8.02x – Problem Set **4** Solutions

Problem 1 (8 points)

a) The force on charge Q_1 due to charge Q_2 is attractive, therefore pointing in the $x+$ direction. The distance between the charges is 1m. The magnitude of the force is

$$F = k\frac{Q_1 Q_2}{d^2} \approx 9.0 \times 10^9 \mathrm{N}. \tag{1}$$

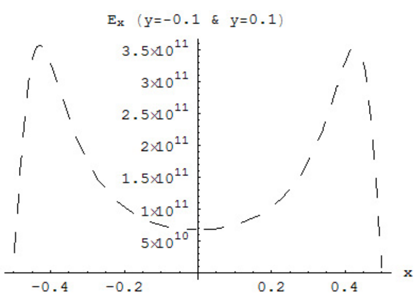
b) The x-component of the electric field will be the smallest exactly in the middle between the charges. If we moved away from $x = 0$, the field from one of the charges would increase more than the amount the field from the other charge would decrease. Therefore it will be the smallest in the middle. It will point in the $x+$ direction, with magnitude

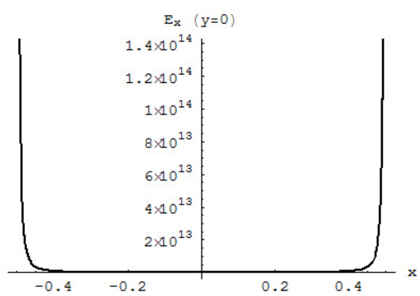
$$E = k\frac{|Q_1|}{(d/2)^2} + k\frac{|Q_2|}{(d/2)^2} = 8k\frac{Q_1}{d^2} \approx 72.0 \times 10^9 \mathrm{V/m}. \tag{2}$$

c) We will plot the components of the electric field, E_x and E_y for $y = -0.1$, $y = 0$ and $y = 0.1$. The electric field has components

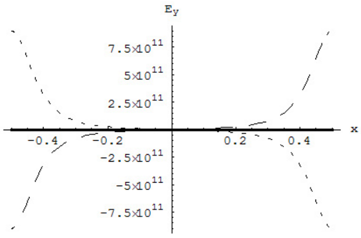
$$\begin{aligned} E_x &= E_x^{(1)} + E_x^{(2)} = &(3)\\ &= k\frac{Q_1}{(x-x_1)^2 + y^2}\frac{x - x_1}{\sqrt{(x-x_1)^2 + y^2}} + k\frac{|Q_2|}{(x-x_2)^2 + y^2}\frac{x - x_2}{\sqrt{(x-x_2)^2 + y^2}}, \\ E_y &= E_y^{(1)} + E_y^{(2)} = &(4)\\ &= k\frac{Q_1}{(x-x_1)^2 + y^2}\frac{y}{\sqrt{(x-x_1)^2 + y^2}} - k\frac{|Q_2|}{(x-x_2)^2 + y^2}\frac{y}{\sqrt{(x-x_2)^2 + y^2}}, \end{aligned}$$

First the E_x component plots (dashed line: $y = -0.1$ and $y = 0.1$, thick line: $y = 0$):

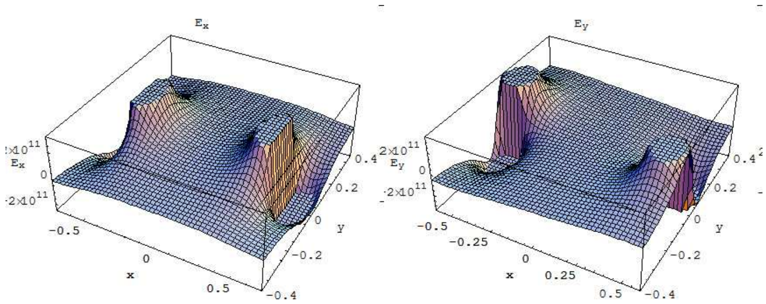




And the E_y plots (dashed line: $y = -0.1$, thick line: $y = 0$, dotted line: $y = 0.1$):



Just as an extra, we also add a 3D plot of E_x and E_y…



d) For the plot of the field lines see the book, picture 21.26(b). From that we can see that the x-component at $y = 0$ goes to infinity at the point charges, and is smaller between them. For $y = \pm 0.1$, the x-component is almost zero at $x = \pm 0.5$, because the field lines are almost vertical. When moving away from the charges at small y, E_x quickly finds its maximum (dense field lines turning towards x) and then decreases, because the field lines become much less dense.

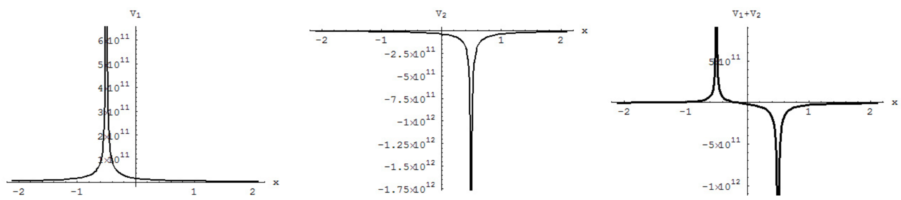
On the other hand, the E_y-component is zero for $y = 0$, nonzero for $y = \pm 10$, and changes sign at $x = 0$, which agrees with our plots.

<u>Problem 2</u> (8 points)

a) We have two charges, $Q_1 = 1$ C and $Q_2 = -2$ C, positioned at x_1 and x_2. The potentials are

$$V_1 = k\frac{Q_1}{|x - x_1|}, \qquad V_2 = k\frac{Q_2}{|x - x_2|}. \tag{5}$$

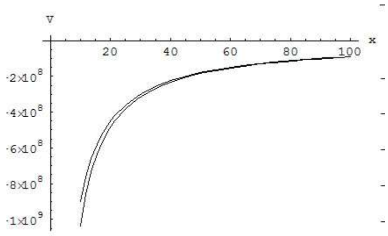
The plot of the potentials V_1, V_2 and both of them combined:



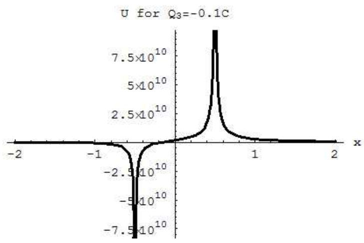
b) For distances $x \gg 1$m, the sum of the potentials can be roughly approximated by a potential from a single charge $Q_1 + Q_2 = -1$ C, located at $x = 0$. We can support our claim by a simple computation:

$$\begin{aligned} V &= V_1 + V_2 = k\frac{Q_1}{|x - x_1|} + k\frac{Q_2}{|x - x_2|} \approx \\ &\approx k\frac{Q_1 + Q_2}{|x|} = V_{\approx}. \end{aligned} \tag{6}$$

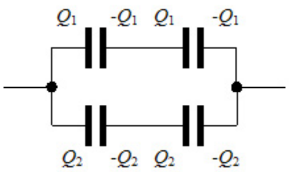
Just to check where this rough approximation makes sense, we plot V and $V_{\approx}$ for large x.



c) The potential energy for $Q_3 = -0.1$C in the potential of the two charges is $U_3 = Q_3(V_1 + V_2)$.



<u>Y&F 24.60</u> (6 points)

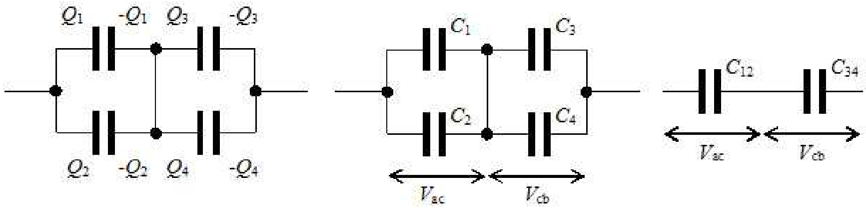


a) With the switch open, the charges on the capacitors distribute themselves as on Fig.3. The capacitance of the top line is the same as the capacitance of the bottom line (two capacitors in series) $C_{top} = 2\mu$F. From the symmetry we find that the charge will distribute itself evenly, $Q_1 = Q_2 = Q/2$. The total capacitance of the net of capacitors is $C = 4\mu$F. From

$$V_{ab} = \frac{Q}{C} = \frac{2Q_1}{C}$$

we get the total charge $Q = 2Q_1 = 210\text{V}4\mu\text{F} = 840\mu\text{C}$. The potential differences V_{ad} and V_{ac} will then be $V_{ad} = Q_1/3\mu\text{F} = 140\text{V}$ and $V_{ac} = Q_1/6\mu\text{F} = 70\text{V}$. We then obtain

$$V_{cd} = V_{ad} - V_{ac} = +70\text{V}. \tag{7}$$



b) When we now close the switch, the situation will look like in the second picture. Because $C_{12} = C_{34} = 9\mu\text{F}$ (C_{12} is $3\mu\text{F}$ and $6\mu\text{F}$ in parallel), we can easily see from symmetry, that $V_{ac} = V_{cb}$. So all the potential differences across all of the capacitors will be the same, $V = V_{ac}$! How big are they?

$$V_{ab} = V_{ac} + V_{cb} = 2V \qquad \Rightarrow \qquad V = V_{ab}/2 = 105\text{V} \tag{8}$$

across each of the four capacitors.

c) How much charge flowed through the line? We can compute the charges on each of the conductors, because we know the potential differences across them.

$$Q_1 = Q_4 = V \times 3\mu\text{F} = 315\mu\text{C}, \qquad Q_2 = Q_3 = V \times 6\mu\text{F} = 630\mu\text{C}.$$

The total charge on the inner plates of the conductors is zero, $-Q_1 - Q_2 + Q_3 + Q_4 = 0$. But before the switch was turned on, the total charge on the inside plates of capacitors 1 and 3 was zero as well. However, now it is

$$Q_{top} = -Q_1 + Q_3 = 315\mu\text{C}, \tag{9}$$

so this amount of charge had to flow through the switch.

<u>Y&F 24.71</u> (8 points)

The plane where the two slabs of dielectric meet, is an equipotential surface. We can imagine a thin metallic plate in there – and nothing would change. The capacitor now looks like a series of two capacitors with thicknesses $d/2$, with capacitances

$$C_1 = K_1 \frac{\epsilon_0 A}{d/2} = 2K_1 \frac{\epsilon_0 A}{d}, \qquad C_2 = K_1 \frac{\epsilon_0 A}{d/2} = 2K_2 \frac{\epsilon_0 A}{d}. \tag{10}$$

The total capacitance is then

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\epsilon_0 A}{d} \left(\frac{2K_1 2K_2}{2K_1 + 2K_2} \right) = \frac{2\epsilon_0 A}{d} \left(\frac{K_1 K_2}{K_1 + K_2} \right). \tag{11}$$