m-1}^{*} \leq 1$. The probability of these events is simply determined by the CDF of the appropriate order statistic. The probability of event $m-1$ is $\operatorname{Pr}\left(x_{m-1}^{*} \leq \theta^{*}\right)=F\left(\theta^{*}\right)^{m-1}$. The probability of event $m$ is $\operatorname{Pr}\left(x_{1}^{*} \geq \theta^{*}\right)=1-\left(1-F\left(\theta^{*}\right)\right)^{m-1}$. For events 1 through
$m-2$ the probability of each event is given by:

$$
\begin{align*}
\operatorname{Pr}(\text { event } k)= & F_{m-1-k}\left(\theta^{*}\right)-F_{m-k}\left(\theta^{*}\right), \forall k \in \mathbb{N} \cap[1, m-2] \\
= & \int_{0}^{F\left(\theta^{*}\right)} \frac{(m-1)!}{(i-1)!(m-1-(m-1-k))!} \\
& \cdot t^{i-1}(1-t)^{m-1-(m-1-k)} d t \\
& -\int_{0}^{F\left(\theta^{*}\right)} \frac{m!}{(i-1)!(m-(m-k))!} \\
& \cdot t^{i-1}(1-t)^{m-(m-k)} d t \\
= & \int_{0}^{F\left(\theta^{*}\right)} \frac{(m-1)!}{(m-k-2)!k!} t^{i-1}(1-t)^{k} d t \\
& -\int_{0}^{F\left(\theta^{*}\right)} \frac{m!}{(m-k-1)!k!} t^{i-1}(1-t)^{k} d t \tag{1}
\end{align*}
$$

For event $m-1$ (which we shall denote as $e v\{m-1\}$ ), with supermajority rule $\gamma / m$, the expected utility of voter $i$ is given by:

$$
\begin{align*}
\mathbb{E}\left[\left.u_{i}^{\gamma / m}\right|_{e v\{m-1\}}\right]= & \int_{0}^{\theta^{*}} \int_{0}^{a_{\gamma}}\left(\begin{array}{c}
-\int_{a_{\gamma-1}}^{a_{\gamma}} 1 f(x) d x \\
+\int_{0}^{a_{\gamma-1}}-\exp \left\{\beta\left(a_{\gamma-1}-x\right)\right\} f(x) d x \\
+\int_{a_{\gamma}}^{1}-\exp \left\{\beta\left(x-a_{\gamma}\right)\right\} f(x) d x
\end{array}\right) \\
& g\left(a_{\gamma-1}, a_{\gamma} \mid \operatorname{ev}\{m-1\}\right) d a_{\gamma-1} d a_{\gamma} \tag{2}
\end{align*}
$$

since if $x_{i}^{*} \geq x_{\gamma}^{*}$ then the ex post social decision will be $x_{\gamma}^{*}$, if $x_{i}^{*} \leq x_{\gamma-1}^{*}$ then $x_{\gamma-1}^{*}$ will be the ex post social decision, and if $x_{\gamma-1}^{*} \leq x_{i}^{*} \leq x_{\gamma}^{*}$ then $x_{i}^{*}$ will prevail. Note that $g\left(a_{\gamma-1}, a_{\gamma} \mid e v\{m-1\}\right)$ is determined by the Change of Variables Theorem applied the the general result for conditional order-statistics:

$$
\begin{equation*}
f_{X_{r} \mid X_{s=v}}(x)=\frac{(s-1)!}{(r-1)!(s-r-1)!} \frac{f(x) F(x)^{r-1}(F(v)-F(x))^{s-r-1}}{F(v)^{s-1}} \tag{3}
\end{equation*}
$$

recalling, as previously mentioned, that order-statistics drawn from a continuous parent form a Markov Chain.

Similarly, the expected utility for event $m$ is:

$$
\begin{align*}
\mathbb{E}\left[\left.u_{i}^{\gamma / m}\right|_{e v\{m\}}=\right. & \int_{\theta^{*}}^{1} \int_{1}^{a_{m-\gamma}}\left(\begin{array}{c}
-\int_{a_{m-\gamma+1}}^{a_{m-\gamma}} 1 f(x) d x \\
+\int_{0}^{a_{m-\gamma}}-\exp \left\{\beta\left(a_{m-\gamma}-x\right)\right\} f(x) d x \\
+\int_{a_{m-\gamma+1}}^{1}-\exp \left\{\beta\left(x-a_{m-\gamma+1}\right)\right\} f(x) d x
\end{array}\right) \\
& g\left(a_{m-\gamma+1}, a_{m-\gamma} \mid \operatorname{ev}\{m\}\right) d a_{m-\gamma+1} d a_{m-\gamma} \tag{4}
\end{align*}
$$

For event $j \in \mathcal{K}$ note that if $m-j-1<\gamma$ and $j<\gamma$ then $\theta^{*}$ is never overturned ${ }^{19}$. Denote the set of these events as $\mathcal{Z}$. In those cases the expected utility of voter $i$ is:

$$
\begin{equation*}
\mathbb{E}\left[u_{i}^{\gamma / m} \mid \mathcal{Z}\right]=\int_{0}^{\theta^{*}}-\exp \left\{\beta\left(\theta^{*}-x\right)\right\} f(x) d x+\int_{\theta^{*}}^{1}-\exp \left\{\beta\left(x-\theta^{*}\right)\right\} f(x) d x \tag{5}
\end{equation*}
$$

Now consider the complement of $\mathcal{Z}$. For events $1, \ldots, m-2$ it is useful to note the partial symmetry between pairs of events. Events $j$ and $m-j-1, \forall j \in\{1, \ldots, m-2\}$ each have $j$ order-statistics on either side of $\theta^{*}$. Denote the set of all the first (lowest numbered) events of such a pair as $P_{1}$ and the set of all others as $P_{2}$. Now let $\widehat{x}$ equal $\theta^{*}$ for $\gamma \geq m-j$ and $x_{\gamma}^{*}$ otherwise.

For the set $P_{1}$, event $j$, and where $\gamma \geq m-j$, the expected utility of voter $i$ is:

$$
\begin{align*}
\mathbb{E}\left[\left.u_{i, P_{1}}^{\gamma / m}\right|_{\operatorname{ev}\{j\}}\right]= & \int_{\theta^{*}}^{1}-\exp \left\{\beta\left(x-\theta^{*}\right)\right\} f(x) d x \\
& +\int_{0}^{\theta^{*}}\binom{\int_{0}^{a_{\gamma-j}}-\exp \left\{\beta\left(a_{\gamma-j}-x\right)\right\} f(x) d x}{-\int_{a_{\gamma-j}}^{\theta^{*}} 1 f(x) d x} \\
& g\left(a_{\gamma-j} \mid \operatorname{ev}\left\{j, P_{1}\right\}\right) d a_{\gamma-j} \tag{6}
\end{align*}
$$

where the conditional density $g\left(a_{\gamma-j} \mid e v\left\{j, P_{1}\right\}\right) d a_{\gamma-j}$ is given by (3) above.

[^14]Where $\gamma<m-j$, the expected utility of voter $i$ is:

$$
\begin{align*}
\mathbb{E}\left[\left.u_{i, P_{1}}^{\gamma / m}\right|_{e v\{j\}}\right]= & \int_{0}^{\theta^{*}} \int_{0}^{a_{\gamma}}\left(\begin{array}{c}
-\int_{a_{\gamma-1}}^{a_{\gamma}} 1 f(x) d x \\
+\int_{0}^{a_{\gamma-1}}-\exp \left\{\beta\left(a_{\gamma-1}-x\right)\right\} f(x) d x \\
+\int_{a_{\gamma}}^{1}-\exp \left\{\beta\left(x-a_{\gamma}\right)\right\} f(x) d x
\end{array}\right) \\
& g\left(a_{\gamma}, a_{\gamma-1} \mid e v\left\{j, P_{1}\right\}\right) d a_{\gamma} d a_{\gamma-1} \tag{7}
\end{align*}
$$

For the set $P_{2}$, event $j$, and where $\gamma \geq m-j$, the expected utility of voter $i$ is:

$$
\begin{align*}
\mathbb{E}\left[\left.u_{i, P_{2}}^{\gamma / m}\right|_{\operatorname{ev}\{j\}}\right]= & \int_{0}^{\theta^{*}}-\exp \left\{\beta\left(\theta^{*}-x\right)\right\} f(x) d x \\
& +\int_{\theta^{*}}^{1}\binom{\int_{a_{\gamma-j+1}}^{1}-\exp \left\{\beta\left(x-a_{\gamma-j+1}\right)\right\} f(x) d x}{-\int_{\theta^{*}}^{a_{\gamma-j+1}} 1 f(x) d x} \\
& g\left(a_{\gamma-j+1} \mid \operatorname{ev}\left\{j, P_{2}\right\}\right) d a_{\gamma-j+1} \tag{8}
\end{align*}
$$

Where $\gamma<m-j$, the expected utility of voter $i$ is:

$$
\begin{align*}
\mathbb{E}\left[u_{i, P_{2}}^{\gamma / m} \mid e v\{j\}\right]= & \int_{\theta^{*}}^{1} \int_{1}^{a_{\gamma-1}}\left(\begin{array}{c}
-\int_{a_{\gamma-1}}^{a_{\gamma}} 1 f(x) d x \\
+\int_{0}^{a_{\gamma-1}}-\exp \left\{\beta\left(a_{\gamma-1}-x\right)\right\} f(x) d x \\
+\int_{a_{\gamma}}^{1}-\exp \left\{\beta\left(x-a_{\gamma}\right)\right\} f(x) d x
\end{array}\right) \\
& g\left(a_{\gamma}, a_{\gamma-1} \mid e v\left\{j, P_{2}\right\}\right) d a_{\gamma} d a_{\gamma-1} \tag{9}
\end{align*}
$$

Equations (6)-(9), in conjuction with (1), completely define the expected utility of a given voter under any "interior" supermajority rule.

## 6 Appendix B: Proofs

This appendix contains proofs of each of the results stated in the text.
Proposition 1. Assume A1. Then the optimal social decision is to set $\theta$ equal to the median of $f(x)$.

Proof. The ex ante optimal social decision solves:

$$
\min _{\theta} \int_{0}^{1}|\theta-x| f(x) d x
$$

The solution to this problem is:

$$
\widehat{\theta}=\underset{\theta}{\arg \min } \int_{0}^{1}|\theta-x| f(x) d x
$$

Which is precisely the definition of the median of $f(x)$.
Proposition 2. Assume A1. Then the optimal supermajority rule is the interval $\alpha=\left[\frac{1}{2}, 1\right]$.

Proof. Step1. Suppose $\alpha<1 / 2$. This implies that there is a coalition, $\mathcal{C}$, of voters with outer measure $\gamma<1 / 2$ who are able to change the social decision. But any change which makes them better off makes the remainder of the voters worse off and will be blocked. Contradiction.

Step 2. Any $\alpha>1 / 2$ also leads to no change to the social decision since any voter beyond the median voter will also be affected by such a change.

Proposition 3. Assume A1, $\beta>0$ and $M$ infinite. Then the optimal social decision requires the following condition to hold:

$$
\int_{\theta}^{1} \beta \exp \{\beta(x-\theta)\} f(x) d x+\int_{0}^{\theta}-\beta \exp \{\beta(\theta-x)\} f(x) d x=0
$$

Proof. The ex ante optimal social decision solves:

$$
\min _{\theta} \int_{0}^{1}-\exp \{\beta|\theta-x|\} f(x) d x
$$

Which is equivalent to the following problem:

$$
\min _{\theta} \int_{0}^{\theta}-\exp \{\beta(\theta-x)\} f(x) d x+\int_{\theta}^{1}-\exp \{\beta(x-\theta)\} f(x) d x
$$

Using Leibniz's Rule the first-order condition is:

$$
\int_{\theta}^{1} \beta \exp \{\beta(x-\theta)\} f(x) d x+\int_{0}^{\theta}-\beta \exp \{\beta(\theta-x)\} f(x) d x=0
$$

Proposition 4. Assume A1, $\beta>0$ and $M$ infinite. Then the optimal
supermajority rule lies in the interval $\alpha=\left[\frac{1}{2}, 1\right]$, and increases as $f(x)$ becomes more skewed to the right.

Proof. First note that $\alpha<1 / 2$ can never be optimal by 2. By 3 if $f(x)$ is symmetric then the ex ante decision is $\theta^{*}=1 / 2$ and any $\alpha \in\left[\frac{1}{2}, 1\right]$ will prevent a change. As $f(x)$ becomes more skewed to the right $\theta^{*}$ becomes larger than the median, thus requiring a larger $\alpha$ to prevent a change.

Proposition 5 Assume A1-A3 and $M$ finite. Then the optimal supermajority rule is increasing in the coefficient of importance, $\beta$.

Proof. First consider $m$ odd.
Step 1: Recall that the expected utility under majority rule is given by:

$$
=\frac{1}{\Gamma\left(\frac{m-1}{2}\right)^{2}}\left(\begin{array}{c}
\Gamma(m) \int_{0}^{1} \int_{0}^{a} a_{\bar{m}}\left[\begin{array}{c}
\int_{0}^{a_{\underline{m}}}-\exp \left\{\beta\left|x-a_{\underline{m}}\right|\right\} f(x) d x \\
+\int_{a_{\bar{m}}}^{1}-\exp \left\{\beta\left|x-a_{\bar{m}}\right|\right\} f(x) d x \\
+\int_{\bar{m}}^{a_{\bar{m}}}-1 f(x) d x \\
\cdot F\left(a_{\underline{m}}\right)^{\frac{m-3}{2}}\left(1-F\left(a_{\bar{m}}\right)\right)^{\frac{m-3}{2}}
\end{array} f\left(a_{\underline{m}}\right) f\left(a_{\bar{m}}\right) d a_{\underline{m}} d a_{\bar{m}}\right. \tag{10}
\end{array}\right)
$$

and that under unanimity we have:

$$
\left.\begin{array}{c}
F\left(\theta^{*}\right)^{m-1}\left[\begin{array}{c}
-\int_{0}^{\theta^{*}}-e^{\beta\left(\theta^{*}-x\right)} f(x) d x \\
+\int_{0}^{\theta^{*}}\left(\begin{array}{c}
\int_{0}^{a_{m-1}}-\exp \left\{\beta\left(a_{m-1}-x\right)\right\} f(x) d x \\
+\int_{a_{m-1}}^{\theta^{*}}-1 f(x) d x \\
\cdot g\left(a_{m-1} \mid B\right) d a_{m-1}
\end{array}\right)
\end{array}\right]+ \\
\frac{1}{F\left(\theta^{*}\right)-1}[\cdot  \tag{11}\\
{\left[F\left(\theta^{*}\right)+\left(1-F\left(\theta^{*}\right)\right)^{m}-1\right)} \\
+\int_{\theta^{*}}^{1}\left[\begin{array}{c}
\int_{\theta^{*}}^{1}-\exp \left\{\beta\left(x-\theta^{*}\right)\right\} f(x) d x \\
2 \int_{0}^{\theta^{*}}-\exp \left\{\beta\left(\theta^{*}-x\right)\right\} f(x) d x \\
1 \\
a_{m-1}-\exp \left\{\beta\left(x-a_{m-1}\right)\right\} f(x) d x \\
+\int_{0}^{a_{m-1}-1 f(x) d x}
\end{array}\right]
\end{array}\right] .
$$

Note that $\mathbb{E}\left[U^{U}\right]$ and $\mathbb{E}\left[U^{M}\right]$ are both monotonically decreasing in $\beta$. It follows from (10) and (11) that $\exists \widehat{\beta}<\infty$ s.t. $\mathbb{E}\left[U^{U}(\widehat{\beta})\right]>\mathbb{E}\left[U^{M}(\widehat{\beta})\right]$.

Step 2: Recall that the expected utility for "interior" supermajority rules is given by equations (2),(4)-(9), in conjuction with (1).

Now note that $\exists \widehat{\gamma}<\infty$ s.t. $\forall \widehat{\beta}>\beta, \mathbb{E}\left[U^{S \widehat{\gamma}}(\widehat{\beta})\right]>\mathbb{E}\left[U^{S \widehat{\gamma}}(\beta)\right]$.

Step 3: It also follows that $\exists \widehat{\beta}<\infty$ s.t. $\forall \widehat{\gamma}>\gamma, \mathbb{E}\left[U^{S \widehat{\gamma}}(\widehat{\beta})\right]>\mathbb{E}\left[U^{S \gamma}(\widehat{\beta})\right]$
Substantially similar reasoning establishes the $m$ even case. This completes the proof.

Proposition 6 Assume A1-A3, $\beta>0$ and $M$ finite. Then the optimal supermajority rule is decreasing in the number of voters, $m$.

Proof. First consider $m$ odd.
Step 1: Note from (10) and (11) that $\mathbb{E}\left[U^{U}\right]$ and $\mathbb{E}\left[U^{M}\right]$ are both monotonically decreasing in $m$. Note also that $\exists \widehat{m}<\infty$ s.t. $\mathbb{E}\left[U^{U}(\widehat{m})\right]<\mathbb{E}\left[U^{M}(\widehat{m})\right]$.

Step 2: Recall that the expected utility for "interior" supermajority rules is given by equations (2),(4)-(9), in conjuction with (1).

Now note that $\exists \widehat{\gamma}<\infty$ s.t. $\forall \widehat{m}>m, \mathbb{E}\left[U^{S \widehat{\gamma}}(\widehat{m})\right]<\mathbb{E}\left[U^{S \widehat{\gamma}}(m)\right]$.
Step 3: It also follows that $\exists \widehat{m}<\infty$ s.t. $\forall \widehat{\gamma}>\gamma, \mathbb{E}\left[U^{S \widehat{\gamma}}(\widehat{m})\right]<\mathbb{E}\left[U^{S \gamma}(\widehat{m})\right]$
Substantially similar reasoning establishes the $m$ even case. This completes the proof.

Proposition 7 Assume A1-A3, $\beta>0$ and $M$ finite. Then the optimal supermajority rule is larger for a distribution of voter types, $\widehat{F}(x)$ than for the distribution $F(x)$ if $\widehat{F}(x)$ is Rothschild-Stiglitz Riskier than $F(x)$.

Proof. Follows immediately from the proof of Proposition 5 by the definition of Rothschild-Stiglitz Riskier and the duality between $\beta>\widehat{\beta}$ and $\widehat{F}(x)$ RothschildStiglitz Riskier than $F(x)$.


[^0]:    *Department of Economics, Harvard University, 1875 Cambridge Street, Cambridge, MA, 02138. email: rholden@fas.harvard.edu. I wish to thank Attila Ambrus, Murali Agastya, Philippe Aghion, Jerry Green, Oliver Hart, Markus Möbius, Hervé Moulin and Demian Reidel for helpful discussions, comments and suggestions. Special thanks is owed to Rosalind Dixon.

[^1]:    ${ }^{1}$ For instance, in Australia $90 \%$ of shareholders must accept a takeover offer for the bidder to be able to move to compulsory acquisition. In the UK the requirement is $75 \%$, and it is a simple majority in the US. Since 2002 the requirement in Germany is $95 \%$.
    ${ }^{2}$ We shall use the terms "supermajority requirement" and "supermajority rule" interchangeably.

[^2]:    ${ }^{3}$ "The solution of the social welfare problem may lie in some generalization of the unanimity condition..." (quoted in Caplin and Nalebuff (1988))

[^3]:    ${ }^{4}$ In fact any requirement from $50 \%$ to unanimity is optimal.

[^4]:    ${ }^{5}$ See, for instance, Ray and Vohra (1999).

[^5]:    ${ }^{6}$ And as $\beta \rightarrow \infty$ expected utilty $\rightarrow-\infty$.

[^6]:    ${ }^{7}$ For an absolutely continuous population the joint density of two order statistics $i<j$, from $n$ statistics, is given by:

    $$
    \frac{n!}{(i-1)!(j-i-1)!(n-j)!} F\left(x_{i}\right)^{i-1}\left(F\left(x_{j}\right)-F\left(x_{i}\right)\right)^{j-i-1}\left[\left(1-F\left(x_{j}\right)\right)^{n-j} f\left(x_{i}\right) f\left(x_{j}\right)\right]
    $$

    (See Balakrishnan and Rao (1998)). For the uniform distribution this implies:

    $$
    f\left(x_{i}, x_{j}\right)=\frac{n!}{(i-1)!(j-i-1)!(n-j)!} u_{i}^{i-1}\left(u_{j}-u_{i}\right)^{j-i-1}\left(1-u_{j}\right)^{n-j}
    $$

[^7]:    ${ }^{9}$ In fact, this result is quite general. The conditional pdf of an order-statistic is given by:

    $$
    f_{X_{r} \mid X_{s=v}}(x)=\frac{(s-1)!}{(r-1)!(s-r-1)!} \frac{f(x) F(x)^{r-1}(F(v)-F(x))^{s-r-1}}{F(v)^{s-1}}
    $$

[^8]:    ${ }^{10}$ For the uniform distribution the density of the $i t h$ order statstic is:

    $$
    f_{i}(u)=\frac{n!}{(i-1)!(n-i)!} u^{i-1}(1-u)^{n-i}
    $$

[^9]:    ${ }^{11}$ Note that the joint density of $\left(x_{1}, x_{2}\right)$ where there are just two order statistics is simply 2 .

[^10]:    ${ }^{12}$ Another notable constitution drafted in recent times is, of course, that of the European Union.
    ${ }^{13}$ The United Nations has been influential in championing / mandating this mechanism.
    ${ }^{14}$ In particular see Dixon (2002) and the analysis therein.

[^11]:    ${ }^{15}$ The public voted for a constituent assembly (basically by proportional representation) which then "debated" and ratified a constitution with no further public involvement. This process was largely the same in Namibia and Cambodia.
    ${ }^{16}$ For instance, it contained proportional representation in the lower house (rather than geographically based constinuencies), which meant that it would almost certainly win fewer seats. It also provided for an upper house, rather than a unicameral legislature (Dixon (2002)).

[^12]:    ${ }^{17}$ This was largely to deal with "exit consents" whereby holders accepting new bonds in an exchange provide their consent, while exiting the existing bonds, to amend the non-payment terms, which typically do not require unanimity. This tactic is frequently used in so called "exchange offers".

[^13]:    ${ }^{18}$ Perhaps on option for some creditors to purchase the entitlements of other creditors in an event of default would achieve this, however. Although this may give rise to other problems.

[^14]:    ${ }^{19}$ There are $j$ order statistics to the right of $\theta^{*}$ and $m-j-1$ order statistics to the left of $\theta^{*}$. Because $j$ and $\gamma$ are integers this means that if both $j$ and $m-j-1$ are less than $\gamma$ then no matter what the realization of $x_{i}^{*}$ is, the ex ante social decision will never be changed.

