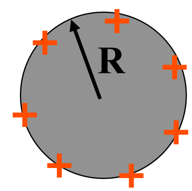
News

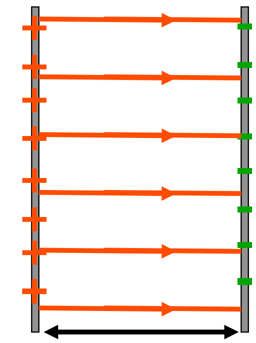
- Quiz #2: Monday, 3/14, 10AM
- Same procedure as for quiz 1
  - Review in class Fri, 3/11
  - Evening review, Fri, 3/11, 6-8PM, 54-100
  - 2 practice quizzes (+ practice problems)
  - Formula sheet

Charge and Potential



Charged Sphere

$$\underbrace{V=1/(4\,\pi\,\varepsilon_0\,R)}_{\text{Geometry!}}\,Q$$



Parallel Plate Capacitor

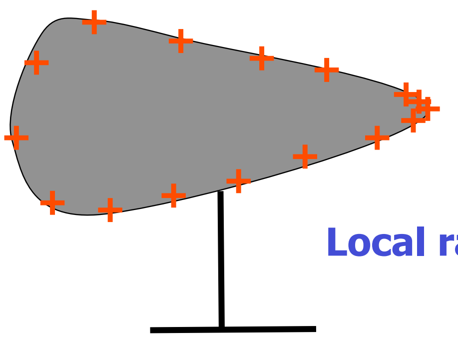
$$V=\underbrace{d/(A\,\varepsilon_0)}\,Q$$

Geometry!

Charge Density

Demo:

Application: Lightning rod - Biggest E near pointy tip!



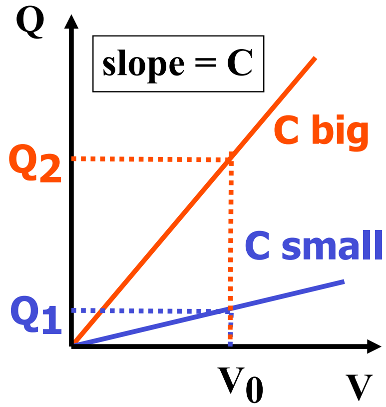
$$\Rightarrow \boxed{E \propto \frac{1}{r} \propto \sigma}$$

Local radius of curvature

Charge and Potential

- For given geometry, Potential and Charge are proportional
- Define
  - $\boxed{Q = C\,V}$ **-> C is Capacitance**
- Measured in [F] = [C/V] : Farad
- C tells us, how easy it is to store charge on it (V = Q/C)

Capacitance

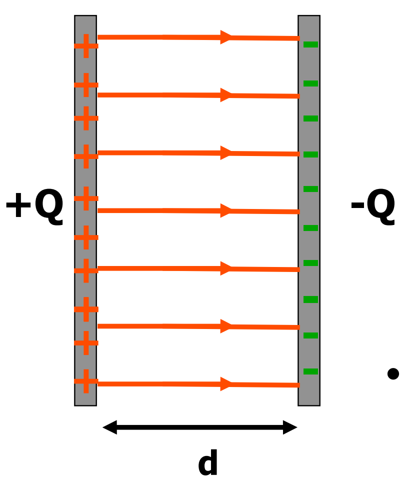


C bigger -> Can store more Charge!

Capacitor

- Def: Two conductors separated by insulator
- Charging capacitor:
  - take charge from one of the conductors and put on the other
  - separate + and - charges

Parallel Plate Capacitor



$$C = \frac{Q}{V(a)-V(b)} = \frac{Q}{E\,d}$$
$$= \frac{Q}{\frac{\sigma}{\epsilon_0}\,d} = \frac{Q}{\frac{Q}{A\epsilon_0}\,d} =$$
$$\boxed{= \epsilon_0 \frac{A}{d}}$$

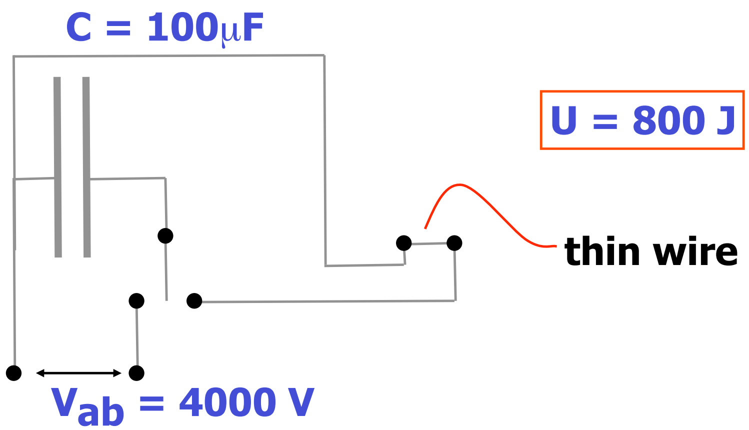
- To store lots of charge
  - make A big
  - make d small

Energy stored in Capacitor

$$W_{tot} = \int_{Q_{initial}}^{Q_{final}} V\,dq = \int_0^Q V\,dq$$
$$= \int_0^Q q/C\,dq = \frac{1}{C}\int_0^Q q\,dq$$
$$= \frac{1}{C}\frac{Q^2}{2}$$

- Work W = 1/2 Q^2/C = 1/2 C V^2 needed to charge capacitor
- Energy conserved
- But power can be amplified
  - Charge slowly
  - Discharge very quickly

Demo (Loud...)

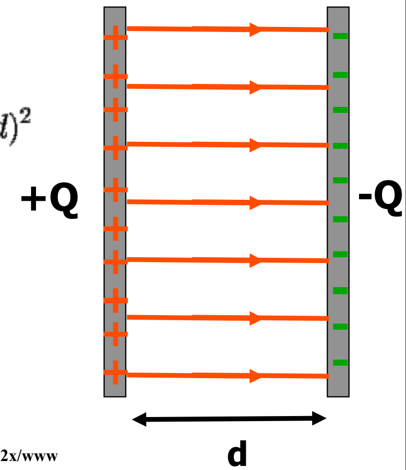


Where is the energy stored?

- Energy is stored in Electric Field

$$U_{stored} = \frac{1}{2}CV^2 = \frac{1}{2}(\epsilon_0\frac{A}{d})(E\ d)^2$$
$$= \boxed{\frac{1}{2}\epsilon_0 E^2\ Volume}$$

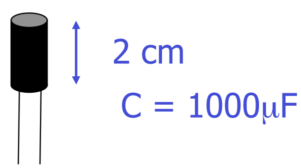
- **E^2** gives Energy Density:
- **U/Volume = ½ e_0 E^2**



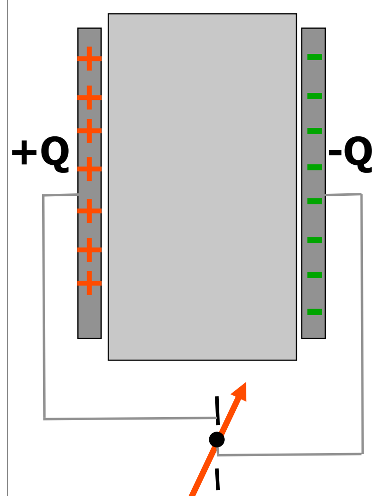
Dielectrics

- Parallel Plate Capacitor:
  - **C = ε_0 A/d**
  - Ex. **A = 1m^2, d=0.1mm -> C ~ 0.1μF**
- How can one get small capacitors with big capacity?

In your toolbox:



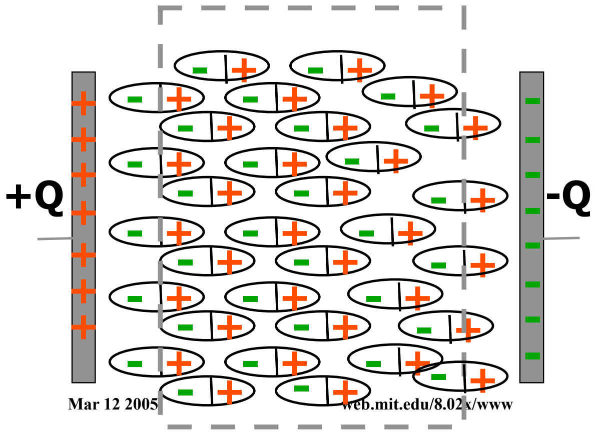
Dielectric Demo



- Start w/ charged capacitor
- **d** big -> **C** small -> **V** large
- Insert Glass plate
- Now **V** much smaller
- **C** bigger
- But **A** and **d** unchanged !
- Glass is a **Dielectric**

Microscopic view

Polarization $\vec{P} = \text{const. } \vec{E} = \varepsilon_0 \chi \vec{E}$



Dielectric Constant

- Dielectric reduces field E_0 ($K > 1$)
  - $E = 1/K \; E_0$
- Dielectric increases Capacitance
  - $C = Q/V = Q/(E\,d) = K\,Q/(E_0 d)$
- This is how to make small capacitors with large **C** !

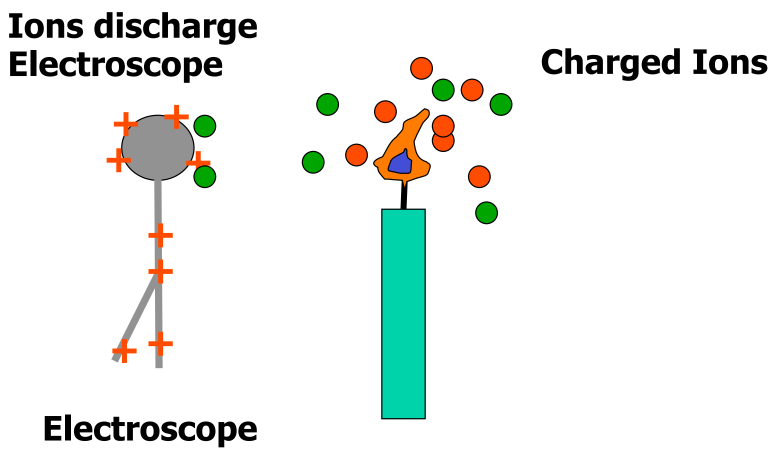
Electric Current

- We left Electrostatics
  - Now: Charges can move in steady state
- Electric Current I:
  - $I = dQ/dt$
  - Net amount of charge moving through conductor per unit time
- Units:
  - **[I] = C/s = A (Ampere)**

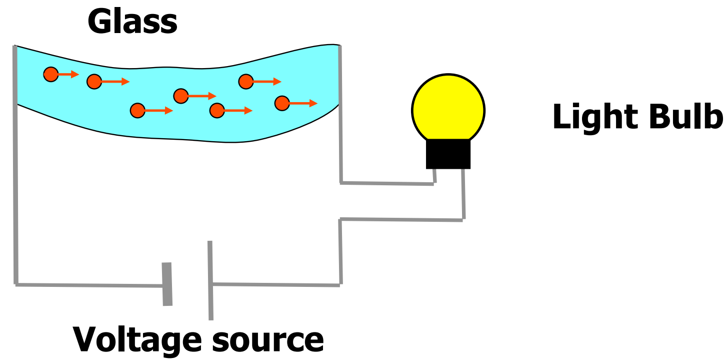
Electric Current

- Current $I = dQ/dt$ has a direction
  - Convention: Direction of flow of positive charges
  - In our circuits, I carried by electrons
- To get a current:
  - Need mobile charges
  - Need $|E| > 0$ (Potential difference)

Demo I

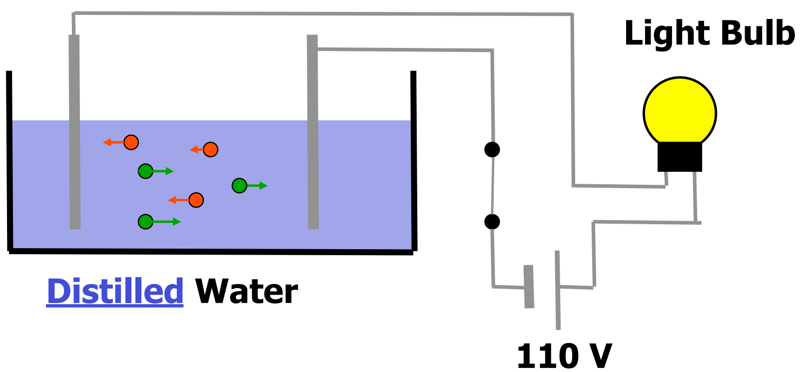


Demo II



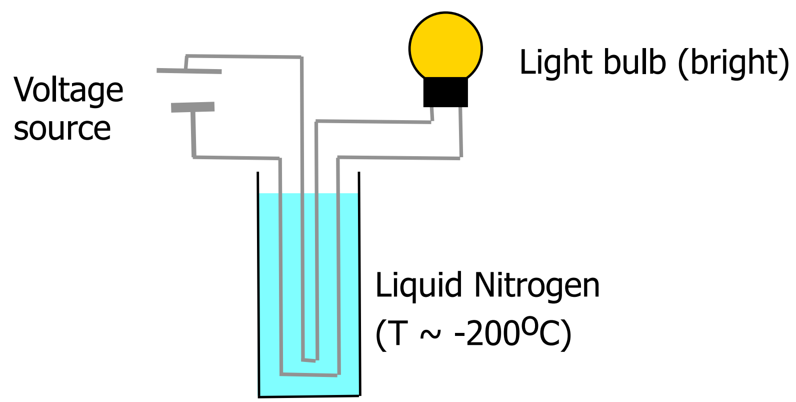
Molten glass: Charge carriers become mobile -> Current flows -> Bulb lights up!

Demo III



Add NaCl: Dissociates into Na^+ and Cl^-
Charge carriers are available -> Current flows -> Bulb lights up

Demo IV



Wire cold -> less resistance -> more current -> bulb burns brighter

Resistivity

- Interplay of scattering and acceleration gives an average velocity v_D
- v_D is called 'Drift velocity'
- How fast do the electrons move?
  - Thermal speed is big: $v_{th} \sim 10^6$ m/s
  - Drift velocity is small: $v_D \sim 10^{-3}$ m/s
- All electrons in conductor start to move, as soon as $E > 0$

Resistance

- Define $R = V/I$: Resistance
- $R = \rho\, L/A$ for constant cross section A
- R is measured in Ohm [Ω] = [V/A]

- Resistivity ρ is property of material (e.g. glass)

- Resistance R is property of specific conductor, depending on material (ρ) and geometry

Ohm's law

$$\boxed{V = R\, I}$$

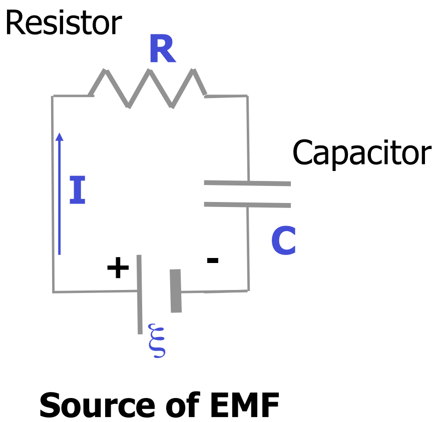
- Conductor is 'Ohmic', if R does not depend on V,I
- For real conductors, that is only approximately true (e.g. $R = R(T)$ and $T = T(I)$)
- Approximation
- valid for resistors in circuits
- not valid for e.g. light bulbs

Electric Power

- Use moving charges to deliver power

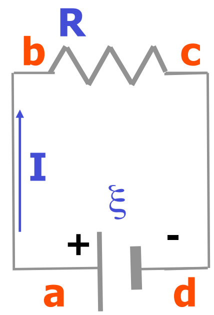
Power = Energy/time = dWdt
= (dq V)/dt =
dq/dt V = $\underline{I\, V}$ = $I^2R = V^2/R$

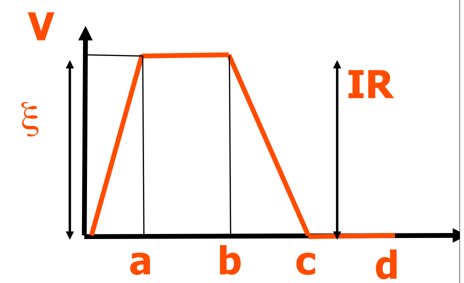
Electric circuits



Electromotive Force EMF

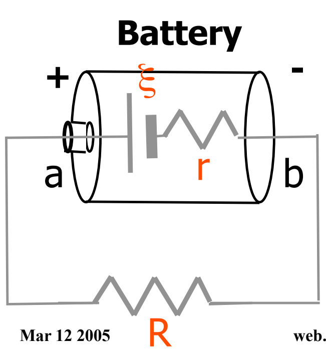
- Def: ξ = Work/unit charge
- ξ is 'Electromotive Force' (EMF)
- Units are [V]





Internal resistance

- Sources of EMF have internal resistance r
- Can't supply infinite power



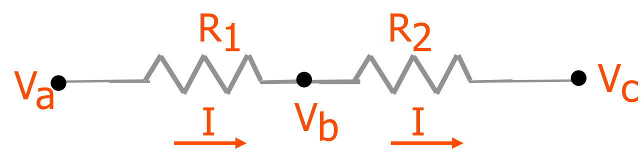
$$\boxed{V_{ab} = \xi - I r}$$

$$= IR$$

-> $I = \xi /(r+R)$

Electric Circuits

Resistors in series

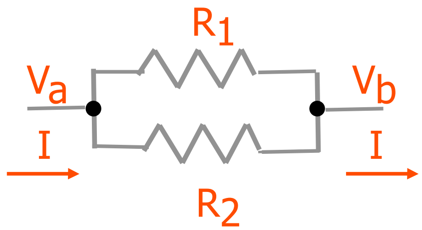


$$V_{ac} = V_{ab} + V_{ac} = I R_1 + I R_2 = I (R_1 + R_2)$$

$= I R_{eq}$ for $\boxed{R_{eq} = (R_1 + R_2)}$

Electric Circuits

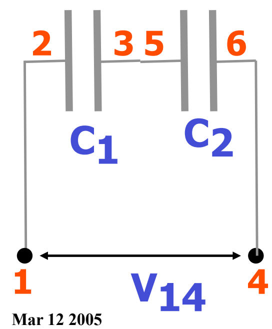
Resistors in parallel



$I = I_1 + I_2 = V_{ab}/R_1 + V_{ab}/R_2 = V_{ab}/R_{eq}$

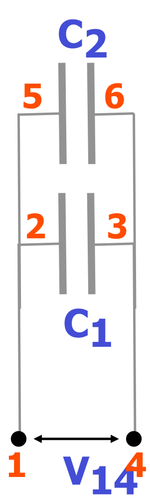
-> $\boxed{1/R_{eq} = 1/R_1 + 1/R_2}$

Electric Circuits



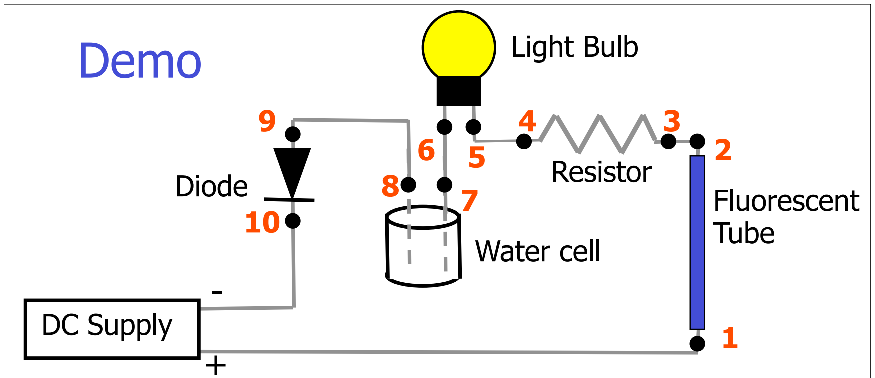
- Two capacitors in **series**
- $V_{14} = V_{23} + V_{56}$
- $Q = Q_1 = Q_2$
- $V_{tot} = Q_1/C_1 + Q_2/C_2 = Q/(C_1+C_2)$
- $1/C_{tot} = 1/C_1 + 1/C_2$
- **Inverse Capacitances add!**

Electric Circuits



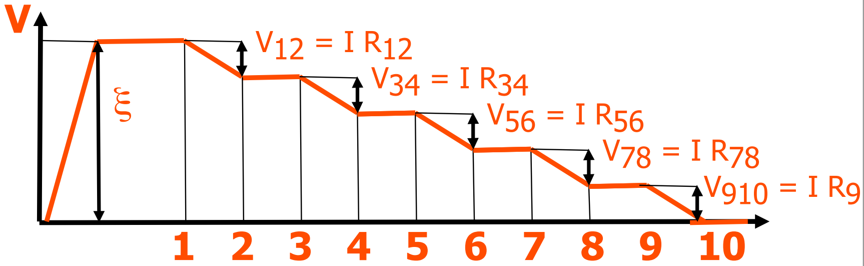
- Two capacitors in **parallel**
- $V_{56} = V_{23} = V_{14}$ (after capacitor is charged)
- $Q_1/C_1 = Q_2/C_2 = V_{14}$
- $Q_{tot} = Q_1 + Q_2$
- $C_{tot} = (Q_1 + Q_2)/V_{14} = C_1 + C_2$
- Capacitors in **parallel -> Capacitances add!**

Demo



$$\text{EMF } \xi \overset{?}{=} V_{12} + V_{34} + V_{56} + V_{78} + V_{910}$$

Loop Rule



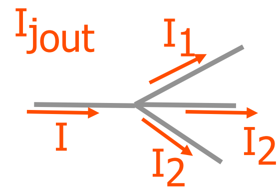
In general, $\Sigma V_j = 0$ <u>Loop Rule</u>

Kirchoff's rules

- Junction rule

At junctions:

$\Sigma I_{iin} = \Sigma$



<u>Charge conservation</u>

- Loop rule

Around closed loops:

$\Sigma \Delta V_j = 0$

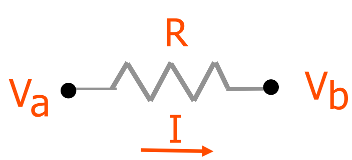
ΔV for both EMFs

and Voltage-drops

<u>Energy conservation</u>

Kirchoff's rules

- Kirchoff's rules allow us to calculate currents for complicated DC circuits
- Main difficulty: Signs!
- Rule for resistors:



$\Delta V = V_b - V_a = - I R$, if we go in the direction of I (voltage drop!)

Kirchoff's rules

- Kirchoff's rules allow us to calculate currents for complicated DC circuits
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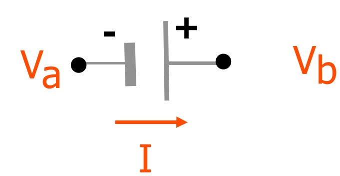
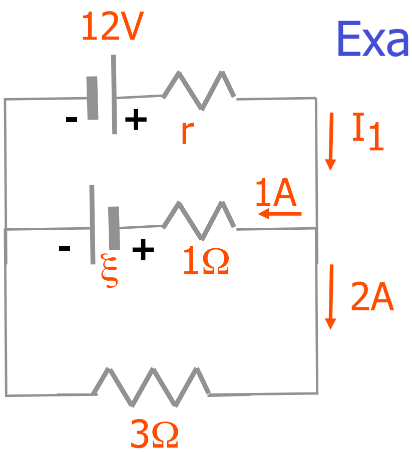


$\Delta V = V_b - V_a = \xi$, if we go in the direction of I

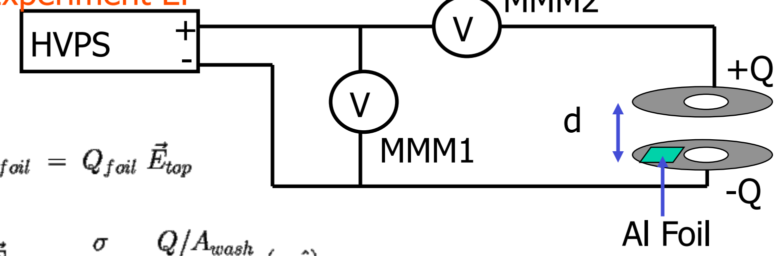


Example

- Pick signs for I_1, ξ
- Junction rule
  I_1 = 1A + 2A = 3A
- Loop rule (1)
  12V – 6V – 3A r = 0
  -> r = 6/3 Ω = 2 Ω
- Loop rule (2)
  12V – 6V – 1V - ξ = 0
  -> ξ = 5V

r, ξ, I_1 ? 3 unknowns

Experiment EF



$$\vec{F}_{onfoil} = Q_{foil}\,\vec{E}_{top}$$

$$\vec{E}_{top} = \frac{\sigma}{2\epsilon_0} = \frac{Q/A_{wash}}{2\epsilon_0}(-\hat{y})$$

$$Q_{foil} = -Q\frac{A_{foil}}{A_{wash}}$$

Q = CV = ε_0A/d V

$$\Rightarrow \vec{F}_{onfoil} = -Q\frac{A_{foil}}{A_{wash}}\frac{Q}{A_{wash}2\epsilon_0}(-\hat{y}) = \frac{Q^2}{A_{wash}^2}\frac{A_{foil}}{2\epsilon_0}\hat{y}$$

$$\Rightarrow \vec{F}_{onfoil} = \frac{(\epsilon_0 V A_{wash}/d)^2}{A_{wash}^2}\frac{A_{foil}}{2\epsilon_0}\hat{y} = \boxed{\frac{\epsilon_0 V^2}{2d^2}A_{foil}\,\hat{y}}$$