THE ECONOMIC AND POLICY CONSEQUENCES OF CATASTROPHES*

by

Robert S. Pindyck
Massachusetts Institute of Technology

Neng Wang
Columbia University

This draft: January 14, 2011

Abstract: What is the likelihood that the U.S. will experience a devastating catastrophic event over the next few decades that would substantially reduce the capital stock, GDP and wealth? Can the probability and possible impact of such an event be inferred from economic data? And how much should society be willing to pay to reduce the probability or impact of a catastrophe? We address these questions using a general equilibrium model that describes production, capital accumulation, and household preferences, and includes the possible arrival of catastrophic shocks. Calibrating to economic and financial data shows the annual mean arrival rate of shocks to be about 1.5% and the expected loss from a shock to be about 30%. We use the model to calculate the tax on consumption society would accept to reduce the probability of a shock, and the cost to insure against its actual impact.

JEL Classification Numbers: H56; G01, E20

Keywords: Catastrophes, disasters, rare events, economic uncertainty, consumption tax, catastrophe insurance, national security.

---

*We thank Ben Lockwood and Jinqiang Yang for their outstanding research assistance, and Robert Barro, Patrick Bolton, Hui Chen, Pierre Collin-Dufresne, Chaim Fershtman, Itzhak Gilboa, François Gourio, Dirk Krueger, Lars Lochstoer, Greg Mankiw, Jim Poterba, Julio Rotemberg, Suresh Sundaresan, three anonymous referees, and seminar participants at Columbia, M.I.T., the NBER, and Tel-Aviv University for helpful comments and suggestions.

1 Introduction.

What is the likelihood that the U.S. will experience a devastating catastrophic event over the next few decades? Can the probability and possible impact of such an event be inferred from the behavior of economic variables such as investment, interest rates, and equity prices? And how much should society be willing to pay to reduce the probability or likely impact of a catastrophic event, or to insure against its actual impact should it occur?

By "catastrophic event," we mean something national or global in scale that would substantially reduce the capital stock and/or the productive efficiency of capital, thereby substantially reducing GDP, consumption, and wealth. Examples that we have in mind (you can come up with your own) include a nuclear or biological terrorist attack (far worse than even 9/11), a highly contagious "mega-virus" that spreads uncontrollably, a global environmental disaster, or a financial and economic crisis on the order of the Great Depression. Unlike more locally contained events such as Hurricane Katrina or the Asian tsunami, as terrible as they were, the events of concern to us would destroy part of the country's (or the world's) physical and/or human capital and raise future costs of operating the remaining capital.[1]

Our approach to analyzing the economics of catastrophes differs considerably from the existing literature. We do not try to estimate the mean arrival rate and impact distribution of catastrophic events from historical data, nor do we use the estimates of others. Instead, we develop an equilibrium model of the economy and estimate these characteristics as a calibration *output* of our analysis. In effect, we are assuming that the calibrated characteristics of catastrophes are those perceived by firms and households, in that they are consistent with behavior, and thus with the data for key economic variables.[2]

Behavioral reactions to possible catastrophic events depend in part on preferences. Like

[1]Those readers who are incurable optimists and/or have limited imaginations should read Posner (2004), who provides additional examples and argues that society fails to take these risks sufficiently seriously, and Sunstein (2007). For a sobering discussion of the likelihood and possible impact of nuclear terrorism, see Allison (2004). In an excellent review article of Posner's book, Parson (2007) points out the need for a general cost-benefit framework to address these risks in a consistent way.

[2]In related work, Russett and Slemrod (1993) used survey data to show how beliefs about the likelihood of nuclear war affected savings behavior, and argue that such beliefs can help explain the low propensity to save in the U.S. relative to other countries. Also, see Slemrod (1990) and Russett and Lackey (1987).

some other recent studies, we assume that households have recursive preferences, which involve three behavioral parameters: the rate of time preference, the index of relative risk aversion, and the elasticity of intertemporal substitution. There is little agreement among economists regarding the "correct" values for these parameters, but our calibration exercise provides insight into their plausible ranges and relative magnitudes.

We specify an AK model of production more general than those used by others in that it includes adjustment costs, so consumption and investment goods are not freely interchangeable and Tobin's q can exceed one. Adjustment costs are crucial, and enable us to generate endogenous consumption-investment and consumption-wealth ratios that match U.S. data.

We model catastrophes as Poisson events with some mean arrival rate, and an impact characterized by a one-parameter power probability distribution. Thus the characteristics of catastrophes are captured by two parameters. Leaving these two parameters unconstrained, we calibrate our model to fit post-war U.S. data for productivity, the consumption-investment ratio, the risk-free interest rate, the equity premium, Tobin's q, and the average real growth rate. We thereby calculate the implied characteristics of catastrophes, and also determine how those characteristics depend on preference parameters.[3] In addition, our model yields the equilibrium price of catastrophe insurance that would prevail in a market for risk trading, and to our knowledge is the first to do so for a production economy.

We find the mean annual arrival rate of catastrophes to be about .015, with a possible range (based on our sensitivity analysis) of .005 to .027. This range is fairly narrow given that catastrophes are indeed "rare" events. We also find that conditional on the index of risk aversion, the expected loss from a catastrophe is about 17 to 30 percent of the capital stock. Finally, our results yield information about the values of behavioral parameters. In particular, we provide evidence that the elasticity of intertemporal substitution is below 0.5.

We use our calibrated model to address the third question that we raised at the outset: How much should society be willing to pay to reduce the probability or likely impact of a

[3] A more general version of the model would allow the impact of a catastrophe to be temporary by assuming that following an initial drop, productivity mean-reverts to its original level. This would introduce a third parameter to the characterization of catastrophes — the rate of mean reversion. The three catastrophe parameters could still be calculated as an output of the model's calibration.

catastrophic event? We calculate a tax-based measure of "willingness to pay" (WTP). In our model a permanent tax on consumption is non-distortionary, equivalent to a lump-sum tax, and equivalent to a reduction in the current capital stock by an amount equal to the tax rate. Thus our WTP is the permanent percentage tax rate that society should be willing to accept in order to reduce the mean arrival rate of a catastrophic event from its calibrated value to a lower value. This approach lets us avoid estimating the cost of reducing the mean arrival rate, which is presumably a convex function of the size of the reduction.

The questions we address have been the focus of a growing literature, the roots of which go back to the observation by Rietz (1988) that low-probability catastrophes might explain the equity premium puzzle first noted by Mehra and Prescott (1985), i.e., could help reconcile a relatively large equity premium (5 to 7%) and low real risk-free rate of interest (0 to 2%) with moderate risk aversion on the part of households. Rietz's article received little attention until the recent work of Barro (2006, 2009) and Weitzman (2007). Barro (2006) assembled data on "consumption disasters," defined as reductions in real GDP of 15% or more, for a panel of 35 countries over the past century. He estimated the Poisson arrival rate of such events (just under 2% per year) and the distribution of the drop in GDP. Using a pure exchange model of the economy similar to that of Rietz, Barro showed that these numbers are roughly consistent with the observed equity premium and real risk-free rate in the U.S.[4]

Barro (2009) extended his earlier work by generalizing his model to include an AK production technology and Epstein-Weil-Zin (EWZ) recursive preferences, thereby endogenizing savings and investment and disentangling the index of risk aversion from the elasticity of intertemporal substitution. Using his earlier estimates of the mean arrival rate and loss impact distribution of catastrophes, the model could again match the observed equity premium and risk-free rate. However, the model's calibration is inconsistent with other basic economic variables. For example, it predicts a consumption-investment ratio of about 1:3, instead of matching the roughly 3:1 ratio in the data. Also, because consumption and investment

[4]Concurrently, Weitzman (2007) showed that the equity premium and real risk-free rate puzzles could be explained by "structural uncertainty" in which one or more key parameters, such as the true variance of equity returns, is estimated through Bayesian updating.

goods are freely interchangable in his model, Tobin's q is (unrealistically) always one.

Several authors have extended Barro's work. Gourio (2008) used an exchange economy model with recursive preferences but allowed disasters to have limited duration. He found that the effect of recoveries on the equity premium could be positive or negative, depending on the elasticity of intertemporal substitution. Gabaix (2008) and Wachter (2008) showed that a time-varying Poisson arrival rate could explain the high volatility of the stock market (in addition to the equity premium and real risk-free rate).

Other studies have sought improved estimates of the event arrival rate and impact. For example, Barro and Ursúa (2008) exploit an extended dataset based on consumption instead of GDP, and Barro, Nakamura, Steinsson, and Ursúa (2009) estimate a more general model that accounts for recoveries. While these studies provide a better understanding of the characteristics of historical "consumption disasters," they are limited in two respects. First, many of the included disasters are manifestations of three global events — the two World Wars and the Great Depression. Second, the possible catastrophic events that we think are of greatest interest today have little or no historical precedent — there are no data, for example, on the frequency or impact of nuclear or biological terrorist attacks.

Consider the forty-year period beginning around 1950 and ending with the breakup of the Soviet Union. During that time there was one potential catastrophic event that dominated all others: the possibility of nuclear war between the U.S. and the Soviet Union. The fear of such of an event was based partly on the possibility of a mistake: One side might see something threatening on a radar screen, and, unable to get sufficient reassurance from a phone call, launch its own missiles. What was the likelihood of such an event and the probability distribution for its impact? Although the Department of Defense, the RAND Corporation, and others did studies to address these questions, there was no historical precedent on which to base estimates. Thus we take a very different approach and ask what event arrival rate and impact distribution are implied by basic economic data.

In the next section we lay out a general equilibrium model that incorporates catastrophic shocks, discuss its solution, and explain how its calibration yields information about shocks. Section 3 shows calibration results, and discusses their implications for the nature of catastrophic shocks, and for household preferences. Section 4 discusses our application of the model to policy analysis, and in particular, the calculation of WTP. Section 5 concludes.

2 Framework.

We construct a general equilibrium model in which: (i) a representative consumer has recursive preferences; (ii) output is given by an AK technology; (iii) investment involves adjustment costs reflecting the expense and time needed to install capital, so $q \neq 1$; (iv) catastrophic shocks are Poisson arrivals, and cause the loss of a random fraction of the capital stock. Despite its generality, the model yields closed-form solutions for equilibrium allocations and pricing.

2.1 Building Blocks.

Preferences. We use the Duffie and Epstein (1992) continuous-time version of EWZ preferences, so that a representative consumer has homothetic recursive preferences given by:

$$V_t = \mathcal{E}_t \left[\int_t^{\infty} f(C_s, V_s) ds \right] \ , \tag{1}$$

where

$$f(C, V) = \frac{\rho}{1 - \psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^{\omega}}{((1-\gamma)V)^{\omega-1}} \ . \tag{2}$$

Here ρ is the rate of time preference, ψ the elasticity of intertemporal substitution (EIS), γ the coefficient of relative risk aversion, and $\omega = (1-\psi^{-1})/(1-\gamma)$. Unlike time-additive utility, recursive preferences separate risk aversion from the elasticity of intertemporal substitution.[5]

Note that if $\gamma = \psi^{-1}$ so that $\omega = 1$, we have the standard constant-relative-risk-aversion (CRRA) expected utility, represented by additively separable aggregator:

$$f(C, V) = \frac{\rho C^{1-\gamma}}{1-\gamma} - \rho V. \tag{3}$$

One of the questions we address is whether γ is close to ψ^{-1}, so that the simple CRRA utility function is a reasonable approximation for modeling purposes. In particular, we examine how

[5]Note that the marginal benefit of consumption is $f_C = \rho C^{-\psi^{-1}}/[(1-\gamma)V]^{\omega-1}$, which depends not only on current consumption but also (through V) on the expected trajectory of future consumption.

equilibrium allocation and pricing constrains the model's parameters, including the elasticity of intertemporal substitution and index of risk aversion.

Production. Aggregate output has an AK production technology:

$$Y = AK \ , \tag{4}$$

where A is a constant. Here K is the *total* stock of capital; it includes physical capital as traditionally measured, but also human capital and firm-based intangible capital.

Catastrophic Shocks. We model catastrophic shocks as Poisson arrivals with mean arrival rate λ. There is no limit to the number of catastrophic shocks; the occurrence of a shock does not change the likelihood of another.[6] When a catastrophic shock does occur, it permanently destroys a stochastic fraction $(1 - Z)$ of the capital stock K, where the remaining fraction, Z, follows a well-behaved probability density function (pdf) $\zeta(Z)$ with $0 \leq Z \leq 1$. By well-behaved, we mean that the moments $\mathcal{E}(Z^n)$ exist for $n = 1$, $1 - \gamma$, and $-\gamma$. As we will see, these are the only moments of Z that are relevant for our analysis.

Investment and Capital Accumulation. Letting I denote aggregate investment, the capital stock K evolves as:

$$dK_t = \Phi(I_t, K_t)dt + \sigma K_t dW_t - (1 - Z)K_t dJ_t \ . \tag{5}$$

Here the parameter σ captures diffusion volatility, W_t is a standard Brownian motion process, and J_t is a jump process with mean arrival rate λ that captures catastrophic events; if a jump occurs, K falls by the random fraction $(1 - Z)$. The adjustment cost function $\Phi(I, K)$ captures effects of depreciation and costs of installing capital and making it productive. We assume $\Phi(I, K)$ is homogeneous of degree one in I and K and thus can be written as:

$$\Phi(I, K) = \phi(i)K \ , \tag{6}$$

[6]Shocks to capital have been widely used in the growth literature with an AK technology, but unlike the existing literature, we examine the economic effects of shocks to capital that involve discrete (catastrophic) jumps. See Jones and Manuelli (2005) for a survey of endogenous growth models with with stochastic AK technology, and Greenwood, Hercowitz and Krusell (2000) for a related study of the role of investment-specific technological change over business cycles.

where $i = I/K$ and $\phi(i)$ is increasing and concave. Unlike other models of catastrophes, we explicitly account for the effects of adjustment costs on equilibrium price and quantities.[7]

2.2 Equilibrium.

Our recursive formulation of the economy requires that markets are effectively complete, which can be achieved via dynamic trading. The following securities are therefore needed at each point in time: (i) a risk-free asset, (ii) a tradeable claim on the value of capital of the representative firm, and (iii) insurance claims for catastrophes with every possible recovery fraction Z. Because our model allows for catastrophic jumps in the capital stock (unlike typical production-based equilibrium models), market completeness requires that agents can trade insurance claims on every possible catastrophic event with recovery fraction Z.

Catastrophic Risk Insurance. Catastrophic risk can be traded through the use of catastrophic insurance swaps (CIS), defined as follows. A CIS for the survival fraction in the interval $(Z, Z + dZ)$ is a swap contract in which the buyer makes a continuum of payments $p(Z)dZ$ to the seller and in exchange receives a lump-sum payoff if and only if a catastrophe with survival fraction in $(Z, Z + dZ)$ occurs. That is, the buyer stops paying the seller if and only if the defined catastrophic event occurs and collects one unit of the consumption good as a payoff from the seller. Note the close analogy between our CIS and the widely used credit default swap (CDS) contracts. Unlike the pricing models for CDS contracts, ours is a general equilibrium model with an endogenously determined risk premium.

Competitive Equilibrium. We define the recursive competitive equilibrium as follows: (1) The representative consumer dynamically chooses investments in the risk-free asset, risky equity, and various CIS claims to maximize utility defined in (1) by taking the equilibrium prices of all assets and investment/consumption goods as given. (2) The firm maximizes its market value, which is the present discounted value of future cash flows, using the equilibrium stochastic discount factor. (3) All markets clear. In particular, (i) the net supply of the risk-

[7] Homogeneous adjustment cost functions are analytically tractable and have been widely used in the investment/q-theory literature. Hayashi (1982) showed that with homogenous adjustment costs and perfect capital markets, marginal and average q are equal. Jermann (1998) integrates this type of adjustment costs into an equilibrium business cycle/asset pricing model.

free asset is zero; (ii) the demand for the claim to the representative firm is equal to unity, the normalized aggregate supply; (iii) the net demand for the CIS of each possible recovery fraction Z is zero; and (iv) the goods market clears, i.e., $I_t = Y_t - C_t$ at all $t \geq 0$.

These market-clearing conditions are standard in the equilibrium pricing literature. When all markets are available for trading by investors and firms, the prices of claims such as the risk-free asset and CIS claims are at levels implying zero demand in equilibrium. With these conditions, we can invoke the welfare theorem to solve the social planner's problem and obtain the competitive equilibrium allocation, and then use the representative agent's marginal utility to price all assets in the economy. We emphasize that CIS insurance markets are crucial to dynamically complete the markets. This is a fundamental difference from models based purely on diffusion processes without catastrophic risk.

In the next section, we summarize the solution of the model via the social planner's problem, leaving the details of the solution to Appendix A. In Appendix B, we derive the decentralized competitive market equilibrium and show that it yields the same solution.

2.3 Model Solution via the Social Planner's Problem.

The Hamilton-Jacobi-Bellman (HJB) equation for the social planner's allocation problem is:

$$0 = \max_{C} \left\{ f(C,V) + \Phi(I,K)V'(K) + \frac{1}{2}\sigma^2 K^2 V''(K) + \lambda \mathcal{E}\left[V(ZK) - V(K)\right] \right\} , \qquad (7)$$

where $V(K)$ is the value function and the expectation is with respect to the density function $\zeta(Z)$ for the survival fraction Z. We have the following first-order condition for I:

$$f_C(C,V) = \phi'(i)V'(K) \ . \qquad (8)$$

The left-hand side of eqn. (8) is the marginal benefit of consumption and the right-hand side is its marginal cost, which equals the marginal value of capital $V'(K)$ times the marginal efficiency of converting a unit of the consumption good into a unit of capital, $\phi'(i)$.

We conjecture that the value function is homogeneous and takes the following form:

$$V(K) = \frac{1}{1-\gamma}\left(bK\right)^{1-\gamma} , \qquad (9)$$

where b is a coefficient determined as part of the solution. Let $c = C/K = A - i$. (Lower-case letters in this paper express quantities relative to the capital stock K.) Appendix A shows that b is related to the equilibrium level of investment by:

$$b = b(\lambda) = (A - i^*)^{1/(1-\psi)} \left(\frac{\rho}{\phi'(i^*)}\right)^{-\psi/(1-\psi)} . \tag{10}$$

For clarity, we have made the dependence of b on the catastrophe likelihood λ explicit by denoting it as $b(\lambda)$. The key determinant of b is the equilibrium investment-capital ratio i^*, which solves the following non-linear equation:

$$A - i = \frac{1}{\phi'(i)} \left[\rho + (\psi^{-1} - 1)\left(\phi(i) - \frac{\gamma\sigma^2}{2} - \frac{\lambda}{1-\gamma}\mathcal{E}\left(1 - Z^{1-\gamma}\right)\right)\right] . \tag{11}$$

Note that in equilibrium, the optimal investment-capital ratio $I/K = i^*$ is constant.

Consider the special case of no adjustment costs as in Barro (2009), for which our adjustment cost function becomes linear and is given by $\phi(i) = i - \delta$, where δ can be interpreted as the expected rate of stochastic depreciation. It is straightforward to show that

$$i = \delta + \psi \left[A - \delta - \rho + (\psi^{-1} - 1)\left(\frac{\gamma\sigma^2}{2} + \frac{\lambda}{1-\gamma}\mathcal{E}\left(1 - Z^{1-\gamma}\right)\right)\right] . \tag{12}$$

Investment in this special case depends on $A - \delta - \rho$. Thus the introduction of adjustment costs in our model lets us separate the effects of A from those of δ and the subjective discount rate ρ, in addition to generating rents for capital with $q \neq 1$.

Equilibrium capital accumulation in our model is given by:

$$dK_t/K_t = \phi(i^*)dt + \sigma dW_t - (1 - Z)dJ_t \ , \tag{13}$$

where i^* is the solution of eqn. (11). Let g denote the expected growth rate (conditional on no catastrophic shocks), i.e., $g = \phi(i^*)$. The expected growth rate inclusive of catastrophes is then $\phi(i^*) - \lambda\mathcal{E}(1 - Z)$, where the second term is the expected downward adjustment of capital stock growth due to possible catastrophes.

We show in the Appendix that the solution to the social planner's problem can be decentralized via the goods-market clearing condition and two first-order conditions (FOC) for the consumer and the producer:

$$i = A - c \tag{14}$$

$$q = \frac{1}{\phi'(i)} \tag{15}$$

$$c = \left[\rho + (\psi^{-1} - 1)\left(g - \frac{\gamma\sigma^2}{2} - \frac{\lambda}{1-\gamma}\mathcal{E}\left(1 - Z^{1-\gamma}\right)\right)\right] q \tag{16}$$

Eqn. (14) is simply an accounting identity that equates saving and investment. Eqn. (15) is a first-order condition for producers. Re-writing it as $\phi'(i)q = 1$, it equates the marginal benefit of an extra unit of investment (which at the margin yields $\phi'(i)$ units of capital, each of which is worth q) with its marginal opportunity cost (1 unit of the consumption good).

Similarly, eqn. (16) is a first-order condition for consumers. It equates consumption (normalized by the capital stock) to the marginal propensity to consume (MPC) out of wealth (everything in the square brackets) times q, the marginal value of a unit of capital. Note that the entire capital stock is marketable and its value is qK. What drives the MPC, c/q? Looking inside the square brackets, if $\psi = 1$, wealth and substitution effects just offset each other, and $c/q = \rho$, the rate of time preference. More generally, if $\psi < 1$, the wealth effect is stronger than the substitution effect, and hence the MPC increases with the growth rate g and decreases with risk aversion and volatility. The opposite holds if $\psi > 1$.

This equilibrium resource allocation has the following pricing implications for the risk-free asset, the equity claim, and the CIS claims.

$$r = \rho + \psi^{-1}g - \frac{\gamma(\psi^{-1}+1)\sigma^2}{2} - \lambda\mathcal{E}\left[\left(Z^{-\gamma} - 1\right) + \left(\psi^{-1} - \gamma\right)\left(\frac{1 - Z^{1-\gamma}}{1-\gamma}\right)\right] \tag{17}$$

$$rp = \gamma\sigma^2 + \lambda\mathcal{E}\left[(1 - Z)\left(Z^{-\gamma} - 1\right)\right] \tag{18}$$

$$p(Z) = \lambda Z^{-\gamma}\zeta(Z) \tag{19}$$

Eqn. (17) for the interest rate r is a generalized Ramsey rule. If $\psi^{-1} = \gamma$ so that preferences simplify to CRRA expected utility, and if there were no stochastic changes in K, the deterministic Ramsey rule $r = \rho + \gamma g$ would hold. The third term captures the precautionary savings effect under recursive preferences of continuous stochastic fluctuations in K, and the remainder adjusts for catastrophic risk. Note that the first term in the square brackets is the reduction in the interest rate due to catastrophic risk under expected utility (and does not depend on ψ). The second term gives the additional effects of catastrophic risk for non-expected utility; when $\psi^{-1} > \gamma$, catastrophic risk further lowers the equilibrium interest rate from the level implied by standard CRRA utility.

Eqn. (18) describes the equity risk premium, rp. The first term on the RHS is the usual risk premium in diffusion models (see, e.g., Breeden (1979) and Lucas (1978)), and the second term is the increase in the premium due to jumps in K. When a jump occurs, $(1 - Z)$ is the fraction of loss, and $(Z^{-\gamma} - 1)$ is the percentage increase in marginal utility from that loss, i.e., the price of risk. The jump component of the equity risk premium is given by λ times the expectation of the product of these two random variables. Note that the fraction of loss and the increase of marginal utility are positively correlated, which substantially contributes to the risk premium. (In the limiting case where the loss is close to 100%, the increase in marginal utility approaches infinity.) Also note that the risk premium depends only on the index of risk aversion, and does not depend on the EIS or rate of time preference.

Finally, eqn. (19) gives the stream of payments that the CIS buyer must make to insure against a catastrophe with loss fraction $(1 - Z)$; should that catastrophe occur, the buyer would receive one unit of the consumption good. Not surprisingly, the higher the arrival rate of a catastrophe with survival fraction Z, $\lambda\zeta(Z)$, the higher the corresponding CIS payment. The multiplier $Z^{-\gamma}$ in eqn. (19) measures the insurance risk premium; the higher is γ and the bigger is the loss (the lower is Z), the more expensive is the insurance.

3 Calibration.

In this section we explain our calibration procedure, including the distribution for the survival fraction Z, and describe the data for the model's inputs. We present a baseline calibration and then do additional sensitivity calibrations. Lastly, we turn to the role of adjustment costs and compare our results with those of Barro (2009). This helps to show the importance of adjustment costs and the implications of certain parameter choices.

First, note that obtaining the probability and impact distribution of catastrophes as a calibration output as we do is similar to the earlier pioneering approach of Rietz (1988).[8]

[8]Rietz used the Great Depression as a reference point for arguing for the plausibility of his calibration,

Unike Rietz, our list of target moments goes beyond the equity premium and risk-free rate.[9]

3.1 The Distribution for Shocks.

The solution of the model presented above applies to any well-behaved distribution for recovery Z. We assume that Z follows a power distribution over (0,1) with parameter $\alpha > 0$:

$$\zeta(Z) = \alpha Z^{\alpha-1} \;\; ; \;\; 0 \leq Z \leq 1 \;, \tag{20}$$

so that $\mathcal{E}(Z) = \alpha/(\alpha+1)$. Thus a large value of α implies a small expected loss $\mathcal{E}(1-Z)$.

The distribution given by eqn. (20) is general. If $\alpha = 1$, Z follows a uniform distribution. For any $\alpha > 0$, eqn. (20) implies that $-\ln Z$ is exponentially distributed with mean $\mathcal{E}(-\ln Z) = 1/\alpha$. Eqn. (20) also implies that the inverse of the remaining fraction of the capital stock follows a Pareto distribution with density function $\alpha(1/Z)^{-\alpha-1}$, $(1/Z) > 1$. The Pareto distribution is fat-tailed and often used to model extreme events.

The power distribution for Z given in (20) simplifies the solution of the model. We need three moments of Z, namely $\mathcal{E}(Z^n)$ where $n = 1$, $1-\gamma$, and $-\gamma$. Eqn. (20) implies

$$\mathcal{E}(Z^n) = \alpha/(\alpha+n) \;, \tag{21}$$

provided that $\alpha + n > 0$. Since the smallest relevant value of n is $-\gamma$, we require $\alpha > \gamma$, which ensures that the expected impact of a catastrophe is sufficiently limited so that the model admits an interior solution for any level of risk aversion γ. Thus $\mathcal{E}(1-Z) = 1/(\alpha+1)$ is the expected loss if an event occurs, and $\mathcal{E}(Z^{-\gamma}-1) = \gamma/(\alpha-\gamma)$ is the expected percentage increase in marginal utility from the loss; both are decreasing in α.

Using eqn. (21), we can rewrite eqns. (16), (17), and (18) as:

$$\lambda = (\alpha+1)\left(\frac{c}{q} - r - rp + g\right) \tag{22}$$

but did not estimate the parameters of the disaster distribution. Instead he considered a range that produced reasonable values for the equity premium and the risk-free rate, given the other parameters of his model.

[9] The effort by Barro (2006) and Barro and Ursua (2008) to obtain independent evidence on the probability and size of disasters attempts to respond to a main criticism of Rietz (1988), namely that the disasters assumed in his model are too large to be plausible. See also Mehra and Prescott (1988) and more recently the discussion by Constantinides (2008).

$$r = \rho + \psi^{-1} g - \frac{\gamma(\psi^{-1}+1)\sigma^2}{2} - \lambda \left[\frac{(\psi^{-1}-\gamma)(\alpha-\gamma) + \gamma(\alpha-\gamma+1)}{(\alpha-\gamma)(\alpha-\gamma+1)} \right] \tag{23}$$

$$rp = \gamma\sigma^2 + \lambda\gamma \left[\frac{1}{\alpha-\gamma} - \frac{\alpha}{(\alpha+1)(\alpha+1-\gamma)} \right]. \tag{24}$$

Finally, for each CIS with survival fraction Z, the required payment is:

$$p(Z) = \lambda\alpha Z^{\alpha-\gamma-1}. \tag{25}$$

3.2 Baseline Calibration.

To calibrate the model we want data covering a time period that is long but relatively stable and free of significant catastrophic events. We therefore use data for the U.S. economy for 1947 to 2008 (except for Tobin's q) to construct average values of the output-capital ratio Y/K, the consumption-investment ratio c/i, Tobin's average q, the real risk-free interest rate r, the equity risk premium rp, the average real growth rate g (without accounting for catastrophes), and the diffusion volatility σ. As discussed in Appendix D, our measure of the capital stock includes physical capital, estimates of human capital, and estimates of firm-based intangible capital (e.g., patents, know-how, brand value, and organizational capital). Thus we obtain a measure of the productivity parameter $A = Y/K$ consistent with the AK production technology of eqn. (4). Likewise, our measure of investment (and GDP) includes investment in firm-based intangible capital. Unless otherwise noted, all rates are annual.

Our value of q comes from estimates by Riddick and Whited (2010), who used firm-level Compustat data for 1972 to 2006. With measurement errors and heterogeneous firms, averaging firm-level data provides a more economically sensible estimate for the q of the representative firm than inferring q from aggregate data. In particular, the firm-level q is more naturally linked to optimal investment by a representative firm, which is the micro-foundation for our model. Averaging the Riddick and Whited estimates across firms and years yields a value of 1.43 for q, which is the number we use in our baseline calibration.[10] A detailed discussion of our calibration inputs, including sources of data, is in Appendix D.

[10]Eberly, Rebelo, and Vincent (2009) also estimate q from firm-level data, but for the period 1981–2003, and for a panel restricted to large firms (the top quartile of firms in the Computstat data). Their average q is lower, only 1.3, but that is due to their exclusion of smaller firms, which tend to have higher q's.

Table 1: SUMMARY OF BASELINE CALIBRATION

Calibration inputs (Annual rates)	Symbol	Value	Calibration outputs	Symbol	Value
output-capital ratio	A	0.113	EIS	ψ	0.181
consumption-investment ratio	c/i	2.84	mean arrival rate	λ	0.015
real growth rate (no shocks)	g	0.02	distribution parameter	α	2.374
diffusion volatility	σ	0.025	expected loss	$\mathcal{E}(1-Z)$	0.296
average q	q	1.43			
risk-free interest rate	r	0.008			
equity risk premium	rp	0.066			
rate of time preference	ρ	0.02			
index of risk aversion	γ	2			

We also need values for two behavioral parameters, the index of risk aversion γ and the rate of time preference ρ. Numbers for γ used in the finance and macroeconomics literature cover a wide range, but values from 1 to 5 are generally considered reasonable. We choose the commonly used value $\gamma = 2$, and likewise set $\rho = .02$. We examine the sensitivity of our results to these choices for γ and ρ.

This leaves five unknowns: the economic variables c and i, the EIS ψ, and the two parameters describing the characteristics of catastrophic shocks, λ and α. We have five equations (14), (15), and (22) to (24), so the model is exactly identified. Also, because we solve for ψ, we can assess the approximation of expected CRRA utility (the constraint $\psi = 1/\gamma$) as a model of preferences.

Table 1 summarizes all of the inputs used in the baseline calibration, and the outputs. The calibration yields a mean arrival rate λ of 0.0150, a value for the distributional parameter α of 2.374, and a value for the EIS ψ of 0.181. This value of α implies that the mean loss $\mathcal{E}(1-Z)$ from a catastrophe is 29.6%. Also, the probability that the loss will be a fraction L or greater, i.e., the probability that $Z \leq 1-L$, is $(1-L)^{\alpha}$. Thus the probability that the loss will be at least 25% is $.75^{2.374} = .51$, at least 50% is .19, and at least 75% is .04. Finally, note that the 1.5% mean arrival rate implies substantial risk; for example, the probability that *no* catastrophe will occur over the next 50 years is only $e^{-.015\times 50} = .47$.

By comparison, Barro and Ursúa (2008) obtained higher values of λ. Using a sample of

24 countries, they estimated λ as the proportion of years in which there was a contraction of real per capital consumption of 10% or more, and for 36 countries, they based λ on contractions of real GDP. They found λ to be 0.038 (for consumption and GDP). But for the U.S. experience (which corresponds to our calibration), there were only two contractions of consumption of 10% or more over 137 years (implying $\lambda = 0.015$), and five GDP contractions (implying $\lambda = 0.036$). They also found an average contraction size (conditional on the 10% threshold, and for the international sample) of 0.22 for consumption and 0.20 for GDP.

More recently, Barro and Jin (2009) independently applied the same power distribution that we used in eqn. (20) to describe the size distribution for contractions. We obtained a value of the distribution parameter α as an output of our calibration; they estimated α for their sample of contractions. In our notation, their estimates of α were 6.27 for consumption contractions and 6.86 for GDP, implying a mean loss of about 14% for consumption and 13% for GDP.[11] As we will see, these mean loss estimates, while smaller than ours, are consistent with a larger value for the index of risk aversion γ, e.g., around 4 or 5 rather than 2.

Estimates of ψ in the literature vary considerably, ranging from the number we obtained to values as high as 2.[12] Our calibrated value of 0.181 is at the low end of the spectrum that appears in the literature. As shown below, however, changing input parameters such as ρ or γ in all cases result in values of ψ below 0.5, and the basic macro data are inconsistent with values of ψ much above 0.5. In fact, as can be seen from eqn. (16), if one's prior is that an increase in the growth rate g should cause the MPC to rise, then ψ must be less than 1.

[11] Eqn. (20) is the distribution for Z, the fraction of the remaining capital stock. It implies that $S = 1/Z$ has the distribution $f_S(s) = \alpha s^{-\alpha-1}$. Barro and Jin (2009) use data on S, conditioned on $S > 1.105$, to estimate α' for the distribution $f_S(s) = \alpha' s^{-\alpha'}$. Thus $\alpha = \alpha' - 1$.

[12] Bansal and Yaron (2004) argue that the elasticity of intertemporal substitution is above unity and use 1.5 in their long-run risk model. Attanasio and Vissing-Jorgensen (2003) estimate the elasticity to be above unity for stockholders, while Hall (1988), using aggregate consumption data, obtains an estimate near zero. Using micro and macro evidence, Guvenen (2006) attempts to reconcile the different estimates. He argues that the elasticity depends on wealth, which is much less evenly distributed than consumption, so that estimates based on aggregate consumption uncover the low of the majority of the population. Also, Yogo (2004) finds that the EIS is less than unity and insignificantly different from zero for eleven developed countries. He argues that weak instruments significantly affect the identification of the EIS through the linearized Euler equation. The Appendix to Hall (2009) provides a brief survey of estimates in the literature.

Table 2: CHANGES IN INPUT VALUES

Cases	ψ	λ	α	$\mathcal{E}(1-Z)$
Note: Unless otherwise indicated, $r = .008$, $c/i = 2.84$, $A = 0.113$, $rp = .066$, $\rho = .02$, $\gamma = 2$, $\sigma = .025$, $g = .020$, $q = 1.43$.				
BASE CASE	0.181	0.0150	2.374	.296
$\rho = .04$	0.315	0.0150	2.374	.296
$\gamma = 4$	0.122	0.0266	4.983	.167
$r = .012$	0.320	0.0013	2.041	.329
$q = 1.35$	0.041	0.0288	2.638	.275
$q = 1.50$	0.291	0.0054	2.152	.317

3.3 Alternative Inputs.

One could argue that the values of some inputs, especially ρ and γ, should differ from those we used. As a robustness check, we re-calibrated the model with changes in various inputs. The results are shown in Table 2. The first row shows the baseline calibration corresponding to the inputs in Table 1. Holding everything else constant in each case, we conduct the following experiments: increasing ρ from .02 to .04, increasing γ from 2 to 4, increasing r from .008 to .012, and changing q from 1.43 to 1.35 and 1.50.

As the table shows, our calibrated values for ψ are generally in the range of 0.1 to 0.3, and for λ are between .005 and .030, a range that is relatively narrow given that we are considering rare events. The estimates of α and thus the expected loss, however, depend strongly on the value of γ. Increasing γ to 4 causes α to approximately double, so that the expected loss falls almost in half. (The expected loss of about 17% is closer to the 13% number that Barro and Jin (2009) estimate.) The reason is that to be consistent with an equity risk premium of 6.6%, more risk aversion requires a thinner tail for the loss distribution, so that there is a lower probability of large losses (and thus smaller value of $\mathcal{E}(1-Z)$).

3.4 Catastrophic Insurance Premium.

Using eqn. (25), we can calculate the cost of insuring against any particular catastrophic risk. For example, to insure against any catastrophe that results in losing a fraction L or more of capital stock (i.e., $1 - Z \geq L$), the required payment per unit of capital is

$$\int_0^{1-L} (1 - Z)p(Z)dZ = \lambda\alpha \left[\frac{(1 - L)^{\alpha-\gamma}}{\alpha - \gamma} - \frac{(1 - L)^{\alpha-\gamma+1}}{\alpha - \gamma + 1} \right]. \tag{26}$$

Thus the required payment per unit of capital to insure against *any* size catastrophe ($L = 0$) is $\lambda\alpha \left[(\alpha - \gamma)^{-1} - (\alpha - \gamma + 1)^{-1} \right]$. Note that unlike the existing catastrophic insurance literature, our model generates the insurance premium in a general equilibrium setting.

Using our baseline calibration ($\gamma = 2$, $\lambda = .0150$, and $\alpha = 2.374$) and eqn. (26), we find the annual CIS payment to insure against any sized catastrophe to be .0693 per unit of capital. We have $A = .113$, so the total annual cost of the insurance would be $.0693Y/.113 = .613Y$, i.e., about 61% of GDP. How much of this large annual CIS payment reflects the expected loss from a catastrophe and how much is a risk premium? We first calculate the expected loss with no risk premium. The implied actuarially fair annual CIS payment is $\int_0^{1-L} (1 - Z)\lambda\zeta(Z)dZ$, which can also be found by setting $\gamma = 0$ in eqn. (26).

Using our baseline parameters and insuring against all possible losses ($L = 0$), the actuarially fair annual payment is only .0044 per unit of capital, so that the average "price" is $.069/.0044 = 15.57$ per unit of actuarially fair insurance. Of course, insuring against more limited potential losses will have different costs. For example, to insure against catastrophes that generate a loss of 25% or more (i.e. $L = 0.25$), the actuarially fair premium is only .0036 (because less insurance is purchased). However, the full annual payment only falls from .0692 to .0679, so that the "price" for such insurance increases to $.0679/.0036 = 19.0$. This insurance is more expensive because it is covering larger losses on average.

Table 3 summarizes both the CIS and actuarially fair payments to cover losses of different amounts for both $\gamma = 2$ and $\gamma = 4$. For a given level of risk aversion, the "price" of risk (the ratio of the CIS payment to the actuarially fair premium) increases with L, the lower bound of the loss fraction that is insured. For example, to insure only against catastrophes that generate a loss of 75% or more (i.e. $L = 0.75$), the cost is .0527 per unit of capital while the actuarially fair rate is only .0005, implying a price of risk of 115.

Note in Table 3 that because of the thinner tail (larger α), the actuarially fair premium

Table 3: LOSS COVERAGE AND COMPONENTS OF CATASTROPHIC INSURANCE PREMIA

	$\gamma = 2$			$\gamma = 4$		
Minimum loss covered, L	CIS	AF	CIS/AF	CIS	AF	CIS/AF
$L = 0.00$ (Full insurance)	0.0692	0.0044	15.57	0.0679	0.0044	15.29
$L = 0.25$	0.0679	0.0036	19.00	0.0638	0.0024	26.81
$L = 0.50$	0.0634	0.0019	33.82	0.0513	0.0005	104.53
$L = 0.75$ (75% or more losses)	0.0527	0.0005	114.77	0.0302	0.00002	1435.78

Note: For each amount of loss coverage, CIS is the required annual insurance payment and AF is the actuarially fair payment. $L = 0.50$ means that only losses of 50% or more are covered.

is lower at all levels of L for $\gamma = 4$ than for $\gamma = 2$.[13] However, the price of risk for larger losses is much higher with more risk aversion. For example, to insure against catastrophes with losses of 75% or more, the average price of insurance is 1,436 for $\gamma = 4$, compared to 115 for $\gamma = 2$. When only large losses are insured, greater risk aversion increases the cost of insurance proportionally more relative to the actually fair price.

3.5 The Role of Adjustment Costs.

How important are adjustment costs? To address this question and do welfare calculations, we must specify an adjustment cost function $\phi(i)$. We use a quadratic function, which can be viewed as a second-order approximation to a more general one:

$$\phi(i) = i - \tfrac{1}{2}\theta i^2 - \delta \ . \tag{27}$$

In our baseline calibration, the resulting value of θ is 10.22, which is determined by eqn. (15): $q = 1/\phi'(i) = 1/(1 - \theta i)$. In our baseline calibration, $q = 1.43$, $i + c = A$, and $c/i = 2.84$, which pins down $\theta = 10.22$.

To explore the role of adjustment costs, we first review Barro's (2009) results and then add adjustment costs to his model. Based on historic "consumption disasters," Barro estimated

[13]Note that the actuarially fair premium for full insurance ($L = 0$) is 0.44%, which is independent of γ. The total expected return on equity, $r + rp$, is the sum of three components: $g + c/q - \lambda\mathcal{E}(1 - Z)$ (see Appendix for details). In our calibration, $c/i = 2.84$ and $A = 0.113$, implying $c = 0.0836$. Substituting $q = 1.43$, $r = .008$, $rp = 0.066$ and $g = 0.02$ into the equation $\lambda\mathcal{E}(1 - Z) = .02 + 0.0836/1.43 - 0.008 - 0.066 = 0.00044$.

Table 4: Effects of Adjustment Costs

(1) Barro (2009) Parameters: $\gamma = 4$, $\psi = 2$, $\rho = .052$, $\sigma = .02$, $A = .174$, $\lambda = .017$, $\mathcal{E}(Z) = .71$, $\mathcal{E}(Z^{1-\gamma}) = 4.05$, $\mathcal{E}(Z^{-\gamma}) = 7.69$						
θ	i	c	c/i	r	q	g
0	**0.126**	**0.048**	**0.381**	**.011**	**1.00**	**.025**
4	0.062	0.112	1.806	-.025	1.33	-.046
8	0.038	0.136	3.579	-.036	1.43	-.068
12	0.027	0.147	5.512	-.041	1.47	-.078
20	0.017	0.157	9.234	-.045	1.51	-.087
(2) Our Parameters: $\gamma = 2$, $\psi = 0.181$, $\rho = .02$, $\sigma = .025$, $A = .113$, $\lambda = .015$, $\mathcal{E}(Z) = .704$, $\mathcal{E}(Z^{1-\gamma}) = 1.723$, $\mathcal{E}(Z^{-\gamma}) = 6.348$						
θ	i	c	c/i	r	q	g
0	0.0303	0.0827	2.72	.038	1.00	.025
4	0.0300	0.0830	2.76	.026	1.14	.023
8	0.0297	0.0833	2.81	.014	1.31	.021
10.22	**0.0294**	**0.0836**	**2.84**	**0.008**	**1.43**	**.020**
20	0.0282	0.0848	3.00	-0.018	2.30	.015

λ to be .017. He set $\gamma = 4$, and using an empirical distribution for consumption declines, estimated the three moments $\mathcal{E}(Z)$, $\mathcal{E}(Z^{1-\gamma})$, and $\mathcal{E}(Z^{-\gamma})$. He also set $\psi = 2$, $\rho = .052$, $\sigma = .02$, and $A = .174$. (Recall that because there are no adjusment costs, only $A - \rho$ can be identified in Barro's model.) As noted earlier, economists differ in their views about ψ, but a value of 2 is at the high end of the range that has appeared in the literature.

The first row of the top panel of Table 4 shows this calibration of Barro's model; there are no adjustment costs so capital is assumed to be perfectly liquid and $q = 1$. The calibration gives a sensible estimate of the risk-free rate r and risk premium rp, but yields a consumption-investment ratio of only 0.38, whereas the actual ratio is about 3. The rest of the top panel shows how the results change as the adjustment cost parameter θ in eqn. (27) is increased. The experiment here is to hold the structural parameters for both preferences and the technology fixed, change only θ, and then re-solve for the new equilibrium price and quantity allocations. First, as we increase θ, q increases because installed capital now earns more rents. Investment becomes more costly (given q, investment is determined by the marginal adjustment cost, i.e., $q = 1/\phi'(i)$), so i falls and $c = A - i$ increases. When

$\theta = 8$, both c/i and q roughly match the data. However, the real risk-free rate falls below -3% and the growth rate falls to around -7%. Basically, given Barro's parameter choices (particularly ψ and ρ) along with the exogenous inputs for λ and the moments of Z, the model cannot match all of the basic economic facts, even allowing for adjustment costs.

The bottom panel of Table 4 shows results using our baseline calibration, but varying the value for θ. (The boldface row corresponds to our calibrated value of θ, 10.22.) As θ increases, the cost of investing increases, so both r and g fall, and q increases because installed capital earns greater rents than newly purchased capital. Our model can match the data, but adjustment costs are crucial. Unlike the top panel, our model generates a low EIS, consistent with realistic numbers for c/i and q, as well as r and g.

4 Policy Consequences.

We now turn to the last question raised in the first paragraph of this paper: What is society's willingness to pay to reduce the probability or likely impact of catastrophic events? Our measure of WTP is the maximum *permanent* consumption tax rate τ that society would be willing to accept if the resulting stream of government revenue could finance whatever activities would permanently reduce the mean arrival rate of a catastrophe.

4.1 Willingness to Pay.

We want to determine the effect of a permanent consumption tax. Given investment I_t and output Y_t, households pay $\tau(Y_t - I_t)$ to the government and consume the remainder:

$$C_t = (1 - \tau)(Y_t - I_t) \ . \tag{28}$$

How large a tax would society accept to reduce λ to λ'? Households would be indifferent between (1) no tax and a likelihood of catastrophe λ and (2) paying a permanent tax at rate τ to reduce the likelihood to $\lambda' < \lambda$ if and only if the following condition holds:

$$V(K; \lambda', \tau) = V(K; \lambda, 0) \ , \tag{29}$$

where $V(K;\lambda,\tau)$ is the representative household's value function from eqn. (9) when consumption is taxed at rate τ. In Appendix C, we show that this condition implies the following:

$$b(\lambda';\tau) = (1-\tau)b(\lambda';0) = b(\lambda;0)\ , \tag{30}$$

where $b(\lambda;0)$ is the coefficient in the value function with no tax and is given by eqn. (10). Thus to reduce the likelihood of a catastrophe from λ to λ', households would be willing to pay a consumption tax at the following constant rate:

$$\tau(\lambda,\lambda') = 1 - \frac{b(\lambda;0)}{b(\lambda';0)}\ . \tag{31}$$

For the household, a permanent tax at rate τ is equivalent to giving up a fraction τ of the capital stock. This is because the tax is non-distortionary. The tax is proportional to output, so households' after-tax consumption is lowered by the same fraction as the tax rate in all states and in all future periods. Thus households' intertemporal marginal rate of substitution, which determines the equilibrium pricing of risk, remains unchanged for any given value of λ. (Note, however, that the equilibrium pricing and resource allocation depend on λ, and thus will change when the government uses the tax proceeds to reduce λ.) Likewise, the total value of capital (including the taxes paid to the government) is unchanged, and investment is unchanged. A fraction τ of ownership is simply transfered from households to the government. This key result follows from the recursive homothetic preferences and equilibrium property that the economy is on a stochastic balanced growth path.[14]

4.2 Tax Calculations.

Table 5 shows the maximum permanent tax rate society would accept to reduce λ from its starting value to .005 and to 0. The first row applies to our baseline calibration, for which λ is .0150 and the expected loss $\mathcal{E}(1-Z)$ is .2964. A tax rate of about 15% would be warranted to reduce λ to 0, and a tax rate of about 11% would be warranted to reduce λ to .005, which is about a third of its starting value.

[14] We focused on a tax used to reduce λ, but the results also apply if the tax is used to reduce the expected impact of a catastrophic event. By substituting α in place of λ, eqn. (31) can be used to find the maximum tax rate households would accept to increase the impact distribution parameter from α to $\alpha' > \alpha$. Note from eqn. (21) that increasing α reduces the expected loss $\mathcal{E}(1-Z)$ as well as the variance of the loss.

Table 5: Tax Calculations

Note: $A = .113$, $r = .008$, $c/i = 2.84$, $rp = .066$, $\sigma = .025$, $g = .020$, $q = 1.43$					
γ	ρ	λ	$\mathcal{E}(1-Z)$	$\lambda' = .005$	$\lambda' = 0$
2	.02	.0150	0.2964	$\tau^* = 0.108$	$\tau^* = 0.152$
2	.04	.0150	0.2964	0.110	0.156
4	.02	.0266	0.1671	0.150	0.176
4	.04	.0266	0.1671	0.153	0.181

Given that there is disagreement regarding the index of risk aversion γ and rate of time preference ρ, we recalculated the tax rates for $\gamma = 4$ and $\rho = .04$. As Table 5 shows, changing ρ has no effect on λ and $\mathcal{E}(1-Z)$, and only a miniscule effect on the tax rate τ^*. Increasing γ results in an increase in λ, but a proportionate decrease in the expected loss $\mathcal{E}(1-Z)$. (The product $\lambda\mathcal{E}(1-Z)$ is .0044 for either value of γ.) Thus the warranted tax rate to bring λ to zero (for $\rho = .02$) increases, but only from 15% to 17.6%.[15] Overall, our tax results are quite robust to variations in γ and ρ.

What justifies these tax rates, given that without a tax a catastrophe would occur only every 67 years on average, and should it occur, the expected reduction of the capital stock is about 30%? To answer this, we examine the welfare effects of catastrophic risk. First, we introduce the following risk-adjusted growth rate $\widehat{g}$:

$$\widehat{g} = g - \frac{\gamma\sigma^2}{2} - \frac{\lambda}{1-\gamma}\mathcal{E}\left(1 - Z^{1-\gamma}\right) . \tag{32}$$

Note that $\widehat{g}$ depends on the index of risk aversion, γ, but not on the EIS ψ. It is the reduction in the normal growth rate g due to continuous fluctuations and sudden drops in K.

Now suppose there is no tax. Note that $b(\lambda; 0)$ is a welfare measure in terms of certainty equivalent wealth, specifically the risk-adjusted benefit from a unit of capital. We show in the Appendix that $b(\lambda; 0)$ is related to $\widehat{g}$ and q as follows:

$$b(\lambda; 0) = \rho\left[1 + \left(\frac{\psi^{-1} - 1}{\rho}\right)\widehat{g}\right]^{\frac{1}{1-\psi}} q(\lambda; 0) . \tag{33}$$

[15] The tax to reduce λ to .005 increases by more, but when $\gamma = 4$ the proportional reduction in λ is larger.

Consider what would happen if there were no tax but λ were 0 instead of .015, i.e., catastrophic risk was costlessly eliminated. Other than λ, the "new" economy has the same structural parameter values as the old one, i.e., the preference parameters (ρ, γ, ψ) and production parameters ($A, \sigma, \alpha, \theta$ and δ) are the same as in our baseline calibration. However, the equilibrium price and quantity allocations will be different because of the different values of λ. In the new equilibrium, there will be less investment and more consumption; the investment-capital ratio falls from 0.029 to 0.0196, and q falls from 1.43 to 1.25. The lower precautionary demand for saving gives rise to a much higher interest rate; r increases from .008 to .086. However, the risk premium falls dramatically from .066 to .00125, implying that the expected return on equity only increases from 7.4% to 8.73%. Eliminating catastrophic risk reduces the "normal" growth rate g from .020 to .013, but it increases the risk-adjusted growth rate $\widehat{g}$ from .0085 to .0120. The overall welfare gain is captured by the change in b, which is about 15%, the amount of the tax.

Eliminating or reducing catastrophic risk is fundamentally different from purchasing insurance since the latter is a zero NPV financial transaction, with no gain in value (at least in an M-M world). Using tax proceeds to reduce λ is a more cost-efficient way to manage aggregate risk than purchasing insurance; it would be a positive NPV project and thus welfare enhancing if it could be done at a cost lower than the WTP. For example, if λ could be reduced to zero at an annual cost of only 7% of consumption, consumers would gain an 8% increase in certainty equivalent consumption units. In this case, the marginal benefit of a reduction in catastrophic risk would clearly outweigh the marginal cost.

Note that our cost-benefit analysis is a general equilibrium one, and is fundamentally different from the standard cost-benefit approach in which an NPV is calculated treating input prices and the cost of capital as exogenous. The standard approach is adequate for evaluating the construction of a new bridge, because while the bridge involves a change in cash flows, there is no change in the cost of capital (i.e., the pricing kernel). But when the "project" involves a major change in the economy (e.g., reducing or eliminating catastrophic risk as in our model), prices as well as cash flows change in the new equilibrium, so the "project" can only be evaluated by comparing its cost to its WTP, as we have done above.

Our model of a production economy with adjustment costs is a natural framework for general equilibrium cost-benefit analysis.

Our results have strong policy implications. A 15% tax on consumption is substantial, but is justified if the resulting revenues could be used to completely or largely eliminate the risk of a catastrophe. Naturally, the less costly it is to reduce catastrophic risk, the more desirable it is for the government to intervene. Overall, our results provide quantitative support to the claims by Posner (2004), Parson (2007), Sunstein (2007), Allison (2004) and others that the risk of a national or global catastrophe is significant, and governments should devote greater resources to reducing that risk.

5 Conclusions.

We set out to find the mean arrival rate and impact distribution of possible catastrophic events that are national or global in scale. Rather than use historical data as others have done, we calculated these event characteristics as calibration outputs from a general equilibrium model. Our baseline estimate of the mean arrival rate is .015, but we find that a reasonable range is between .005 and .025. Our estimates of the impact distribution and expected loss should a catastrophe occur depend on the index of risk aversion γ (which we take to be between 2 and 4). However, the expected losses are large: 27% to 32% if $\gamma = 2$.

Our model provides a natural benchmark to quantitatively assess public policy; it fully incorporates general equilibrium quantity and price adjustments by the private sector in anticipation of a policy intervention. We calculated as a "willingness to pay" measure the permanent tax on consumption that society would accept to reduce the annual probability of a catastrophe. We find that a tax of about 15% would be justified if the resulting revenues could be used to reduce the probability to zero. An alternative to a tax is insurance, but we have shown that the cost of insurance to cover a catastrophe of any size is very large. Using tax revenues to eliminate or reduce the likelihood of a catastrophe is especially attractive if the required revenue is less than the WTP, making the social NPV positive.

Our results also have implications for some of the behavioral parameters that are often used in macroeconomic and financial modeling. In all cases the calibrations yield values for the elasticity of intertemporal substitution that are well below 0.5. In addition, our results are generally consistent with the view that restricting preferences to expected CRRA utility is not a bad approximation for modeling purposes.

Some caveats are clearly in order. Our model is intentionally simple and stylized. For example, we solved the social planner's problem for a representative firm with an AK production technology and adjustment costs, and a representative household with rational expectations. This is equivalent to a competitive equilibrium with a large number of identical firms and identical households, with the same production technology and preferences, so that we ignore heterogeneity among firms and households. We also characterize catastrophic events in a simple way — a Poisson arrival with a constant mean arrival rate, and a permanent impact that follows a one-parameter distribution. These simplifications, however, make the model highly tractable, and provide an innovative approach to estimating the characteristics and policy implications of possible catastrophic events.

Appendix

A. Solution of Model.

Substituting the conjectured value function (9) into the consumption FOC (8) yields:

$$\rho C^{-\psi^{-1}} \frac{1}{(bK)^{(1-\gamma)(\omega-1)}} = \phi'(i)(bK)^{-\gamma} b \ . \tag{34}$$

Simplifying and using $c = C/K$, we have

$$c = \left(\frac{\rho}{\phi'(i)}\right)^{\psi} b^{1-\psi} \ . \tag{35}$$

Substituting (35) back into the HJB eqn. (7) yields eqn. (11) for the optimal i^*.

From Duffie and Epstein (1992), the stochastic discount factor (SDF), $\{M_t : t \geq 0\}$, is

$$M_t = \exp\left[\int_0^t f_V(C_s, V_s)\, ds\right] f_C(C_t, V_t) \ . \tag{36}$$

From the equilibrium allocation results,

$$\begin{aligned} f_C(C,V) &= \phi'(i^*) b^{1-\gamma} K^{-\gamma} \,, && (37) \\ f_V(C,V) &= -h \,, && (38) \end{aligned}$$

where

$$h = -\frac{\rho(1-\gamma)}{1-\psi^{-1}} \left[\left(\frac{c^*}{b}\right)^{1-\psi^{-1}} \left(\frac{\psi^{-1}-\gamma}{1-\gamma}\right) - 1\right]. \tag{39}$$

Using the equilibrium relation between b and c^*, we can simplify the above as follows:

$$h = \rho + \left(\psi^{-1} - \gamma\right) \left[\phi(i) - \frac{\gamma\sigma^2}{2} - \lambda \mathcal{E}\left(\frac{1 - Z^{1-\gamma}}{1-\gamma}\right)\right]. \tag{40}$$

Using Ito's lemma and the equilibrium allocation, we have

$$\frac{1}{M_{t-}} dM_t = -h dt - \gamma \left[\phi(i^*) dt + \sigma dW_t\right] + \frac{\gamma(\gamma+1)}{2} \sigma^2 dt + \left(Z^{-\gamma} - 1\right) dJ_t \ . \tag{41}$$

The equilibrium restriction that the expected rate of change of M_t must equal $-r_t$ implies the following formula for the equilibrium interest rate:

$$r = h + \gamma\phi(i^*) - \frac{\gamma(\gamma+1)\sigma^2}{2} - \lambda\mathcal{E}\left(Z^{-\gamma} - 1\right) \ . \tag{42}$$

Let $Q(K)$ denote the value of the capital stock and q denote Tobin's q. By homogeneity, $Q(K) = qK$. The equilibrium dividend is then $D_t = C_t$ for all t. The standard valuation methodology implies that $M_tD_tdt + d(M_tQ_t)$ has an instantaneous drift of zero. Using Ito's lemma and simplifying yields an equation for q:

$$\frac{c^*}{q} = \rho - \left(1-\psi^{-1}\right)\phi(i^*) + \frac{\gamma(1-\psi^{-1})\sigma^2}{2} + \frac{\lambda}{1-\gamma}\mathcal{E}\left[\left(\psi^{-1}-1\right)\left(Z^{1-\gamma}-1\right)\right]. \tag{43}$$

Using (35 and $q = 1/\phi'(i^*)$, we can write the above equation as:

$$b = \rho\left[1+\left(\frac{\psi^{-1}-1}{\rho}\right)\widehat{g}\right]^{\frac{1}{1-\psi}} q\ , \tag{44}$$

where $\widehat{g}$ is defined in (32). The expected rate of return on equity is then

$$r^e = \rho + \psi^{-1}\phi(i^*) - \frac{\gamma(\psi^{-1}-1)\sigma^2}{2} + \lambda\mathcal{E}\left(Z-1\right) + \frac{\lambda}{1-\gamma}\mathcal{E}\left[\left(\psi^{-1}-1\right)\left(Z^{1-\gamma}-1\right)\right]\ . \tag{45}$$

Therefore, the aggregate risk premium rp is given by

$$rp = r^e - r = \gamma\sigma^2 + \lambda\mathcal{E}\left[(Z-1)\left(1-Z^{-\gamma}\right)\right]\ . \tag{46}$$

B. Decentralized Market Solution.

Here, we provide the decentralized market equilibrium solution. First, we find the representative consumer's optimal consumption, portfolio choice and CIS demand. Second, we turn to firm value maximization taking prices as given. Finally, we conjecture and verify equilibrium prices and resource allocation.

Consumer Optimality. Let X denote the consumer's total marketable wealth and π the fraction allocated to the market portfolio. For catastrophe with recovery fraction in $(Z, Z+dZ)$, $\xi_t(Z)X_tdt$ gives the total demand for the CIS over time period $(t, t+dt)$. The total CIS premium payment in the time interval $(t, t+dt)$ is then $\left(\int_0^1 \xi_t(Z)p(Z)dZ\right)X_tdt$.

We conjecture that the cum-dividend return of the market portfolio is given by

$$\frac{dQ_t + D_tdt}{Q_{t-}} = \mu dt + \sigma dW_t - (1-Z)dJ_t\,, \tag{47}$$

where μ is the expected return on the market portfolio (including dividends) but without the effects of catastrophic risk (and will be determined in equilibrium). When a catastrophe occurs, the consumer's wealth changes from X_{t-} to X_t as follows:

$$X_t = X_{t-} - (1-Z)\pi_{t-}X_{t-} + \xi_{t-}(Z)X_{t-}\,. \tag{48}$$

The consumer's wealth accumulation is then given by

$$\begin{aligned} dX_t &= r\left(1-\pi_{t-}\right)X_{t-}dt + \mu\pi_{t-}X_{t-}dt + \sigma\pi_{t-}X_{t-}dW_t - C_{t-}dt \\ &\quad - \left(\int_0^1 \xi_{t-}(Z)p(Z)dZ\right)X_{t-}dt + \xi_{t-}(Z)X_{t-}dJ_t - (1-Z)\pi_{t-}X_{t-}dJ_t\,. \end{aligned} \tag{49}$$

The first four terms in (49) are standard in classic portfolio choice problems (with no insurance or catastrophes). The last three terms capture the effects of catastrophes on wealth accumulation. The fifth term is the total CIS premium paid before any catastrophe. The sixth term gives the CIS payments by the seller to the buyer when a catastrophe occurs. The last term is the loss of consumer wealth from exposure to the market portfolio.

The HJB equation for the consumer in the decentralized market setting is given by[16]

$$\begin{aligned} 0 \;=\; \max_{C,\pi,\xi(\,\cdot\,)} \; & \Big\{ f(C,J) + \Big[rX\,(1-\pi) + \mu\pi X - \Big(\int_0^1 \xi(Z)p(Z)dZ \Big) X - C \Big] J'(X) \\ & + \frac{1}{2}\sigma^2\pi^2X^2J''(X) + \lambda\mathcal{E}\left[J(X-(1-Z)\pi X + \xi(Z)X) - J(X) \right] \Big\} \, . \end{aligned} \tag{50}$$

The FOCs for consumption C, market portfolio allocation as a fraction π of total wealth X, and the CIS demand $\xi(Z)$ for each Z are respectively:

$$f_C(C,J) \;=\; J'(X) \tag{51}$$

$$(\mu - r)XJ'(X) \;=\; -\sigma^2\pi X^2J''(X) + \lambda\mathcal{E}\left[(1-Z)J'(X-(1-Z)\pi X + \xi(Z)X) \right] \tag{52}$$

$$0 \;=\; -Xp(Z)J'(X) + \lambda X\left[J'(X-(1-Z)\pi X + \xi(Z)X) \right] f_Z(Z) \, . \tag{53}$$

The last FOC follows from the point-by-point optimization in (50) for the CIS demand and hence it holds for all levels of Z. Now conjecture that the consumer's value function is

$$J(X) = \frac{1}{1-\gamma}(uX)^{1-\gamma} \, , \tag{54}$$

where u is a constant to be determined. Using the consumption FOC (51) and the conjectured value function (54), we obtain the following linear consumption rule:

$$C = \rho^{\psi}u^{1-\psi}X \, . \tag{55}$$

Imposing the equilibrium outcome in which (1) $\pi = 1$; (2) $\xi(Z) = 0$ for all Z; and (3) the consumer's wealth equals the total value of the market portfolio, $X = Q$, we obtain:

$$0 \;=\; (\mu - r)J'(Q) + \sigma^2QJ''(Q) - \lambda\mathcal{E}\left[(1-Z)J'(ZQ) \right] \tag{56}$$

$$p(Z) \;=\; \lambda\frac{J'(ZQ)}{J'(Q)} f_Z(Z) \tag{57}$$

Using these equilibrium conditions, we can simplify the HJB equation as follows:

$$\begin{aligned} 0 \;=\; & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{\rho}{u}\right)^{\psi-1} - 1 \right] u^{1-\gamma}X^{1-\gamma} + \left(\mu - \rho^{\psi}u^{1-\psi} \right)(uX)^{1-\gamma} - \frac{\gamma}{2}\sigma^2(uX)^{1-\gamma} \\ & + \lambda\mathcal{E}\left[Z^{1-\gamma} - 1 \right] \frac{1}{1-\gamma}(uX)^{1-\gamma} \end{aligned} \tag{58}$$

[16] In writing the HJB equation (7), we use the result that the "normalized" aggregator as defined and derived by Duffie and Epstein (1992) applies to our setting with both jumps and a diffusion. See Benzoni, Collin-Dufresne, and Goldstein (2010).

Eqn. (55) implies $c = \rho^{\psi} u^{1-\psi} q$ under the equilibrium condition $X = Q = qK$. Substituting $c = \rho^{\psi} u^{1-\psi} q$ into (58), we obtain

$$0 = \frac{1}{1-\psi^{-1}} \left(\frac{c}{q} - \rho \right) + \left(\mu - \frac{c}{q} \right) - \frac{\gamma}{2}\sigma^2 + \lambda \mathcal{E}\left[Z^{1-\gamma} - 1 \right] \frac{1}{1-\gamma} \tag{59}$$

Firm Value Maximization. We assume financial markets are perfectly competitive and M-M holds. While the firm can hold financial positions (e.g., CIS contracts), equilibrium pricing implies that there is no value in doing so. We can thus ignore financial contracts and only focus on investment I when maximizing firm value, which is independent of financing. Taking the unique stochastic discount factor (SDF) implied by the equilibrium consumption process as given, the firm maximizes its value by choosing I to solve:

$$\max_{I} \ \mathcal{E}\left[\int_0^{\infty} \frac{M_s}{M_0} \left(AK_s - I_s \right) ds \right], \tag{60}$$

subject to capital accumulation, the production technology, and the transversality condition.

Using the homogeneity property of our model, we conjecture that the SDF is given by a geometric Brownian motion with constant drift, constant volatility and proportional jump for each possible recovery fraction Z, i.e.

$$dM_t = -rM_{t-}dt - \eta M_{t-} dW_t + M_{t-}\left[\left(Z^{-\gamma} - 1 \right) dJ_t - \lambda \mathcal{E}\left(Z^{-\gamma} - 1 \right) dt \right]. \tag{61}$$

The second and the third terms capture diffusion and catastrophic risk respectively. Both terms are martingales. Note that to make the catastrophe term a martingale, we must subtract the expected change of M due to all possible catastrophes. Finally, the first term gives the equilibrium drift of M, which must be $-rM_t$ from the no-arbitrage condition.

No arbitrage implies the drift of $M_t(AK_t - I_t)dt + d\left(M_t Q_t\right)$ is zero. From Ito's Lemma we have the following dynamics for $Q(K)$:

$$dQ(K) = \left(\Phi(I,K) Q_K + \frac{1}{2} Q_{KK} \sigma^2 K^2 \right) dt + \sigma K Q_K dW_t + \left(Q(ZK) - Q(K) \right) dJ_t \,. \tag{62}$$

Again using Ito's Lemma, we have

$$\begin{aligned} &M_{t-}(AK - I)dt + M_{t-}\left(Q_K \Phi(I,K) dt + \frac{1}{2}\sigma^2 K^2 Q_{KK} dt \right) + Q\left[-r - \lambda \mathcal{E}\left(Z^{-\gamma} - 1 \right) \right] M_{t-} dt \\ &-\eta M_{t-} \sigma K Q_K dt + \lambda \mathcal{E}\left(Z^{-\gamma} Q(ZK) - Q(K) \right) M_{t-} dt = 0 \,. \end{aligned} \tag{63}$$

Simplifying the above, we have

$$\begin{aligned} \left[r + \lambda \mathcal{E}\left(Z^{-\gamma} - 1 \right) \right] Q(K) \ &= \ (AK - I) + Q_K \left(\Phi(I,K) - \eta\sigma K \right) + \frac{1}{2}\sigma^2 K^2 Q_{KK} \\ &\quad + \lambda \mathcal{E}\left(Z^{-\gamma} Q(ZK) - Q(K) \right) . \end{aligned} \tag{64}$$

The FOC with respect to investment is therefore

$$1 = \Phi_I(I, K)Q_K\,. \tag{65}$$

Using the homogeneity assumption, we conjecture that firm value is $Q(K) = qK$, where Tobin's q is to be determined. We can thus simplify (64) as follows:

$$\left[r + \lambda\mathcal{E}\left(Z^{-\gamma} - 1\right)\right] q = (A - i) + q\left(\phi(i) - \eta\sigma\right) + \lambda\mathcal{E}\left(Z^{1-\gamma} - 1\right) q\,. \tag{66}$$

The equilibrium dynamic for firm value Q_t is then given by

$$dQ_t = gQ_{t-}dt + \sigma Q_{t-}dW_t - (1 - Z)\,Q_{t-}dJ_t\,. \tag{67}$$

where $g = \phi(i)$ is the expected growth without the effects of catastrophes.

The FOC (65) can be simplified as follows:

$$q = \frac{1}{\phi'(i)}\,. \tag{68}$$

Market Equilibrium. We now verify that the conjectured prices and quantities are consistent with equilibrium market outcomes, and replicate the six key equations (14)-(19) in the text. First, eqn. (14) follows immediately from the goods market clearing condition, $Y = C + I$, and the homogeneity property. Second, eqn. (15) is the FOC for the producer under homogeneity. Third, we obtain eqn. (16) for consumption by comparing the dynamics for firm value on the consumer and firm sides, (47) and (67), to obtain the restriction:

$$\mu = \phi(i) + \frac{c}{q}\,. \tag{69}$$

The expected rate of return (without catastrophes) is $\phi(i)$ plus the dividend yield, which is also the consumption-wealth ratio. Substituting (69) into (59) gives eqn. (16).

Fourth, using the equilibrium consumption and evaluating the SDF via (36), we obtain the equilibrium interest rate r given by (17) and the equilibrium market price of diffusion risk $\eta = \gamma\sigma$. Note that the implied interest rate is also consistent with eqn (66).

Fifth, simplifying (56), we have the following result:

$$0 \;\;=\;\; (\mu - r) - \gamma\sigma^2 - \lambda\mathcal{E}\left[Z^{-\gamma}(1 - Z)\right]\,. \tag{70}$$

Adding the expected loss due to the catastrophic risk, we obtain the following formula for the equity risk premium rp:

$$rp = \mu + \lambda\mathcal{E}(1 - Z) - r = \gamma\sigma^2 + \lambda\mathcal{E}\left[(1 - Z)\left(Z^{-\gamma} - 1\right)\right]\,, \tag{71}$$

which is eqn. (18). Finally, substituting (54) into (57) gives the CIS insurance premium $p(Z)$ of eqn. (19). We have verified that the conjectured equilibrium is indeed consistent with the social planner's solution.

C. Consumption Tax.

When taxes are distortionary the standard welfare theorem argument no longer applies. In our case, however, we will see that a permanent consumption tax is non-distortionary, so the social planner's solution again coincides with the competitive market equilibrium. We thus proceed by solving the social planner's problem.

With a tax, the first-order condition (FOC) for consumption in the planner's problem is

$$(1-\tau)f_C(C,V) = \phi'(i)V'(K) \ . \tag{72}$$

Consider a tax to reduce λ. We conjecture that $V(K;\lambda,\tau)$, the value function for given values of λ and τ, has the homothetic form:

$$V(K;\lambda,\tau) = \frac{1}{1-\gamma}\left(b(\lambda;\tau)K\right)^{1-\gamma}, \tag{73}$$

where $b(\lambda;\tau)$ measures certainty-equivalent wealth (per unit of capital) when consumption is permanently taxed at rate τ. Let $V(K;\lambda,0)$ and $b(\lambda;0)$ denote the corresponding quantities in the absence of a tax as in Section 2. Let $c=(1-\tau)(A-i)$ denote the after-tax consumption-capital ratio. Substituting $V(K;\lambda,\tau)$ given by (73) into the FOC (72) yields:

$$c = \left(\frac{(1-\tau)\rho}{\phi'(i)}\right)^{\psi} b^{1-\psi} \ . \tag{74}$$

Substituting (74) for $i = A - c/(1-\tau)$ into the Bellman eqn. (7) and simplifying, we can write the equilibrium consumption-capital ratio $c^*(\tau)$ as:

$$c^*(\tau) = \frac{1-\tau}{\phi'(i^*)}\left[\rho + (\psi^{-1}-1)\left(\phi(i^*) - \frac{\gamma\sigma^2}{2} + \frac{\lambda}{1-\gamma}\mathcal{E}\left(1-Z^{1-\gamma}\right)\right)\right]. \tag{75}$$

Using the identity $c^* = (1-\tau)(A-i^*)$, the optimal investment-capital ratio i^* solves:

$$A - i^* = \frac{1}{\phi'(i^*)}\left[\rho + (\psi^{-1}-1)\left(\phi(i^*) - \frac{\gamma\sigma^2}{2} + \frac{\lambda}{1-\gamma}\mathcal{E}\left(1-Z^{1-\gamma}\right)\right)\right]. \tag{76}$$

Rewriting (74) and using i^* from solving (76), we have the following equation for b:

$$b(\lambda;\tau) = (c^*)^{1/(1-\psi)}\left[\frac{(1-\tau)\rho}{\phi'(i^*)}\right]^{-\psi/(1-\psi)} . \tag{77}$$

Therefore,

$$b(\lambda;\tau) = (1-\tau)^{1/(1-\psi)}(A-i^*)^{1/(1-\psi)}(1-\tau)^{-\psi/(1-\psi)}\left[\frac{\rho}{\phi'(i^*)}\right]^{-\psi/(1-\psi)} = (1-\tau)b(\lambda;0),$$

where the last equality follows from (10). We have shown that:

$$b(\lambda;\tau) = (1-\tau)b(\lambda;0)\ . \tag{78}$$

Note that $b(\lambda;0)$ in eqn. (10) is evaluated at the equilibrium i^* (without a tax). In this case, the solution to eqn. (11) remains the same. That is, the aggregate investment-capital ratio, aggregate output, and the aggregate capital stock all remain unchanged.

D. Data and Inputs to Calibration.

Unless otherwise indicated, National Income and Product data are from the Dept. of Commerce (www.bea.gov/national/nipaweb), data on fixed reproducible assets are from the Federal Reserve's Flow of Funds (www.federalreserve.gov/releases/), data on T-Bill rates and the CPI are from the Federal Reserve, and returns on the S&P 500 are from Robert Shiller (www.econ.yale.edu/˜ shiller). The data are for the period January 1947 to December 2008. Inputs to the calibration are measured or calculated as follows. (All data and calculations are in a spreadsheet available from the authors on request.)

Capital Stock. Our measure of the total capital stock K_T has three components: physical capital K_P, human capital K_H, and intangible capital held by firms K_I. Physical capital, from the Fed's Flow of Funds data, consists of fixed reproducible assets, including those held by federal and state and local governments. To estimate the stock of human capital, we use an approach suggested by Mankiw, Romer, and Weil (1998), and take the wage premium (the average wage minus the minimum wage) to be the return to human capital. We also assume that physical and human capital earn the same rate of return. Thus the total annual return to human capital is the wage premium as a fraction of the average wage (about .60 on average) times total compensation of employees. To get the rate of return on physical capital, we use total capital income (corporate profits including the capital consumption adjustment, i.e., gross of depreciation, plus rental income, plus proprietors' income) as a fraction of the stock of physical capital. That rate of return (about 7%) is used to capitalize the annual return to human capital.[17] For the stock of intangible capital, we use a weighted average of McGrattan and Prescott's (2005) estimates of the intangible capital stock as a fraction of GDP for 1960–69 and 1990–2001. The result is $K_I = .68Y$. Given annual values for K_P, K_H, and K_I, we calculate annual values for $A = Y/K_T$ and use the average value of A (0.121) as an input to our calibration.

Investment. We need total investment, inclusive of investment in intangible capital, to measure the consumption-investment ratio C/I. In equilibrium, investment in intangible capital is given by $I_I = (\delta_I + g)K_I$, where δ_I is the depreciation rate for intangible capital

[17]For comparison, we also calculated the stock of human capital using the results in Jones, Manuelli, Siu, and Stacchetti (2005), who estimated investment in human capital as a fraction of GDP. Assuming the depreciation rates for human and physical capital are the same, the equilibrium stocks will be in proportion to the investment levels. We obtained similar results (to within 15%) for the stock of human capital.

and g is the real GDP growth rate (.02). The BEA's estimate of the depreciation rate on R&D is 11%, but McGrattan and Prescott (2005) argue that this rate is too high for most non-R&D intangible capital. McGrattan and Prescott (2010) estimate the depreciation rate for intangible capital to be 8%, which is the rate we use. Thus $I_I = .10K_I = .068Y$.[18] Adding this to investment in physical capital yields a consumption-investment ratio of 2.84.

Tobin's q. We use the firm-level estimates of q from Riddick and Whited (2010), which are based on Compustat data for the period 1972 to 2006. Averaging their estimates across firms and years yields a value of 1.43

Real Risk-Free Interest Rate and Equity Risk Premium. We use monthly data on the nominal 3-month T-bill rate net of the percentage increase in the CPI for all items. Averaging over the (annualized) monthly numbers yields $r = 0.008$. For the equity risk premium, we use the monthly return (capital gain plus dividend) on the S&P500, compiled by Shiller, and subtract the nominal 3-month T-bill rate. Averaging over the annualized monthly numbers yields $rp = 0.066$.

Real GDP Growth Rate and Normal Volatility. We use real GDP and population data from the Bureau of Economic Analysis to compute the annual growth rate of real per-capita GDP. Averaging over these annual growth rates yields $g = .020$. Assuming no catastrophes occurred during $1947 - 2008$, our estimate of normal volatility is the sample standard deviation of these annual growth rates, which is $\sigma = .025$.

[18] This is within the range of the Corrado et al. (2005) estimates of investment in intangible capital.

References

Allison, Graham, *Nuclear Terrorism: The Ultimate Preventable Catastrophe*, Henry Holt & Company, 2004.

Attanasio, Orazio P., and Annette Vissing-Jorgensen, "Stock Market Participation, Intertemporal Substitution and Risk Aversion," *American Economic Review*, Papers and Proceedings, May 2003.

Bansal, Ravi, and Amir Yaron, "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 2004. **59**, 1481–1509.

Barro, Robert J., "Rare Disasters and Asset Markets in the Twentieth Century," *Quarterly Journal of Economics*, 2006, **121**, 823–66.

Barro, Robert J., "Rare Disasters, Asset Prices, and Welfare Costs," *American Economic Review*, March 2009, **99**, 243-64.

Barro, Robert J., and Tao Jin, "On the Size Distribution of Macroeconomic Disasters," NBER Working Paper 15247, August 2009.

Barro, Robert J., Emi Nakamura, Jon Steinsson, and José Ursúa, "Crises and Recoveries in an Empirical Model of Consumption Disasters," unpublished working paper, 2009.

Barro, Robert J., and José Ursúa, "Macroeconomic Contractions since 1870," *Brookings Papers on Economic Activity*, Spring 2008. 255–335.

Benzoni, Luca, Pierre Collin-Dufresne, and Robert S. Goldstein, "Explaining Asset Pricing Puzzles Associated with the 1987 Market Crash," Columbia Univ. working paper, 2010.

Breeden, Douglas T., "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics*, 1979, **7**, 265–296.

Constantinides, George, "Comments on 'On the Size Distribution of Macroeconomic Disasters'," *Brookings Papers on Economic Activity*, Spring 2008, 341-349.

Corrado, Carol, Charles Hulten, and Daniel Sichel, "Measuring Capital and Technology: An Expanded Framework," in Corrado, Haltiwanger, and Sichel, eds., *Measuring Capital in the New Economy*, University of Chicago Press, 2005.

Duffie, Darrell, and Larry G. Epstein, "Stochastic Differential Utility," *Econometrica*, 1992, **60**, 353–394.

Eberly, Jan, Sergio Rebelo, and Nicolas Vincent, "Investment and Value: A Neoclassical Benchmark," 2009. Working paper, Northwestern University.

Gabaix, Xavier, "Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance," NBER Working Paper No. 13724, January 2008.

Gourio, François, "Disasters and Recoveries," *American Economic Review*, May 2008, **98**, 68–73.

Greenwood, Jeremy, Zvi Hurcowitz, and Per Krusell, "The role of investment-specific technological change in the business cycle," *European Economic Review*, 2000, **44**, 91-115.

Guvenen, Fatih, "Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective," *Journal of Monetary Economics*, October 2006, **53**, 1451–72.

Hall, Robert E., "Intertemporal Substitution in Consumption," *Journal of Political Economy*, April 1988, **96**, 339–357.

Hall, Robert E., "Reconciling Cyclical Movements in the Marginal Value of Time and the Marginal Product of Labor," *Journal of Political Economy*, April 2009, **117**, 281–323.

Jones, Larry E. and Rodolfo E. Manuelli, 2005, "Neoclassical Models of Endogenous Growth: The Effects of Fiscal Policy, Innovation and Fluctuations," *Handbook of Economic Growth*, ed. Phillippe Aghion and Steven Durlauf, 13-65. Amsterdam: North-Holland.

Jones, Larry E., Rodolfo E. Manuelli, Henry E. Siu, and Ennio Stacchetti, "Fluctuations in Convex Models of Endogenous Growth, I: Growth Effects," *Review of Economic Dynamics*, 2005, **8**, 780–804.

Lucas, Robert E, Jr, "Asset Prices in an Exchange Economy," *Econometrica*, 1978, **46**, 1429–45.

Mankiw, N. Gregory, David Romer, and David N. Weil, "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics*, May 1992.

McGrattan, Ellen R., and Edward C. Prescott, "Taxes, Regulations, and the Value of U.S. and U.K. Corporations," Federal Reserve Bank of Minneapolis Research Staff Report 309, May 2005.

McGrattan, Ellen R., and Edward C. Prescott, "Technology Capital and the U.S. Current Account," *American Economic Review*, Sept. 2010, **100**, 1493–1522.

Mehra, Rajnish, and Edward C. Prescott, "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, (1985), **15**, 145–161.

Mehra, Rajnish, and Edward C. Prescott, "The Equity Premium: A Solution?," *Journal of Monetary Economics*, (1988), **22**, 133–136.

Parson, Edward A., "The Big One: A Review of Richard Posner's *Catastrophe: Risk and Response*, *Journal of Economic Literature*, March 2007, **45**, 147–164.

Posner, Richard A., *Catastrophe: Risk and Response*, Oxford University Press, 2004.

Rietz, Thomas A., "The Equity Risk Premium: A Solution," *Journal of Monetary Economics*, (1988), **22**, 117–131.

Riddick, Leigh A., and Toni M. Whited, "The Corporate Propensity to Save," *Journal of Finance*, forthcoming.

Russett, Bruce, and Miles Lackey, "In the Shadow of the Cloud: If There's No Tomorrow, Why Save Today?" *Political Science Quarterly*, Summer 1987, **102**, 259–272.

Russett, Bruce, and Joel Slemrod, "Diminished Expectations of Nuclear War and Increased Personal Savings: Evidence from Individual Survey Data," *American Economic Review*, Sept. 1993, **83**, 1022–1033.

Slemrod, Joel, "Fear of Nuclear War and Intercountry Differences in the Rate of Saving," *Economic Inquiry*, Oct. 1990, **28**, 647–657.

Sunstein, Cass R. *Worst-Case Scenarios*, Harvard University Press, 2007.

Wachter, Jessica, "Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?" NBER Working Paper No. 14386, October 2008.

Weitzman, Martin L., "Subjective Expectations and Asset-Return Puzzles," *American Economic Review*, Sept. 2007, **97**, 1102–1130.

Weil, Philippe, "Nonexpected Utility in Macroeconomics," *Quarterly Journal of Economics*, 1990, **105**, 29-42.

Yogo, Motohiro, "Estimating the Elasticity of Intertemporal Substitution When Instruments Are Weak," *Review of Economics and Statistics*, August 2004, **86**: 797-810.