Massachusetts Institute of Technology *Spring Term 2005*

8.02X Electricity and Magnetism

Practice-Quiz #4b

Solutions

Problem 1 (25 points)

In the HVPS experiment, you built a "transformer" by winding 6 loops of wire around a tightly wound red coil.

(a) Which side of the transformer was the primary side in this setup?

The 6 Loops

(b) Assume that in your setup the inner (red) coil had length L_1, number of windings N_1 and radius R_1. The outer coil (wire loops) had length L_2, number of windings N_2 and radius R_2. Derive an expression for the mutual inductance of the two coils. Show work!

(c) Based on the known output voltages of LVPS and HVPS, estimate (within a factor of 2) a numerical value for the number of windings of the red coil (ignore the different length for primary and secondary coil).

(d) Assume a current $I_2(t) = I_0 * \cos(\omega t)$ was flowing through the outer coil. What would the voltage across the red coil $\Delta V_1(t)$ be?

b) $$EMF_{Coil} = -M\frac{dI_1}{dt}$$

$$= -\underbrace{N_2 \cdot \pi R_1^2 \cdot \frac{dB_1}{dt}}_{\frac{d\Phi_{B_2}}{dt}} = -\underbrace{N_2 \cdot \pi R_1^2 \cdot \frac{N_1}{L_1} \cdot \mu_0}_{M} \cdot \frac{dI}{dt}$$

c) $V_{LVPS} \approx 10V$

$V_{HVPS} \approx 1000V$

$$\frac{V_{LVPS}}{V_{HVPS}} = \frac{N_{LVPS}}{N_{HVPS}} \Rightarrow N_{HVPS} \approx 6000$$

($V_{LVPS} \to 10V$, $N_{LVPS} \to 6$, $V_{HVPS} \to 1000V$)

d) $V_1(t) = -M \frac{dI_2}{dt}$; $I(t) = I_0 \cdot \cos(\omega t)$

$= -M \cdot (-I_0 \cdot \omega \cdot \sin(\omega t))$

$= M \cdot I_0 \cdot \omega \cdot \sin(\omega t)$

with $M = -N_2 \cdot \pi R_1^2 \cdot \frac{N_1}{L_1} \cdot \mu_0$

Problem 2 (25 points)

Shown below is a circuit that is connected to a DC power supply with an output voltage V_0. For times t < 0, the switch is in position 1 and a current is flowing through the inductor (inductance L), the resistor (resistance R) and the power supply. Assume the switch has been closed for a very long time and the resistance of the inductor is negligible. Assume also that for t < 0, the capacitor (Capacity C) is discharged (Q=0).

At t=0, the switch is moved to position 2 and the power supply and resistor are therefore removed from the circuit.

(a) At t=0, what is the total energy stored in the circuit formed by capacitor and inductor?

$$U = \frac{1}{2} L I^2 + \underbrace{\frac{1}{2} \frac{Q^2}{C}}_{=0} = \frac{1}{2} L \left(\frac{V_0}{R}\right)^2$$

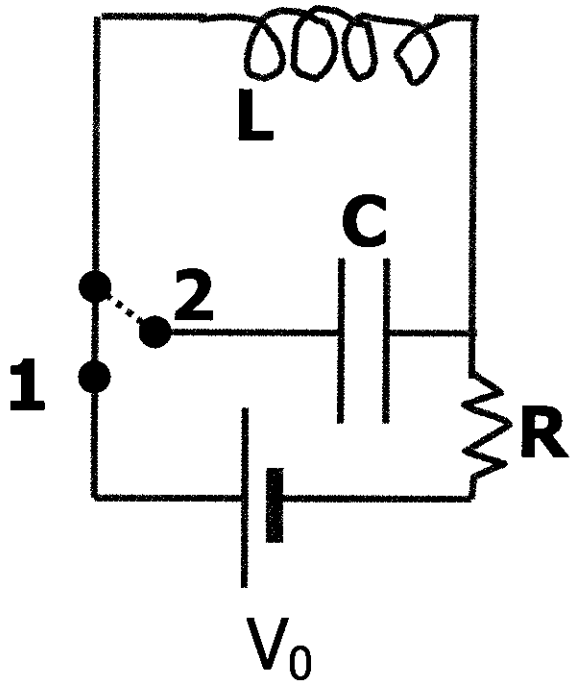
(b) Give an example (sketch) of a mechanical system that corresponds to the circuit formed by the inductor and capacitor (after t=0). Identify which elements in the mechanical system correspond to which circuit elements.

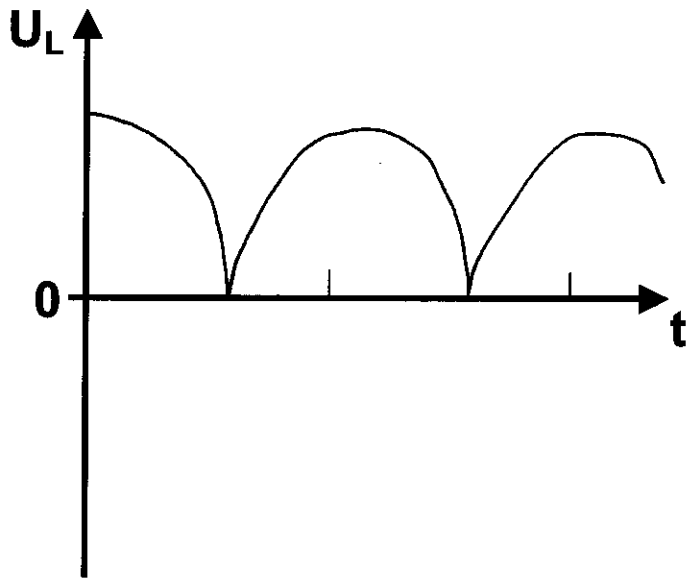
Mass on a spring: Inertia: Mass ↔ Inductor
Restoring Force: Spring ↔ Capacitor

(c) How will the charge Q(t) on the capacitor vary with time? Give an equation in terms of the quantities defined above.

$Q(t) = Q_0 \cdot \sin(\omega t)$ with $\omega = \sqrt{\frac{1}{LC}}$ and $Q_0 = \sqrt{LC} \cdot I$

(d) On the graph below, sketch how the energy in the inductor varies with time after t=0.


L
C
2
1
R
V_0


U_L
0
t

Problem 3 (25 points)

Consider a plane wave with an amplitude that is described by the following equations:

$A_x = 0$
$A_y = 0$
$A_z = A_0 \cos(\omega t - (2\pi/ 3m)\, x)$

(a) Which direction is the wave traveling in?

(b) How big is the wave length of the wave?

(c) Could these equations describe a sound wave? Explain your answer.

(d) If the wave was electromagnetic, what would the frequency f be?

a) + x direction

b) $\lambda = 3m$

c) No. $\vec{A}$ is $\perp$ to $\vec{v}$ → transverse wave

d) $\lambda \cdot f = 3 \cdot 10^8 \frac{m}{s} \Rightarrow f = 10^8 Hz$

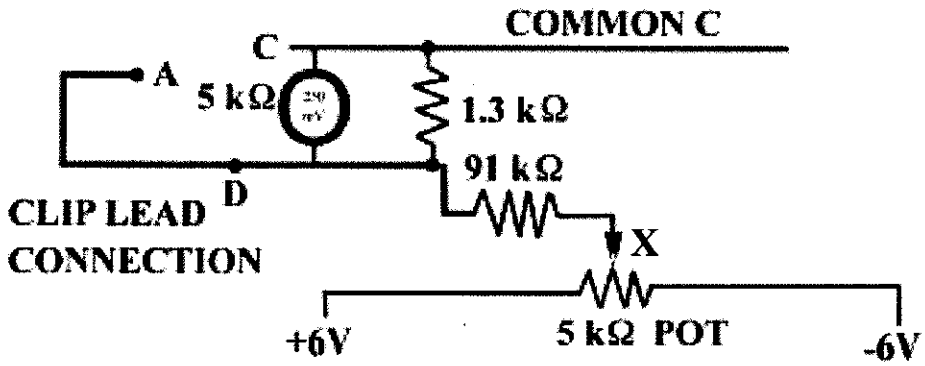
Problem 4 (25 points) AMP experiment

(a) What is the purpose of the AMP experiment? (1-2 sentences)

(b)How did you calibrate the AMP setup? What does the calibration curve tell you? (2-3 sentences)

(c) Shown below is a calibration circuit like that on the AMP experiment. All voltages a measured relative to the common line C, which is defined as 0V. What is the voltage at point X when the slider of the potentiometer is 1/2 way between the extreme positions?

(d)What are the maximum and minimum voltages at point D relative to C, when the slider is moved from one extreme position to the other?

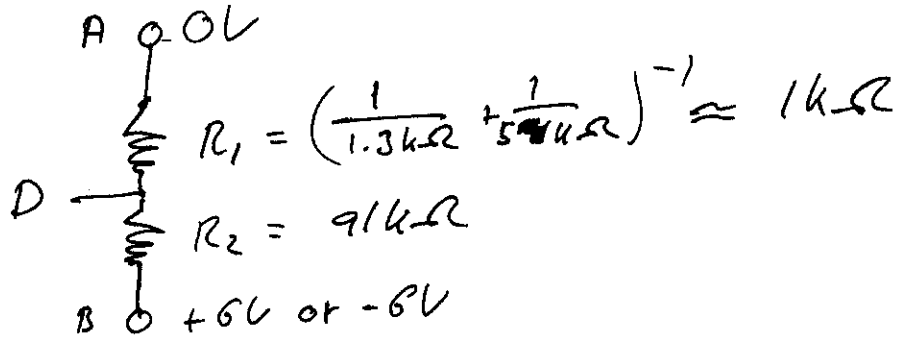


a) Purpose: To "amplify" the input voltage, i.e. create an output signal that is proportional to the input, but larger a factor "g", the gain.

b) First shorted the input and zeroed the output using 100 kΩ pot, with MMM on the 250 mV setting. Then connected calibration output D to Input A. Vary input voltage from −67 mV to 67 mV and record output voltage as a function of input voltage.

c) By symmetry $V(x) = 0$

d) Equivalent diagram



$$V_D = \frac{R_1}{R_1+R_2} \cdot V_{AB} = \frac{1}{92} \cdot 6V \quad \text{(slider left)}$$

$$= -\frac{1}{92} \cdot 6V \quad \text{(slider right)}$$