Ballistic Impact of Textile Structures

DAVID ROYLANCE, ANTHONY WILDE, AND GREGORY TOCCI[1]

*Polymers and Composites Division, Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172, U. S. A.*

ABSTRACT

Previous work on transverse impact of single textile fibers is reviewed and extended to model orthogonal weaves in which fiber crossovers are simplified as pin joints. A dynamic finite-element computer technique previously developed for single fibers is extended to model the woven panel, and this method is shown to produce results which are in substantial agreement with experimental observations of ballistic nylon panels. Impact of a woven textile panel is shown to exhibit substantial differences compared to the equivalent impact of a single fiber, primarily in that the propagating strain waves experience pervasive and complex interactions due to the influence of the fiber crossovers. The vast majority of ballistic energy is seen to be deposited in the orthogonal fibers passing through the impact point, while the other fibers are essentially ineffective, which suggests possible improvements in the design of textile structures intended for dynamic impact applications.

KEYWORDS

Single fibers; woven panels; nylon fabric; textile structures. Armor design; transverse impact; ballistic impact. Wave propagation; dynamic finite-element analysis; dynamic constitutive relations; dynamic failure criteria; stress-strain. Rate-independent theory; numerical analysis; algorithm; Fortran.

Introduction

Although many applications exist for textile structures of high dynamic strength—automotive seat belts as just one example—the most demanding of these continues to be the military personnel armor system, where a lightweight textile structure is designed to offer protection against fragmentation threats. To date, development of such systems has been almost entirely empirical; new materials or novel weaves are fabricated based on little more than educated experience and then tested under some sort of controlled fragment-simulator impact. Such a technique is not only costly, but, of course, is not guaranteed the development of an optimum system in any finite number of trials. Ultimately, one must resort to actual test, but any analytical technique which offers valid guidance as to parametric changes would be highly valuable. This paper presents results based on a computer treatment of ballistic impact of a woven textile panel, which does serve this purpose and offers promise as a design aid in new impact-resistant structures.

The dynamic response of a single fiber subjected to transverse impact has been the subject of a great many studies over approximately the past three decades. Our laboratory has reviewed and extended this work as a means of elucidating the materials—as opposed to structural parameters bearing on ballistic resistance.

A good deal of valuable information has been obtained in these studies, but we were eventually forced to the conclusion that one could not separate the structural response from the ballistic event and still obtain, thereby, any sort of useful correlation between fiber properties and ballistic resistance of a panel woven from that fiber. The computer technique mentioned above is our response to this problem; it is primarily a structures code into which the fiber materials properties can be input. As will be seen, the structural response is a function of these materials properties as well as the structural geometry itself. The code output then provides a quantitative measure of the impact performance of a given system, but perhaps even more valuable is the quantitative feel it gives as to the basic mechanics of impact of a textile structure.

Transverse Impact of Single Fibers

As the response of a textile structure is built upon the response of the fibers with which it is woven, the mechanics of single-fiber impact will be reviewed briefly to provide a starting point for our discussion of woven-panel impact.

Rate-Independent Theory

Since the early work of Rakhmatulin and Taylor, several authors have formulated the mechanics of transverse impact, usually assuming a rate-independent

material constitutive response. Reviews of this work can be found in the books of Rakhmatulin [6] and Cristescu [2]. The salient features of this theory can be stated with reference to Figure 1. Upon impact,
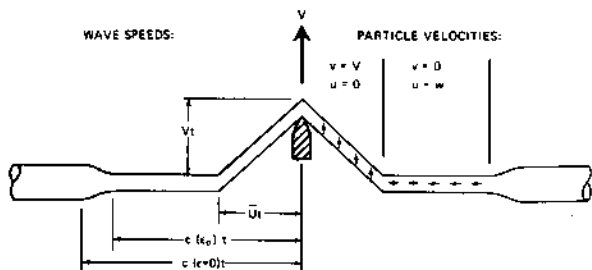

FIG. 1. Wave propagation in a transversely impacted fiber.

longitudinal strain waves are propagated outward from the point of impact. The increments of strain ϵ comprising these waves propagate at speeds $c(\epsilon)$ corresponding to the slope of the dynamic stress-strain curve at that strain: $c(\epsilon) = \sqrt{k(dT/d\epsilon)} = \sqrt{kE(\epsilon)}$. (Here the material density is included implicitly by using the textile units of grams per denier for tension T and modulus E, and k is a units conversion factor. For speeds in meters per second, $k = 8.82 \times 10^4$.) Depending on the shape of the stress-strain curve, these strain waves may contain both dispersive and shock components. Behind the longitudinal waves, material flows inward toward the point of impact at a constant velocity w and strain ϵ_0. In addition to the longitudinal waves, transverse waves are propagated outward from the point of impact. The transverse wave usually propagates more slowly than the final longitudinal wavelet and can usually be characterized as follows. At the transverse wave-front, the inward material flow velocity ceases abruptly and is replaced by a transverse particle velocity v equal to that of the projectile. The strain and tension are unchanged across this wavefront, but both the longitudinal and transverse particle velocities experience discontinuities there; in this re-

gard, the transverse wave is a shock. (The apparently unbalanced tensions on either side of the transverse wavefront are compensated by the change in particle momentum as the wave propagates.) Behind the transverse wavefront, all particle velocities are equal in magnitude and direction to the projectile velocity, and the fiber configuration is a straight line at a constant inclination θ from the longitudinal direction.

Smith [10] has presented a comprehensive summary of the rate-independent theoretical relations between these variables, as well as the modifications which are necessary to describe such complications as shock formation and interference between the longitudinal and transverse waves. For a given dynamic stress-strain curve and impact velocity V, these relations can be used to calculate the final strain ϵ_0, the final tension T_0, the longitudinal particle velocity w between the final strain wavelet and the transverse wave, the transverse wave speed U and $\bar{U}$ (in Lagrangian and laboratory coordinates, respectively), and the fiber inclination θ. These relations are an exceedingly valuable guide to the basic elements of fiber impact, and can be used to make approximate parametric studies of the effects of materials properties. As an example, when one makes the not-too-incorrect assumption of linear constitutive response (stress-strain behavior characterized by a constant modulus E), one obtains the relation between tension and velocity, with E as a parameter.

$$V^2 = \epsilon_0(1 + \epsilon_0)E - [\sqrt{\epsilon_0(1 + \epsilon_0)E} - \epsilon_0\sqrt{E}]^2 \quad (1)$$

with

$$T_0 = E\epsilon_0 \quad (2)$$

Although this relation is somewhat unwieldy in closed form, it is easily graphed; the result is shown in Figure 2. As an example of the sort of qualitative design information, such relations offer; one sees here that an increase in fiber modulus has the effect of increasing the tension level which will be generated upon impact at a given velocity. This effect may well mitigate the beneficial increased wave speed, which tends to spread the impact energy over a wider volume of material.
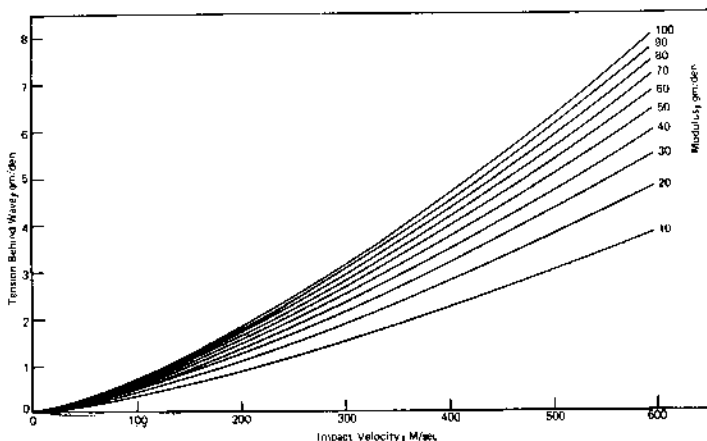

FIG. 2. Theoretical relation between impact velocity and tension for various fiber moduli.

Development of Dynamic Stress-Strain Curves

The rate-independent theoretical relations which use the dynamic stress-strain curve to predict the fiber's response upon impact can be worked backwards. If one knows experimentally the relation between the fiber inclination angle θ or transverse wave speed $\bar{U}$ vs impact velocity V, then he can use the rate-independent theory to infer the dynamic stress-strain relation. Papers by Smith [9] and Schultz [8] illustrate the technique using $\bar{U} - V$ and $\theta - V$ data, respectively. A paper by Roylance [7] extends the technique to account for viscoelastic response. The technique will not be detailed here, but it is conceptually similar to split-Hopkinson bar tests, where a one-dimensional wave equation is used to infer a dynamic stress-strain curve from oscilloscope load-time traces; in our case, multiflash high-speed photographs of the fiber configuration after impact take the place of the oscilloscope traces.

Some typical results of this technique are shown in Figures 3 and 4, which present static and dynamic curves, respectively, of a series of chemically identical nylon yarns which have been drawn to differing ratios so as to produce a graded series of tenacities. As with all dynamic constitutive tests, one must exercise some caution with respect to the accuracy of these curves. How-

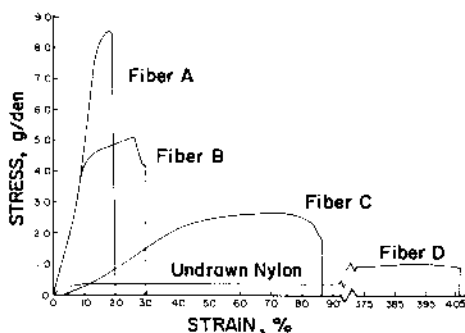FIG. 4. Dynamic stress-strain curves for same nylon fibers.

FIG. 3. Static stress-strain curves for series of nylon fibers.

ever, the fundamental conclusions drawn from them are of considerable interest to the polymer scientist interested in dynamic materials response. In general, the dynamic curve is displaced upward with respect to the static curve, reflecting the rate-dependence of the molecular response to stress. This displacement, however, is not especially significant to the fiber's response. It is the variation between static and dynamic breaking strain which is highly significant. At these dynamic rates, the very long static extension exhibited by the lowest-tenacity fiber simply does not have time to occur, and its energy absorption to fracture is markedly reduced. The highest-tenacity fiber, on the other hand, does not make use of these slow molecular mechanisms anyway, and its energy absorption is essentially the
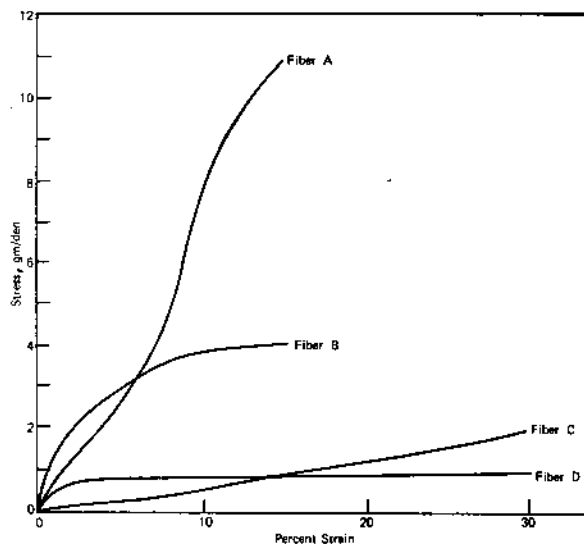
same at dynamic rates as at static rates. The result is a complete reversal of energy absorption capabilities; the low-tenacity fiber is much tougher than the high-tenacity fiber at static rates, but the reverse is true at dynamic rates.

Direct Numerical Analysis

As valuable as described above is the rate-independent theory of transverse impact, one quickly develops situations where the closed-form mathematical approach becomes intractable. Such situations, which must be considered in real cases, include unloading waves resulting from projectile slowdown and wave interactions such as occur when the longitudinal strain wave reflects from the fiber clamp and collides with the ongoing transverse wave. And as will be seen, the problem of an impacted woven panel is hopelessly intractable in closed form. For these situations, we have resorted to extensive use of numerical computer approximations and have found them highly valuable in these problems.

In recent years, Davids *et al.* [4, 11] have developed a dynamic form of finite-element analysis which will be referred to here as "direct analysis." The mechanics of wave propagation are usually formulated by applying an impulse-momentum balance and a condition of continuity to an incremental volume of material; when the size of the volume element is reduced to the limit, a system of hyperbolic partial differential equations results which, in conjunction with the boundary values and the material constitutive law, describes the space-time response of the physical system. The combined system of equations is then attacked by analytic mathematical techniques, such as Laplace transform methods

or the method of characteristics, or by replacing the partial derivatives with finite divided differences so as to effect a computation solution using a digital computer. Direct analysis is a computer-oriented technique, but it differs from the finite-difference approach in that the incremental volume element is never taken to the limit; the original governing relations are used directly and the development of the differential equation is dispensed with.

Direct analysis has several advantages over these other methods. Its conceptual simplicity leads to an easily written and debugged computer program, and boundary conditions and materials properties can be changed with only very minor program alterations. In addition, the problems of wave interactions, reflections, etc, which render closed form approaches intractable, are incorporated automatically simply by specifying the appropriate boundary conditions. Direct analysis was first applied to transverse fiber impact by Lynch [3]. His paper showed that many diverse aspects of the problem can be examined by this method: energy loss of the impacting projectile, energy partition in the impacted fiber, nonlinear and time-dependent stress-strain curves, and impact of a flexible membrane. His computation scheme was then used by Roylance [7] to assess the effects of viscoelastic relaxation during the impact. The essential algorithm employed is similar in both single-fiber and woven-panel impact; the case for single fibers will be presented briefly here for illustration.

A fiber of half-length L, to be impacted at zero obliquity at its mid-point with a projectile of mass M and velocity V, is considered as consisting of n finite elements. Associated with the i-th element are laboratory coordinates x_i and y_i, a scalar strain ϵ_i, and vector quantities T_i tension and v_i velocity. The tension T_i has the same direction as the element itself (approximating the fiber's assumed inability to sustain a bending moment), while v_i is not restricted in direction. These variables are then related by simple governing laws such as impulse-momentum, continuity, etc. The program is then written so as to employ these relations sequentially and effect a recursive algorithm for proceeding from one element to the next over the length L and then repeating the process at a new increment of time. Even though the fiber motion takes place in two space dimensions, the computer solution is referenced to a Lagrangian frame attached to and extending with the fiber; this essentially reduces the problem to one dimension. The components of the vector quantities with respect to the laboratory coordinates are computed by means of the element's inclination angle.

$$\theta_i = \tan^{-1}[(y_{i+1} - y_i)/(x_{i+1} - x_i)] \qquad (3)$$

As the Fortran coding of the algorithm would be too space-consuming to include here, the program logical four steps will be described verbally. A program listing and sample runs are available from the authors upon request.

Step 1. Specify input parameters n, L, M, V, maximum time, etc.

Step 2. Define increment sizes. The length increment is just the fiber half-length divided by the number of elements n, and the time increment is related to the length increment and the wave speed by the Courant stability criterion for hyperbolic systems [1].

$$c = \sqrt{kE} \qquad (4a)$$

$$\Delta L = L/n \qquad (4b)$$

$$\Delta t = \Delta L/c \qquad (4c)$$

The accuracy generally improves as n is increased, but the stability criterion causes the run time to increase as the square of the number of elements. The mesh size is, therefore, chosen so as to balance the conflicting requirements of economy and accuracy.

Step 3. Propagation procedure to be repeated for $i = 1$ to $i = n$ over the length of the fiber.

Step 3a. Using the existing values of tension at either end of the element, an impulse-momentum balance is used to compute the acceleration of the element. This acceleration is then integrated to find new values for velocity and displacement. Initially (at zero time), the tensions in each element are set to zero. Thereafter, the boundary conditions are included explicitly; at $i = 1$, the velocity is set equal to the projectile velocity, and at $i = n$ (the clamp), the velocity is set to zero.

Step 3b. Now knowing the new displacements, a new element strain is computed using a continuity (strain-displacement) condition.

Step 3c. A new tension is now computed from the strain using the material constitutive law. This law is included as a subroutine to permit easy implementation of various laws.

Step 4. When values of tension, strain, velocity, and position are known for all elements of fiber, the time is incremented and the propagation procedure (Step 3) repeated. At each time step, a new projectile velocity is computed by using the tension and element inclination at $i = 1$. Steps 3 and 4 are repeated for a specified number of times corresponding to the time range of interest, or a failure criterion can be incorporated into the program to terminate computation at a time corresponding to fiber failure.

Although the above description is for single-fiber impact, the program logic does not change appreciably for the woven-panel case. Here, the essential difference is the choice of propagation path for Step 3; i.e., when the program variables are known at one length element, there is a choice as to which to compute next. Each

nodal point is also a fiber crossover point, so that four fiber elements intersect there. In the fabric code presently in use, the solution is advanced along a front inclined 45° to the orthogonal fibers passing through the impact point. This requires that the algorithm calculate the program variables at the node diagonal to known node. The Fortran coding becomes considerably more lengthy in the fabric code, due primarily to the necessity of keeping track of three rather than two space dimensions; all vector quantities must be resolved by two angles similar to Equation 3. The run time also increases very substantially for this code, since the number of mesh points to be computed per time step is squared relative to the single-fiber case. A related problem is the sheer bulk of data produced by the fabric code, since strain, tension, velocity, strain energy, kinetic energy, and position (the vector quantities requiring a data triplet for specification) are computed for every mesh point and at every time increment. We have made the maximum possible use of graphic output, such as three-dimensional computer plots, to reduce the volume of data to comprehensible size.

In both the fiber and the fabric codes, neither the detailed formulation of the governing equations nor their sequential ordering is unique; the algorithms used represent just one possibility which generates reasonably stable and accurate results. Typical data from the fiber program is shown in Figure 5. This is the
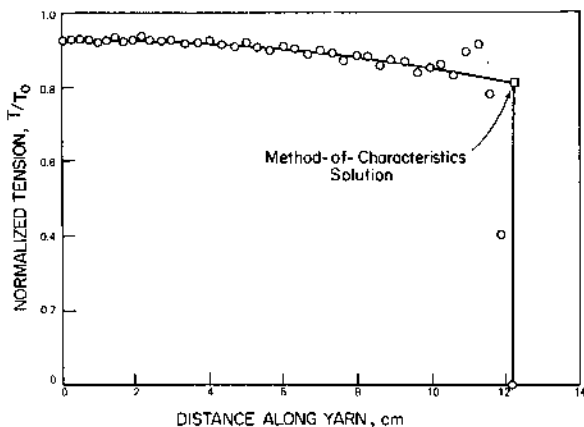

FIG. 5.   Fiber code prediction of normalized tension distribution in a viscoelastic fiber impacted transversely (from ref. [7]).

normalized stress distribution in a viscoelastic fiber, which shows the effects of stress relaxation. Considerable numerical overshoot is evident at the wavefront, but the distribution extrapolates to the analytical value given by a method-of-characteristics solution [7].

Transverse Impact of Woven Panels

Given the many assumptions which are made in the development of the numerical method (neglecting effect of missile geometry, fiber bending stresses, slippage at crossovers, etc), one must first ask if the computer's results have any physical significance at all. Certainly, it is more than possible to write an algorithm which produces divergent and meaningless results. We have come to trust the fiber program almost completely, largely since it can be checked with the analytic theory. Such a check is not available for the fabric program, but its results agree with experimental observations to such a point as to develop a similar confidence in its accuracy. As an example, the $V_s - V_r$ curve produced by the fabric code is essentially identical to the experimental curve; see Figure 6 for data for ballistic nylon panels of equivalent areal density. (Here V_s is the impact velocity and V_r is the residual projectile velocity after penetration.) These curves encompass many aspects of
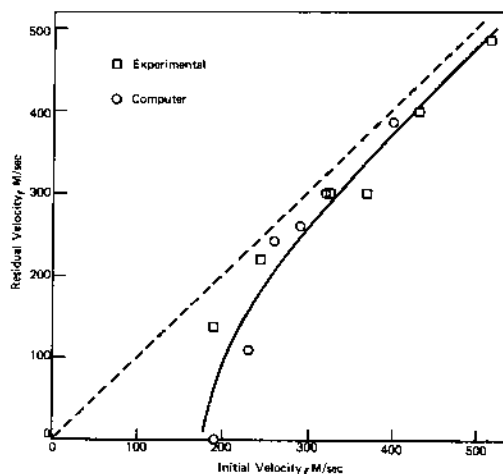

FIG. 6. $V_s - V_r$ data for ballistic nylon; experimental and computer predictions.

the ballistic event—generation and propagation of stress waves, projectile deceleration, prediction of failure time—so that agreement here is a rather convincing demonstration of the code's applicability. These curves, and most of those to follow, are for a single layer of 2200 den yarn woven in a 2×2 basketweave and having 44 crossovers per inch.

The essential difference between fiber and fabric impact is the extensive and pervasive wave interactions which are generated in the latter case due to the presence of the fiber crossovers. At each node, a portion of the onward-propagating strain wave will be reflected, thus attenuating the magnitude of the wavefront while increasing the strain level in the material behind the wave. When a single fiber is impacted, a

plateau of constant strain is developed behind the strain wave; fracture does not occur until the wave reflects from the clamp or the projectile to increase the strain. In the impacted panel, however, the arrival at the impact point of strain wavelets reflected from the cross-overs causes the strain to increase here continuously with time after impact. This effect is plotted for ballistic nylon in Figure 7, which shows the strain versus time history at the impact point for various impact velocities. The impact point is always the point of maximum strain in the fabric, and penetration occurs when the strain here exceeds the dynamic breaking strain of the fiber. The initial strain generated in Figure 7 is identical to that predicted by the analytical theory for fibers (Eq. 1); the rate of strain increase, thereafter, is a complicated function of wave speed (fiber modulus), missile mass, and fabric geometry.

FIG. 8.   Strain at impact point and projectile velocity vs time after impact.

nylon at 290 m/sec. The wavefront attenuation is evident, as is the increase of strain at the impact point with time after impact. The shape of the distributions is unexpected, but even more striking is the effect of the crossovers on the wave speed itself. The longitudinal wave speed for the single fibers (75 gpd) as given by $c = \sqrt{kE}$ is 2572 m/sec, but the measured wavespeed in the computer simulation of the fabric is 1819 m/sec, which is slower than the expected value by a factor of $\sqrt{2}$. This effect is attributed to the effective increase in lineal density due to the crossovers (similar to the technique of manipulating wavespeed without changing material properties by weighting the fiber with strips of tape [5]), but it was not anticipated prior to the computer studies. The ability of the fabric code to model this effect is another evidence of its reliability.

Figure 10 is an example of the three-dimensional maps produced by the code. Here we are looking at the distribution of strain in the set of parallel fibers running in the direction of the x-axis (the one coming "out of the
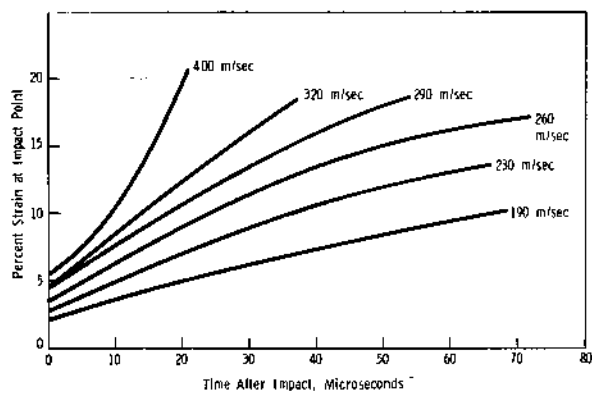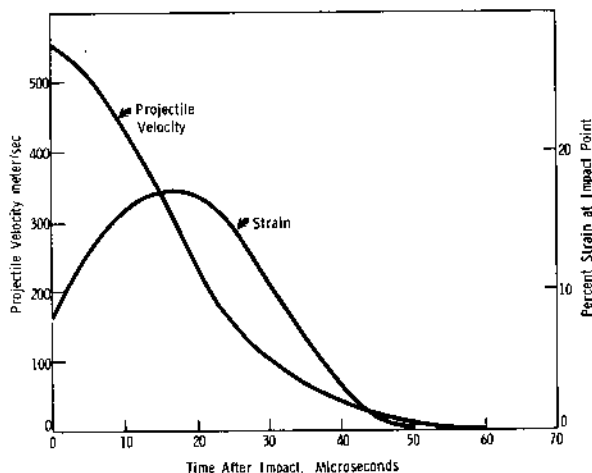

FIG. 7.   Strain at point of impact vs time after impact at various velocities.

In real fabric panels, the mass of fiber is sufficient to cause appreciable projectile slowdown at ballistic rates. The unloading waves generated by this slowdown are extremely difficult to handle analytically, but are generated automatically by the numerical codes. Although the fabric partitions the absorbed projectile energy in complicated distributions of strain and kinetic energy, the projectile itself sees only the tension and inclination angle at the point of impact; one should, therefore, expect a relation between these two variables. Such a relation is seen in Figure 8, which is for impact of 12 layers of ballistic nylon at a velocity below the penetration velocity for that weight of fabric. It is noted that the projectile velocity experiences an inflexion at the same time as the maximum in strain.

Figure 9 shows the effect of the crossover interactions on the propagation of longitudinal strain waves along the orthogonal fibers passing through the point of impact. These are the strain distributions along these fibers for three different times after impact of ballistic
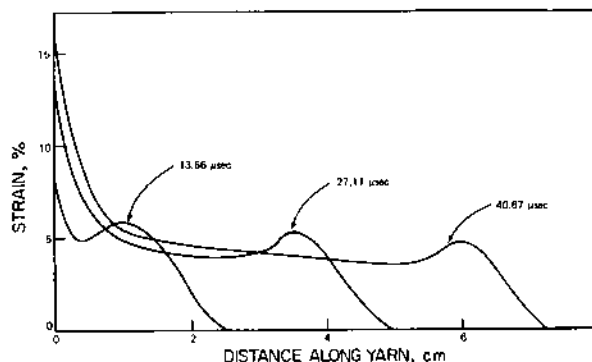

FIG. 9.   Strain distributions along orthogonal fiber running through impact point at various times after impact.

paper"). An identical map, but rotated 90° in the fiber plane, exists for the orthogonal set of parallel fibers. Here the high strains at the impact point are clearly evident, as is the fact that the *x*-direction strain in the *x*-fibers does not reach very far along the *y*-axis. This map is for the same impact event as Figure 9, at 40.67 μsec after impact. The wave distribution for this time
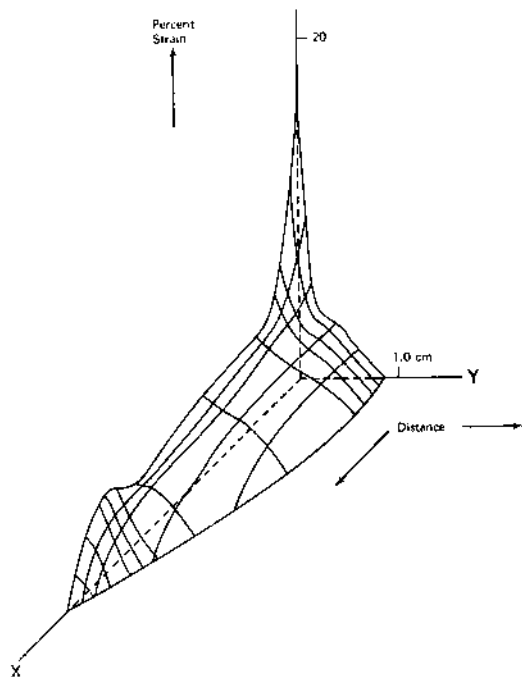

FIG. 10. Strain map in impacted fabric of ballistic nylon, 40.67 μsec after 290 m/sec impact.

in Figure 9 can be seen as the part of the map in Figure 10 directly above the *x*-axis.

Similar maps can be produced for all of the program variables at each time after impact, and indeed this would be a good way to do parametric studies of the effect of materials and structural variations. But for intuitive grasp of the nature of fabric impact, the energy map shown in Figure 11 is especially valuable by itself. The vertical axis here represents the total energy of that element of fabric directly below; this total is the sum of fiber strain energy and kinetic energy. (The numerical values are somewhat arbitrarily normalized on the basis of the mass of fabric in the 20 × 20 cm test panel used in the experimental shots.) It is clearly shown here that the vast majority of ballistic energy is deposited in the orthogonal fibers running through the impact point, while the rest of the fibers are essentially ineffective. This may well be a finding of considerable importance in the design of impact-resistant fabric structures. If by special weaving techniques or use of isotropic felts one can distribute the energy more equitably, an improvement in ballistic performance might be expected.
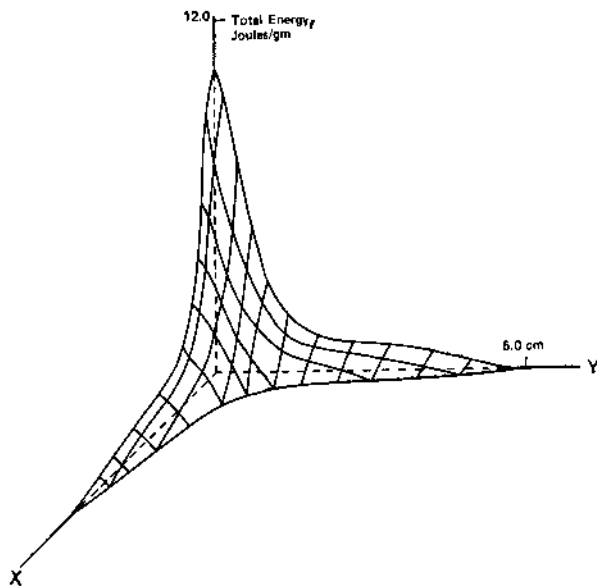

FIG. 11. Energy map, same conditions as Fig. 10.

Figure 12 shows the variation with time of the energy contributions making up the energy map of Figure 11. The curve for *y*-kinetic energy (that portion of the scalar kinetic energy arising from the *y*-component of the velocity vector) is identical to the *x*-component. For this case, again the impact of ballistic nylon at 290 m/sec, the kinetic components are twice as large as the strain energy in the fabric.

Summary and Conclusion

The primary purpose of the above discussion is to demonstrate the feasibility of a unified analytic-experi-
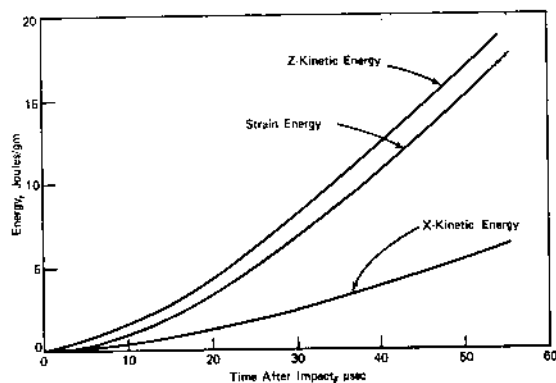

FIG. 12. Energy partition in impacted fabric vs time; ballistic nylon at 290 m/sec.

mental study of ballistic impact in textile structures. The single fiber work is highly valuable as a means of determining the elemental response of the material, the

determination of the dynamic stress-strain curves being an important example. But as seen in the results from the fabric code, the materials properties and the fabric geometry combine to produce a structural response which cannot be determined from a knowledge of the single-fiber properties alone.

While the results of the fabric code are not so rigorously accurate as to obviate the necessity for experimental confirmation, they are extremely useful in developing an intuitive feel for the nature of fabric impact. This understanding is a great aid in limiting the number of parameters which must be included in an experimental test matrix.

In the interest of space, this paper has presented only characteristic results which lead to this understanding of fabric impact. Work is in progress to complete a computer parametric study of the detailed effects of various materials properties, namely, modulus, tenacity, breaking strain, and fracture energy. It is expected that none of the qualitative features reported in this paper will be altered in this detailed study, but only variations of a quantitative nature.

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