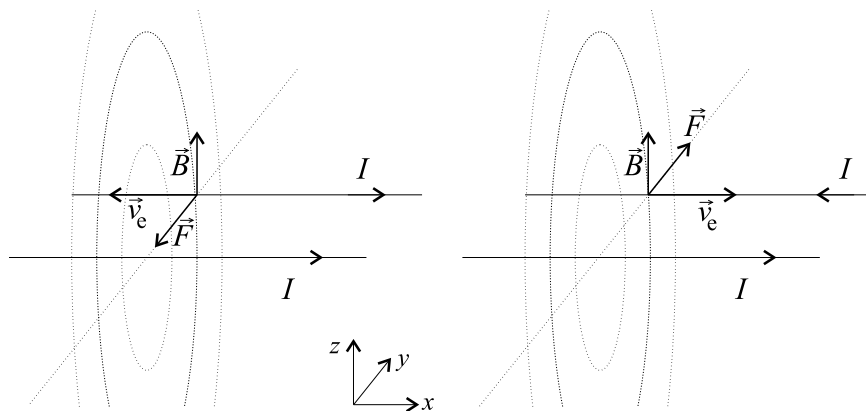

8.02x – Problem Set 8 Solutions

Problem 1 (10 points)

a) The currents run in the same direction \hat{x} . The field created by the first wire has direction \hat{z} at the place of the second wire. The *electrons* in the second wire move to $-\hat{x}$ (opposite the current), so the Lorentz force on them has direction $-\hat{y}$, towards the first wire. The wires attract each other.

b) Now the currents run in opposite directions. Replace the velocity of *electrons* in the second wire by $-\vec{v}$, so we get force $-\vec{F}$ (in the \hat{y} direction). The wires thus repel each other.



Problem 2 (10 points)

The electrons in the CRT tube are deflected to the right, when looking from the front. This could be caused by an electric field pointing horizontally to the left, or a magnetic field pointing vertically upwards. Let's try to distinguish these cases by moving the apparatus around.

If we rotate the apparatus in the horizontal plane, and the beam is still deflected to the right \Rightarrow the problem-causing field is a *vertical magnetic field*.

If we rotate the apparatus in the horizontal plane and find that the deflection changes, and that there are two (opposite) orientations for which it is zero \Rightarrow the villain is the *horizontal electric field*.

Problem 3 (10 points)

a) Each nucleus has energy $E = 208 \times 2.8 \text{TeV} = (208 \times 2.8 \times 10^{12})(1.6 \times 10^{-19}) \text{J} = 9.3 \times 10^{-5} \text{J}$. Using the relativistic equation $E^2 = p^2 c^2 + m_0^2 c^4$ for the energy and momentum, we get

$$\begin{aligned} p &= \sqrt{\frac{E^2}{c^2} - m_0^2 c^2} = \\ &= \sqrt{\left(\frac{9.3 \times 10^{-5} \text{J}}{3 \times 10^8 \text{m/s}}\right)^2 - [208(1.67 \times 10^{-27} \text{kg})(3 \times 10^8 \text{m/s})]^2} \approx \\ &\approx 3.1 \times 10^{-13} \frac{\text{kg}\cdot\text{m}}{\text{s}}. \end{aligned} \quad (1)$$

All of p comes just from the first term. We could have seen this much easier in the eV units. The rest mass (and energy) of the lead nucleus is $E_0 = 208(1.67 \times 10^{-27} \text{kg})(3 \times 10^8 \text{m/s})^2 = 3.1 \times 10^{-8} \text{J} = 0.2 \text{TeV}$. The total energy when accelerated is $208 \times 2.8 \text{TeV}$. Obviously, almost all of the energy comes from momentum. Thus $p \approx 208 \times 2.8 \text{TeV} = (208 \times 2.8)(1.6 \times 10^{-19}) / (3 \times 10^8 \text{m/s}) = 3.1 \times 10^{-13} \text{kg}\cdot\text{m/s}$.

b) The radius of the path of a charged particle in a magnetic field is $R = \frac{p}{qB}$, so the magnetic field needed to keep the nuclei on their trajectory is

$$B = \frac{p}{qR} = \frac{3.1 \times 10^{-13} \text{kg}\cdot\text{m/s}}{(1.602 \times 10^{-19} \text{C})(27 \text{km})} \approx 71 \text{T}. \quad (2)$$

c) A linear accelerator of length $L = 100 \text{m}$ would have to have a potential difference of $V = E/q \approx 6 \times 10^{14}$ between its ends. This would mean an electric field of magnitude $E_{\text{field}} = V/d \approx 6 \times 10^{12} \text{V/m}$, which is ridiculously high. For comparison, the fields inside atoms have strengths of order

$$E_{\text{atomic}} \approx \frac{kq}{r_a^2} \approx \frac{(9.0 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2)(1.602 \times 10^{-19} \text{C})}{(10^{-10} \text{m})^2} \approx 10^{10} \frac{\text{V}}{\text{m}}. \quad (3)$$