Penetration Mechanics of Textile Structures: Influence of Non-Linear Viscoelastic Relaxation

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A numerical simulation of ballistic impact and penetration on woven textile panels is described which can easily incorporate a wide variety of realistic constitutive and fracture models. The use of this model in assessing viscoelastic relaxation effects is illustrated, and is further extended to include non-linear viscoelastic effects. Since a variety of non-linear models is presently available and there is insufficient evidence to indicate the superiority of any single one in this instance, the Eyring non-linear model was chosen arbitrarily to indicate the ease with which these models may be implemented into the numerical treatment. The results obtained using the non-linear model are compared with comparable computer experiments using linear elastic and linear viscoelastic models.

INTRODUCTION

Textile structures have been used to provide protection against ballistic threats since the Second World War, with the development then of flak jackets for aircraft crewmen. Now used widely by military and police personnel, these devices have been constructed principally of ballistic nylon or impregnated fiberglass. In recent years, however, improved devices have been developed using aramid fibers (DuPont's Kevlar® 29 or 49), and these are being considered for such additional applications as aircraft engine rotor-blade burst containment. Development and design of these devices has been largely empirical, and considerable effort has been expended to develop rational analytical tools which may be used in design, or at least in improving the designer's intuition.

Although closed-form mathematical analyses can be applied to the initial ballistic response of a single fiber (1), late-time effects arise due to stress wave interactions and reflections which make such closed-form analyses intractable. In the case of woven panels, each fiber crossover acts to reflect a portion of the stress wave which is propagating outward from the impact point, so here closed-form treatments are completely inapplicable. The complexity of these phenomena have resulted in the development in our laboratory of a series of computer codes, and these numerical treatments have proven to be of great value in understanding the ballistic event. These codes do not involve the idealizing approximations needed in many other treatments, such as modeling the woven panel as a membrane, so that the user is able to proceed directly from fiber material properties, weave geometry, projectile impact velocity, etc.

NUMERICAL ANALYSIS OF TEXTILE IMPACT

Computational Scheme

The computer method used in the present analysis of textile impact is an outgrowth of a technique pioneered by Davids, *et. al.* (2) and applied successfully to a variety of wave propagation problems. This approach, which is similar in final form to finite-difference analysis but markedly different in derivation, was first used by Lynch (3) to analyze transverse impact of single fibers and later extended by Roylance, *et al.* to the study of viscoelastic fiber impact (4) and impact of woven textile panels (5). As indicated in *Fig. 1*, the woven panel is first idealized as an assemblage of pin-jointed, flexible fiber elements, each having a mass which makes the areal density of the idealized mesh equal to that of the panel being simulated. The initial projectile velocity is imposed on the node at the impact point, which causes a strain to develop in the adjacent elements. The tension resulting from this strain is computed from the constitutive law, and this tension is used to calculate an acceleration in the

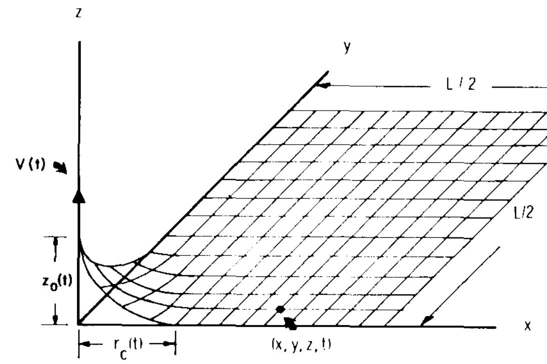

Fig. 1. Idealization of fabric mesh for numerical analysis.

neighboring elements. The computer proceeds outward from the impact point in this manner, using a momentum-impulse balance, a strain-displacement condition, and a constitutive equation to compute for each element the current values of tension, strain, velocity, position and such ancillary but important quantities as strain energy and kinetic energy. At the end of these calculations, a new projectile velocity is computed from the tensions acting on the projectile from the fibers, and the process is repeated for a new increment of time.

In the development of dynamic codes of this sort, due attention must be given to matters of numerical stability and accuracy. Stability—the absence of spurious numerical oscillations which grow without bound during the computations—is insured by selecting the time increment to match the time needed for the propagating stress waves to traverse the length of the finite element (6). Stability can be monitored by comparing the total energy stored in and dissipated by the fabric with the kinetic energy lost by the projectile. Accuracy is more difficult to insure, since no closed-form analytical solutions of this problem exist for comparison. (The degenerate case of a single fiber impacted transversely does agree closely with closed-form analyses, however). As a means of demonstrating accuracy, therefore, several computer experiments were performed which simulated actual ballistic firing experiments for which high-speed photographic data was available (7). *Figure 2* shows a typical comparison of this type, where here

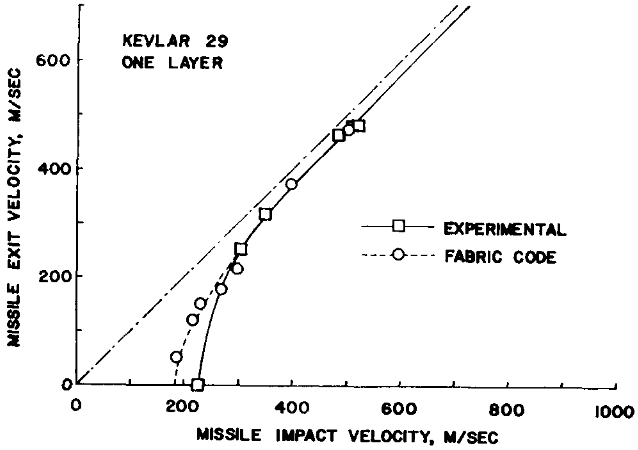

Fig. 2. Residual velocity after penetration: comparison of experimental and numerical results.

impact on a single layer of Kevlar 29 was simulated. Very crude approximations for the material properties were used here: the modulus was assumed to remain constant with strain and was set at the value obtained by quasi-static tension tests, and the failure criterion used was a simple maximum-strain condition. In spite of these idealizations, good agreement with experiment was obtained.

Modeling of Dynamic Material Properties

In spite of the good agreement with experiment obtained using only very simple idealizations of material properties, one would expect that for improved insight to the penetration process a proper simulation of dynamic material properties would be necessary. In particular, two classes of material response must be modeled: constitutive and fracture. Both of these can be expected to be dependent on both time and temperature in the case of polymeric textiles. Brief descriptions of currently employed models will be given here.

The numerical algorithm is finally terminated by simulated rupture of the fibers. Since the strain and tension histories are computed for each element in the mesh, a variety of failure criteria may be easily incorporated. The use of Eyring-type rate process fracture criteria (8) are particularly attractive, since they are computationally convenient and still provide good simulation of time and temperature effects. A simple but very useful such criterion is that due to Zhurkov, who states that the lifetime τ of a solid subjected to a constant stress σ is:

$$\tau = \tau_o \exp\left(\frac{U - \gamma\sigma}{kT}\right) \tag{1}$$

where k is Boltzman's constant and T is the absolute temperature. τ_o, U and γ are material constants related to the dissociation kinetics of the atomic bonds and the internal defect structure of the material. For time-varying stresses and/or temperatures, one may assume superpossibility and write Zhurkov's equation in the form

$$\int_o^\tau \frac{dt}{\tau_o \exp\left\{\dfrac{[U - \gamma\,\sigma(t)]}{kT(t)}\right\}} = 1 \tag{2}$$

In the present treatment, the current value of the above integral is computed at each node. The time and location of rupture is determined when the integral value reaches unity at any node.

In the course of the iterative calculations, a constitutive material law must be evoked at each element in order to compute the element tension from its strain history. One would expect that a model incorporating viscoelastic effects would be necessary for proper simulation of polymeric structures. With nylon fibers, for instance, there is considerable evidence that relaxation does indeed occur in the ballistic time frame (9). This is expected in light of the dynamic mechanical spectrum of nylon, in which a beta relaxation is observed having an apparent activation energy of approximately 60 KJ/mole (10); this relaxation is calculated to occur in approximately five microseconds at room temperature.

A general viscoelastic model well suited for computing tensions from prescribed strains is the Wiechert model, depicted schematically in *Fig. 3*. This model takes the polymer response to be that of the shown array of Newtonian dashpots of viscosity η_j and Hookean springs of stiffness k_j. The differential tension-strain law for the jth arm of the model is

$$\dot{\epsilon} = \frac{1}{k_j}\dot{\sigma}_j + \frac{1}{\eta_j}\sigma_j \qquad (3)$$

where the dots indicate time differentiation, σ is the tensile stress and ϵ is the strain. Casting this equation in finite difference form relative to a discrete time increment Δt and solving:

$$\sigma_j{}^t = \frac{1}{[1 + (\Delta t/\tau_j)]}\,[\,k_j\,(\epsilon^t - \epsilon^{t-1}) + \sigma_j{}^{t-1}] \qquad (4)$$

where the superscripts t and t-l indicate values at the current and previous times respectively. $\tau_j = \eta_j/k_j$ is a characteristic relaxation time for the j^{th} arm. The total tension at time t is the sum of all the $\sigma_j{}^t$ plus the tension in the equilibrium spring $k_e\epsilon^t$:

$$\sigma^t = k_e\epsilon^t + \sum_j \frac{k_j\,(\epsilon^t - \epsilon^{t-1}) + \sigma_j{}^{t-1}}{1 + (\Delta t/\tau_j)} \qquad (5)$$

This tension-strain calculation is performed at each element node. In addition to storing all the k_j and τ_j, the computer must also store the previous strain and tension values at each node.

Although several spring-dashpot arms are needed to model accurately the distribution of relaxation times inherent even in a "single" polymer relaxation, valuable insight to the influence of viscoelastic effects on fabric impact may be obtained by considering the model obtained by using only one spring-dashpot arm in parallel with the equilibrium spring. This is the "standard linear solid", and it is often used for qualitative studies of viscoelastic phenomena. Typical results obtained using this model are shown in *Fig. 4*, where the model constants have been taken as representative of drawn nylon fibers (k_e = 80 gpd*, k_1 = 20 gpd, τ_1 = 5 μs). *Figure 4* shows the variation in fiber stress along the fibers running through the impact point, 30.4 μs after an impact at 300 m/s, and compares the standard linear solid material with that of a linear elastic material of the same initial modulus. The degree of relaxation λ is the fraction of initial modulus which undergoes relaxation; it is equal to 0.2 for the standard linear solid for the above choice of model constants, and is equal to zero for elastic materials. Note that the stress distribution even for elastic materials is non-uniform, as a result of wave reflection and interaction from fiber crossovers. Also shown in *Fig. 4* is the ratio of the viscoelastic to the elastic stresses (denoted T/T_o); the variation of this parameter with distance shows that the effect of relaxation is most dominant near the wavefront, and assumes a stable value less than unity at distances far behind the traveling wave.

* gpd, or grams per denier, is a commonly used but non-SI unit which is very convenient in wave propagation calculation. The conversion to SI units is: 1 gpd = (88.2ρ) MPa, where ρ is the density in Mg/m^3.

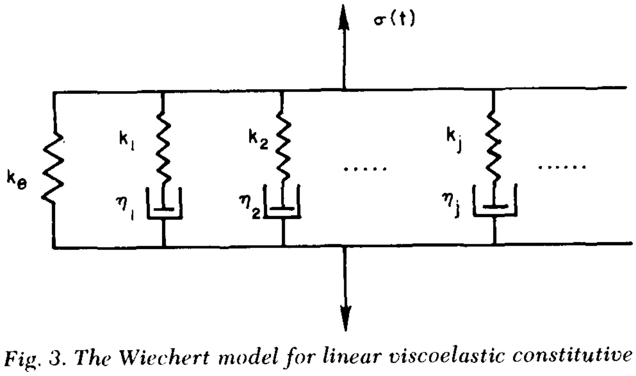

Fig. 3. The Wiechert model for linear viscoelastic constitutive response.

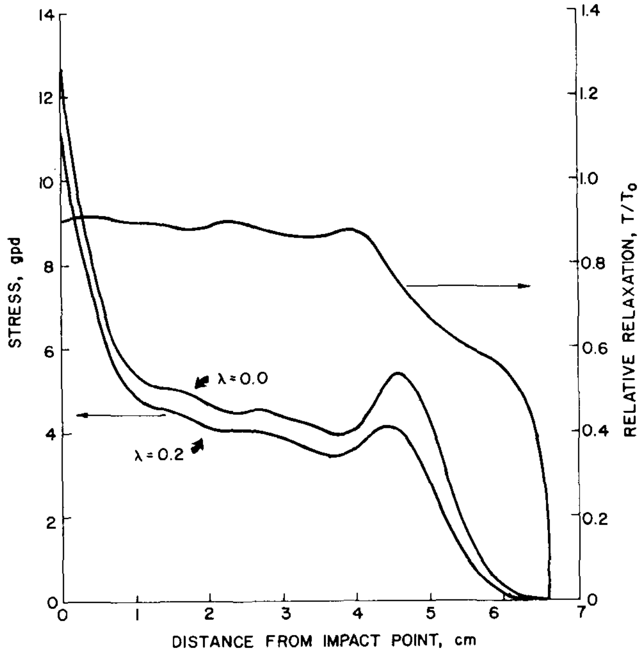

Fig. 4. Stress distributions along orthogonal fibers passing through impact point: comparison of linear elastic and linear viscoelastic fibers.

NON-LINEAR VISCOELASTIC RELAXATION

Although the linear viscoelastic models described in the previous section are useful for illustrating the general features of relaxation on the penetration process, they are certainly not rigorously applicable to the types of materials presently used in impact-resistant textile structures. In fact, although most polymeric materials are time-dependent to various degrees, very few, if any are linear, even at low strains. Beyond stating that linearity is not observed, however, there does not exist at the present time any single generally accepted means of describing nonlinear effects. The reader is referred to the recent books by Ward (10) and Findley, *et al.* (11) for reviews of the several techniques currently in use.

One strong advantage of the direct numerical analysis presented here is the ease with which various constitutive models can be imcorporated: they are employed as a library of subroutines, and no code revision is necessary in order to incorporate new models. As a means of demonstrating this flexibility, a somewhat arbitrarily-selected nonlinear viscoelastic constitutive model has been incorporated. The implementation of this model will be described briefly, and the results obtained there-from will be compared with those of the linear elastic and linear viscoelastic models.

The Wiechert model can be extended to include the effect of material nonlinearity by rendering the spring and/or dashpot elements non-linear. If for instance one uses a power-law spring and Eyring dashpot defined as

$$\sigma = k\epsilon_s^b,\ \dot{\epsilon}_d = A \sinh(\alpha\sigma) \qquad (6)$$

Then the finite-difference equation relating tensions and strains in the j^{th} arm of the model is:

$$\frac{\epsilon^t - \epsilon^{t-1}}{\Delta t} = \frac{1}{b_j k_j}\left(\frac{\sigma_j^t}{k_j}\right)^{\frac{1}{b}-1}\left(\frac{\sigma_j^t - \sigma_j^{t-1}}{\Delta t}\right) + A \sinh(\alpha\sigma_j^t) \qquad (7)$$

A relation such as this requires an iterative numerical solution for σ_j^t at each element and at each time step; the computer effort is increased but the principles of the impact algorithm are straightforward. The principal obstacle to the use of non-linear models in impact analysis is not the incorporation of the models into the solution scheme, but rather the determination of the material parameters (the b's, k's, A's and α's in the above equation) applicable in the microsecond time scale of polymer relaxations.

To illustrate the effect of non-linear constitutive models on panel ballistic response, a series of computer experiments was performed on three different simulations of nylon fabric: one using only linear elastic response (only the equilibrium spring in the Wiechert model), one using the "standard linear solid" model for linear viscoelastic response (the equilibrium spring plus one dashpot arm), and the last being a standard linear solid but with the dashpot made a non-linear Eyring element. The initial modulus was taken as 100 gpd, the relaxed modulus as 80 gpd, and the relaxation time for the standard linear solid as five microseconds. The concept of relaxation time (time to complete 63.2 percent of the total relaxation) has no meaning for the non-linear element since the rate of relaxation changes non-linearly with the stress so the A and α were arbitrarily chosen so as to cause relaxation in approximately the same time scale ($10^3\ s^{-1}$ and 0.1 den/gm respectively). The non-linear constitutive equation was solved by using Muller's method (12), which increased the computation time by roughly one third.

Figure 5 shows the stress distribution along the orthogonal fibers through the impact point for the non-linear viscoelastic and linear elastic fabrics, 20 μs after a 300 m/s impact. As in *Fig. 4*, this figure also depicts the variation in the parameter T_α/T_o, the ratio of the viscoelastic to elastic stress. The forms of this parameter for non-linear and linear viscoelastic fabrics are compared in *Fig. 6*. Although the equilibrium value far from the wavefront is approximately the same in both cases, strong differences are evident near the wavefront. These are due to the relatively more rapid response to higher strain gradients in the non-linear material.

Another important indicator of viscoelastic fabric response is the stress at the point of impact. The stress and strain at the impact point increase with time due to the

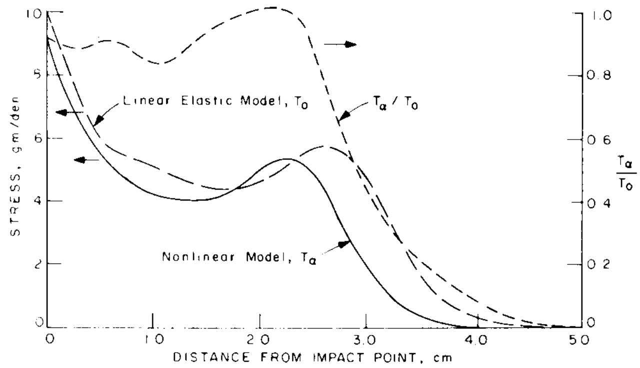

Fig. 5. Stress distribution along orthogonal fibers passing through impact point: comparison of linear elastic and nonlinear viscoelastic fabrics.

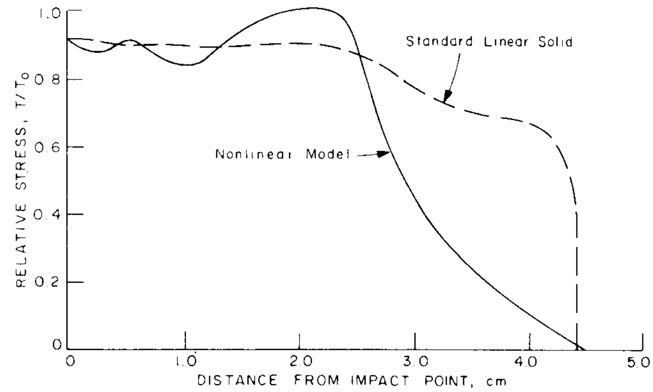

Fig. 6. Stress relaxation along orthogonal fibers passing through impact point, relative to elastic material: comparison of linear and nonlinear viscoelastic fabrics.

continual arrival there of wavelets reflected from fiber crossovers, but in viscoelastic materials both stress relaxation and creep strain are superimposed on this overall increase. In *Fig.* 7 are plotted the stress histories for the three material models, as well as the stress relaxation ratio defined before as the ratio between the viscoelastic and elastic stress. As in the earlier two figures, the linear and non-linear viscoelastic models approach essentially identical equilibrium values at long time, but are markedly different near the initial disturbance due to the more rapid response of the non-linear material to large gradients.

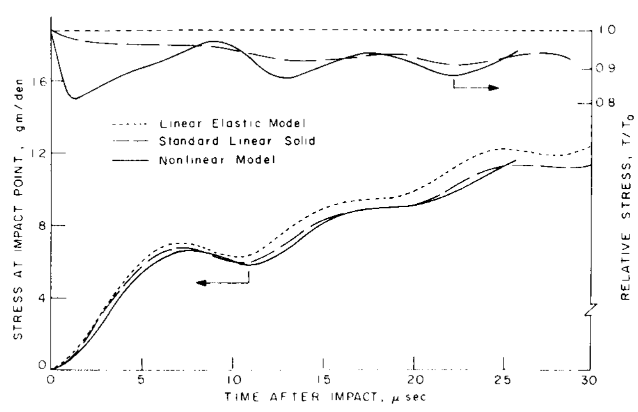

Fig. 7. Stress history at point of impact: comparison between linear elastic, linear viscoelastic, and nonlinear viscoelastic fabrics.

CONCLUSION

The results above demonstrate that non-linear viscoelastic constitutive models can be incorporated into numerical analyses of textile impact with no real difficulty, and that the results are appreciably different than those obtained by linear viscoelastic models. However, it should be emphasized that these specific results were obtained from an arbitrary selection of both the non-linear model and its numerical parameters, so they must be regarded as illustrative only. With this analytical tool in hand, one can now turn to the task of inferring suitable constitutive models and numerical parameters from appropriate experiments.

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