Journal of Experimental Psychology:
Human Perception and Performance
2001, Vol. 27, No. 4, 985–999

0096-1523/01/$5.00 DOI: 10.1037//0096-1523.27.4.985

Visual Search for Orientation Among Heterogeneous Distractors: Experimental Results and Implications for Signal-Detection Theory Models of Search

Ruth Rosenholtz
Xerox Palo Alto Research Center and National Aeronautics and Space Administration Ames Research Center

Three experiments tested a signal-detection theory (SDT) model of visual search (e.g., as described in J. Palmer, C. T. Ames, & D. T. Lindsey, 1993). In Experiment 1, participants searched for a 0° line among distractors at (a) 30°; (b) ⅓ at 30°, ⅔ at 50°; (c) ⅓ at 30°, 50°, and 70°; and (d) ⅓ at 30°, ⅔ at 70°. The SDT model predicts improved performance in the more heterogeneous conditions, as some distractors are more discriminable from the target. In contrast, in Experiment 1 human performance degraded in the more heterogeneous conditions (c and d). In Experiment 2, sparser displays improved the performance of the SDT model. In Experiment 3, search for $\theta°$ among homogeneous $\theta + 20°$ distractors was compared with search for $\theta°$ among $\theta \pm 20°$ distractors. Performance in the latter condition was often worse, relative to performance in the homogeneous condition, than predicted by the SDT model; however, this depended greatly on the identity of the target.

People's daily lives are permeated by the sorts of visual activities that fit under the rubric of visual search. They search their bookcases for a particular book or look for a familiar face in a crowd. In many cases, people search for a target item in very complex scenes with highly heterogeneous distracting items.

Researchers have proposed a number of models for visual search. One popular class of models is an extension of signal-detection theory (SDT) models of detection and discrimination (see, e.g., Green & Swets, 1966; Melsa & Cohn, 1978). The SDT model is an ideal observer model for visual search on the basis of the assumption that the observer makes noisy observations of each stimulus. An ideal observer model makes decisions that are based on a minimum probability-of-error criterion. This criterion has been shown to be equivalent to the maximum a posteriori criterion, and, in the particular case of flat priors (in which each display element is equally likely to be the target element), has also been shown to be equivalent to the maximum-likelihood decision criterion.

In the basic SDT model considered in this article, the observer makes noisy, independent observations of each target or distractor in each display. An important consequence of the noisy representation of the stimulus is that a distractor may be mistaken for the target. A useful intuition is that search becomes more difficult when it is more likely that one of the distractors may be confused with the target.

The basic SDT model has been shown to predict a number of visual search results (e.g., Bennett & Jaye, 1995; Eckstein, Thomas, & Shimozaki, 2000; Palmer, 1994; Palmer, Ames, & Lindsey, 1993; Palmer & McLean, 1995; Shaw, 1980; Verghese & Stone, 1995). However, much of this modeling of visual search results has dealt with search among homogeneous distractors. This article further investigates how well the SDT model predicts results of search among heterogeneous distractors.

How do human observers perform on search tasks with heterogeneous distractors? Duncan and Humphreys (1989) suggested the following general rule for human performance on such tasks: Search difficulty increases as the target–distractor similarity increases and as distractor–distractor similarity decreases.

SDT has been shown (Palmer et al., 1993) to correctly predict the increase in search difficulty as target–distractor discriminability decreases. If adding variability to the distractors (and thus reducing their similarity to each other) reduces the difference between the target and some of the distractors, one would expect that SDT might predict an increase in search difficulty, in agreement with results from psychophysics research and Duncan and Humphreys's (1989) general rule. But, what happens if distractor variability is increased by making some of the distractors more discriminable from the target? Intuitively, SDT would predict that this increase would make the search task easier because those distractors would be less likely to be confused with the target. But according to Duncan and Humphreys's general rule, one should expect human performance to worsen.

In the section titled Experiment 1, I added variability to the distractors by making some of them more distinct from the target. This manipulation did degrade human performance, whereas the SDT model incorrectly predicts improved performance. In the section titled Experiment 2, I investigated whether the basic SDT model continued to make incorrect predictions in a less-crowded display. The section titled Experiment 3 describes an experiment in

Ruth Rosenholtz, Xerox Palo Alto Research Center, Palo Alto, California, and National Aeronautics and Space Administration Ames Research Center, Moffett Field, California.

This research was funded in part by a National Research Council Research Associateship. I thank Tina Beard for her valuable psychophysics advice and John Palmer for fruitful conversations on modifications to the basic signal-detection theory model.

Correspondence concerning this article should be addressed to Ruth Rosenholtz, Xerox Palo Alto Research Center, 3333 Coyote Hill Road, Palo Alto, California 94304. Electronic mail may be sent to rruth@parc.xerox.com.

which I introduced different heterogeneity into the distractor distribution and again compared the results with the predictions of the model. The Modifications to the Basic SDT Model section discusses possible modifications to the model that incorporate some of the intuitions of Duncan and Humphreys's (1989) general rule. The General Discussion section discusses the implications for models of visual search.

The SDT model of visual search, as it has typically been implemented, is inappropriate for modeling the search experiments presented in this article. First, previous implementations have been appropriate for observations with a linear distribution, such as observations of luminance, line length, and so on. However, my experiments involve observations of orientations, which wrap around, so that a line segment at angle θ is the same as one at angle $\theta + 180°$. Thus, orientation has what is known as a *circular* or *directional* distribution (see, e.g., Mardia, 1972). Second, previous implementations are inappropriate for conditions in which the target feature value may not be strictly larger than that of the distractors, like the conditions in Experiment 3, in which distractors were tilted both slightly clockwise and slightly counterclockwise from the target. In this article, the implementation of the SDT model is generalized to make predictions for arbitrary distributions of target and distractor observations. The Appendix elaborates on how these more general predictions were made, and it includes MATLAB (1993) code.

Experiment 1

The first experiment was designed to investigate effects of distractor heterogeneity when variability is added to the distractors by making some of them more distinguishable from the target. Search was for a horizontal line (0°) among lines of different orientations. Percent correct performance was measured on a two-interval forced-choice (2IFC) task for four conditions (see Figure 1): (a) homogeneous task, distractors at 30°; (b) one third of distractors at 30° and two thirds at 50°; (c) one third of distractors at 30°, one third at 50°, and one third at 70°; and (d) one third of distractors at 30° and two thirds at 70°.

Predictions

The internal observation noise is assumed to have a *wrapped-normal distribution* (Mardia, 1972)—an orientation distribution with many of the properties of the normal distribution. Like the normal distribution, the wrapped-normal distribution has two parameters: mean and standard deviation. Figure 2 shows a family of

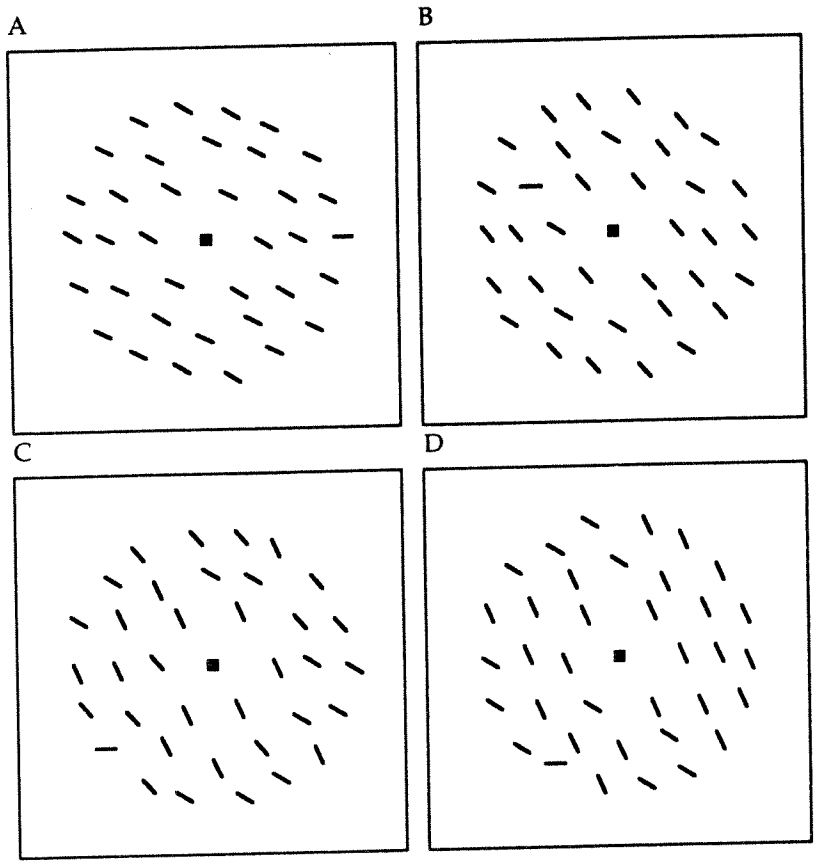

Figure 1. Scale drawings representative of stimuli for Experiment 1. The target was always a horizontal (0°) line. A: Condition 1: Homogeneous distractors at 30°; B: Condition 2: one third of distractors at 30°, two thirds at 50°; C: Condition 3: one third of distractors at 30°, 50°, and 70°; D: Condition 4: one third of distractors at 30°, two thirds at 70°.

curves representing predictions of the basic SDT model for internal noise standard deviations between 4° and 30°. For a wide range of internal noise parameters, the model predicts worst performance in the homogeneous Condition 1, with roughly identical performance in Conditions 2, 3, and 4. Intuitively, in the homogeneous case, there are more confusable distractors (with orientation close to the target orientation) and roughly the same number of most-confusable distractors (at 30°) in the remaining three conditions. For very small amounts of internal noise, the predicted performance saturates at roughly 100% for all four conditions. For very large amounts of internal noise, the predicted performance is roughly at chance for all four conditions.

Method

On each trial, observers viewed a fixation square in the center of the screen, then pressed the space bar on the computer keyboard to begin the trial. One stimulus appeared for 75 ms and was followed by a masking stimulus for 150 ms. Another fixation square followed, for 500 ms, then the second stimulus, also for 75 ms, and the second mask, for 150 ms. A blank screen replaced the mask, and observers indicated which of the two stimuli contained the target (0°) line, by typing *1* or *2*. We used Brainard's (1997b) Psychophysics Toolbox (for further information, see Brainard, 1997a) to design and control the experiments.

Each session consisted of a block of 100 trials for each of the four conditions listed above. The order of the conditions was determined by a Latin square design. At the beginning of each block, observers could freely view a sample stimulus for that block. Each observer participated in three sessions following a training session; thus, the results shown represent 300 trials for each condition.

Each stimulus consisted of 36 high-contrast white lines, 0.25 degrees in length, drawn on a black background and displayed on an Apple Multiple Scan 20 monitor.[1] The lines were arranged in concentric rings at radii equal to three, five, and seven times the line length, with 6, 12, and 18 lines in the three rings, respectively. In addition, the x- and y-positions of each line were jittered, with jitter uniformly distributed over ± line length/4. The total stimulus size was roughly 3.75 degrees in diameter, viewed binocularly from 6 ft (1.83 m). Figure 1 shows representative stimuli for the four conditions (shown with black lines on a white background for ease of reproduction). The target appeared in each location with equal probability.

The mask consisted of "stars" at each jittered line element location in the stimuli; each star was formed by line elements of twice the length used in the stimuli and positioned at every 10 degrees in orientation.

Two observers with corrected-to-normal vision participated in the experiment: myself (R.E.R.) as well as an experienced psychophysics participant (B.L.B.) who was naive to the purpose of the experiment.

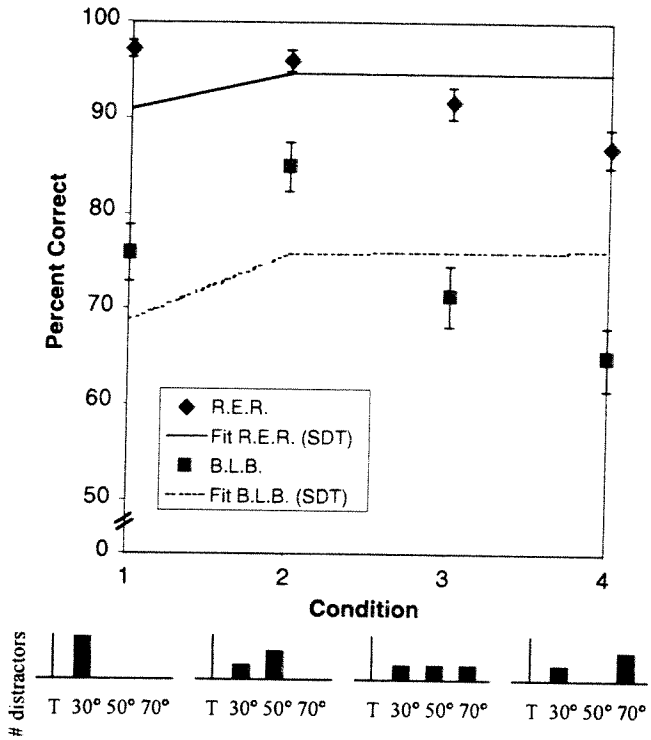

Figure 3. Results of Experiment 1 for observers R.E.R. and B.L.B., along with best-fit predictions from the basic signal-detection theory (SDT) model (solid line: R.E.R., dashed line: B.L.B.). Distributions of the distractors for the four conditions are shown below the x-axis. The target (T) was always a horizontal (0°) line.

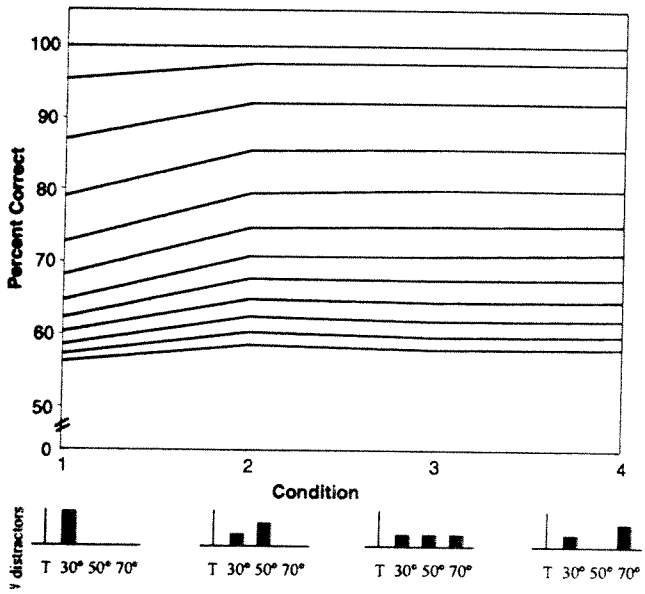

Figure 2. Predictions from the basic signal-detection theory model for Experiment 1. Each curve shows the predictions for a different value of the internal noise parameter, from 4° standard deviation to 30°. Distributions of the distractors for the four conditions are shown below the x-axis. T = target.

Results

The experimental results are shown in Figure 3. R.E.R. performed significantly better in both the homogeneous Condition 1 and the low variability Condition 2 compared with either of the two more heterogeneous conditions, Conditions 3 and 4 ($p < .05$, z test). R.E.R. showed no significant difference between Conditions 3 and 4 ($p > .05$, z test). B.L.B. performed significantly better in Condition 2 than in any of the other conditions, including

[1] When referring to size on the screen, I have used *degrees* (spelled out); when referring to the orientation of lines, I have used the symbol for degrees, °.

the homogeneous one, and significantly better in the homogeneous condition than in Condition 4 ($p < .05$, z test).

Though the 2 observers had different performance in Condition 1 relative to Condition 2, both showed a strong trend of decreasing performance across Conditions 2, 3, and 4, and the basic SDT model did not predict this trend. The curves in Figure 3 show the best-fit SDT model predictions to both observers' data. The SDT model predictions, although significantly correlated with the data ($r = .73$, $v = 6$, $p < .05$), provided a poor qualitative fit. The SDT model has one parameter: the internal noise standard deviation (see Appendix). The value of this parameter in the fit of the data was 9.2° for R.E.R. and 15.6° for B.L.B.

Discussion

The basic SDT model failed to predict the experimental results on search among heterogeneous distractors when distractor variability was added by making some of the distractors more distinct from the target. The SDT model predicted worst performance in the homogeneous condition and roughly equal performance in the remaining conditions; however, observers performed better in the two more homogeneous conditions than in the two more heterogeneous ones.

Palmer, Verghese, and Pavel (2000) advocated testing simple models of visual search under more restricted conditions than those presented in Experiment 1. The stimuli in Experiment 1 were relatively crowded and might have been perceived as textures rather than individual elements, suggesting that perhaps masking may have occurred between elements of the display, or that other sorts of texture processing in the visual system might have come into play. Although the position of the targets and distractors was randomly jittered, their placement was still fairly regular, and perhaps there were some configural effects. Finally, each of the displays was followed by a mask, which might have introduced temporal interactions not accounted for by simple models of visual search.

Palmer et al. (2000) suggested that models of visual search should first be tested using widely separated stimuli, to minimize spatial interactions between elements of the display, and with no mask following the stimuli, to minimize complex temporal interactions. Palmer et al. reasoned that if simple models can adequately predict search results under these restricted conditions, then the models provide a reasonable starting point for generalizing to more complex stimuli. The basic SDT model has accurately predicted the results of visual search experiments within these restricted conditions (Eckstein et al., 2000; Palmer et al., 1993; Palmer & McLean, 1995; Verghese & Stone, 1995). The model has less accurately predicted results for more complex stimuli (Morgan, Ward, & Castet, 1998; Palmer, 1994; Verghese & Nakayama, 1994). Note, however, that a number of models of visual search incorporate interactions between elements of the display as an important part of the model (e.g., Duncan & Humphreys, 1989; Rosenholtz, 1999; Wolfe, 1994). From the point of view of these models, it is arguable that the sparse displays advocated by Palmer et al. (2000) do not provide the critical test of a model of visual search. In Experiment 2, I tested visual search with the same combinations of target and distractors as in Experiment 1, under more restricted conditions, with sparser, less-regular displays and no mask following the stimuli.

Experiment 2

Method

The stimuli and viewing conditions were similar to those used in the first experiment, with a few modifications as described below. The task was again a 2IFC task, with timing as described for Experiment 1, except the duration of each stimulus was shortened to 50 ms, and no mask followed the stimuli. (In a preliminary experiment with no mask and eight display elements, a 75-ms stimulus duration led to performance saturated near 100% correct for all conditions.) Each stimulus consisted of eight high-contrast white lines on a black background. The lines were 0.4 degrees in length and were positioned on a circle with a radius of eight times the line length. The positions were jittered uniformly, in both the x- and y-directions, about $\pm 1.4 \times$ line length. The overall stimulus size was roughly 7.9 degrees in diameter. Representative stimuli are shown in Figure 4. An audible tone provided feedback.

Each session consisted of a block of 100 trials for each of the four conditions described in Experiment 1. The order of the conditions was determined by a Latin square design. Both of the observers participated in a training session followed by three experimental sessions; again, the results shown represent 300 trials for each condition. The observers in this experiment were not those who had participated in Experiment 1. Both had corrected-to-normal vision and were naive as to the purpose of the experiment.

Results

The results are shown in Figure 5, along with curves showing the best-fit predictions of the basic SDT model. The internal noise parameters of the model were 14.5° and 12.9° for observers J.A.K. and J.O.E., respectively. None of the differences between the four conditions are significant. The basic SDT model predicts a difference of roughly 7% or 8% correct between the homogeneous condition and the mean of the heterogeneous conditions—a difference that would be statistically significant—whereas the difference in the experimental data is only 1% or 2% correct. However, this difference between the predictions and the experimental results is rather small and inconclusive. Standard Pearson product–moment correlation squared (r^2) measures of goodness of fit break down in this situation because there is virtually no variability in the data to explain. The error in the fit is larger than the variability in the data.

Discussion

In Experiment 2, the stimuli were designed to be nearly optimal for the basic SDT model, with few display items, sparse displays, and no mask following the displays. In these conditions, the basic SDT model better predicts experimental results for the four conditions tested, though it still arguably predicts subtly different results, both qualitatively and quantitatively. The effect of heterogeneity increases with larger numbers of items and/or more crowding—in some ways this makes sense because with more items, estimates of heterogeneous distributions become more trustworthy. A complete model of visual search will need to explain this dependence of heterogeneity effects on the number or crowding of items in the display.

In Experiment 3, I tried a different manipulation of distractor heterogeneity to attempt a larger effect of heterogeneity versus homogeneity and thus to better test models of visual search. The motivation behind this experiment was consideration of a possible

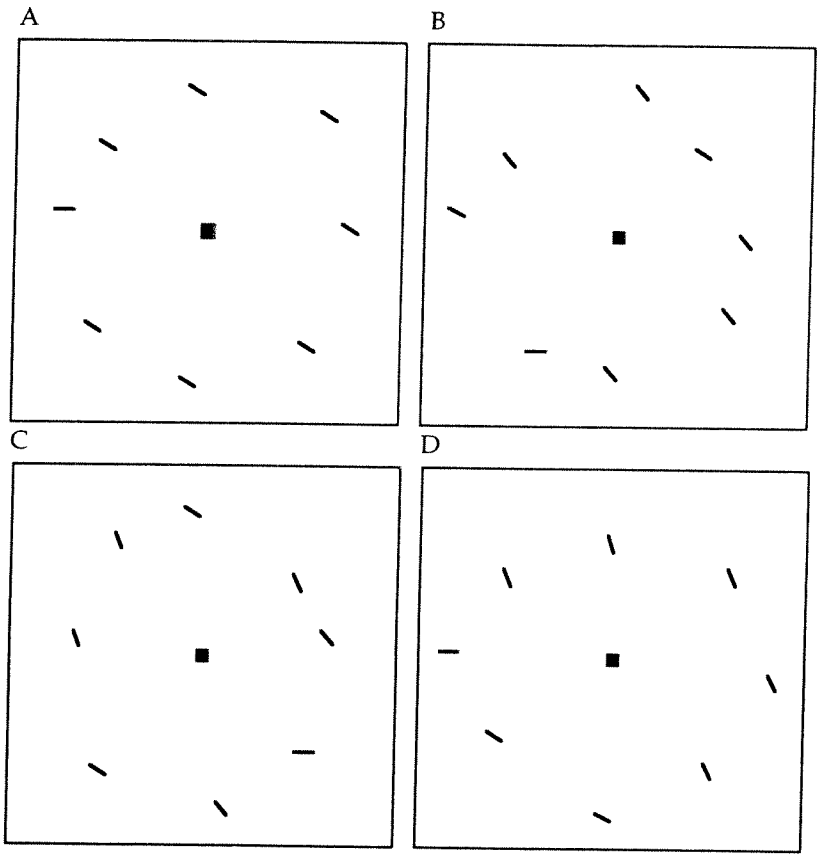

Figure 4. Scale drawings representative of stimuli for Experiment 2, with the same target and distractor distributions as in Experiment 1. A: Condition 1: Homogeneous distractors at 30°; B: Condition 2: one third of distractors at 30°, two thirds at 50°; C: Condition 3: one third of distractors at 30°, 50°, and 70°; D: Condition 4: one third of distractors at 30°, two thirds at 70°.

interpretation of Duncan and Humphreys's (1989) general rule for search among heterogeneous distractors: Search becomes easier as experimenters make either the target more distinguishable from the distractors or as they decrease the variability of the distractors. This suggests a measure in which search ease is related to the difference between the target and the mean of the distractors, relative to the standard deviation of the distractors; this measure is reminiscent of a *d* prime measure of discriminability. In a previous study (Rosenholtz, 1999), I showed that such a saliency measure is a good qualitative predictor of a number of results in motion and color search.

In an experiment in which search for a target at T° among distractors at T + 20° (homogeneous condition) is compared with search for the same target among distractors at T ± 20° (symmetric flanking condition), the saliency rule suggested above would predict much worse performance in the latter case relative to the former; in the former case, the saliency would be positive, and in the latter case, it would be zero. The SDT model should predict only a modest decrease in performance. Wolfe, Friedman-Hill, Stewart, and O'Connell (1992) performed a similar experiment measuring reaction time. In this study, I replicated their experiment in a 2IFC paradigm, measuring percent correct performance, so as to model the results with the basic SDT model while avoiding the complex issues of modeling reaction time data.

To model the results of this experiment with the basic SDT model, it was necessary to use a different implementation of the SDT model. The standard implementation of the basic SDT model assumes that all distractors have feature values less than the feature value of the target, and this is clearly not true in the flanking conditions of Experiment 3.

Experiment 3

Method

The viewing conditions and experimental method were the same as in Experiment 2. We studied four conditions: (a) homogeneous horizontal, target = 0°, distractors = 20°; (b) symmetric flanking horizontal, target = 0°, distractors = ±20°; (c) homogeneous oblique, target = 45°, distractors = 65°; (d) symmetric flanking oblique, target = 45°, distractors = 25°, 65°.

Representative stimuli are shown in Figure 6. Again, the observers participated in one training session with 100 trials per condition, which was followed by three sessions, each consisting of 100 trials per condition. The 4 observers had normal or corrected-to-normal vision, and 3 were naive as to the purpose of the experiment. I was the fourth observer.

Results

The results are shown in Figure 7, along with the best-fit predictions of the basic SDT model. I allowed the model to have

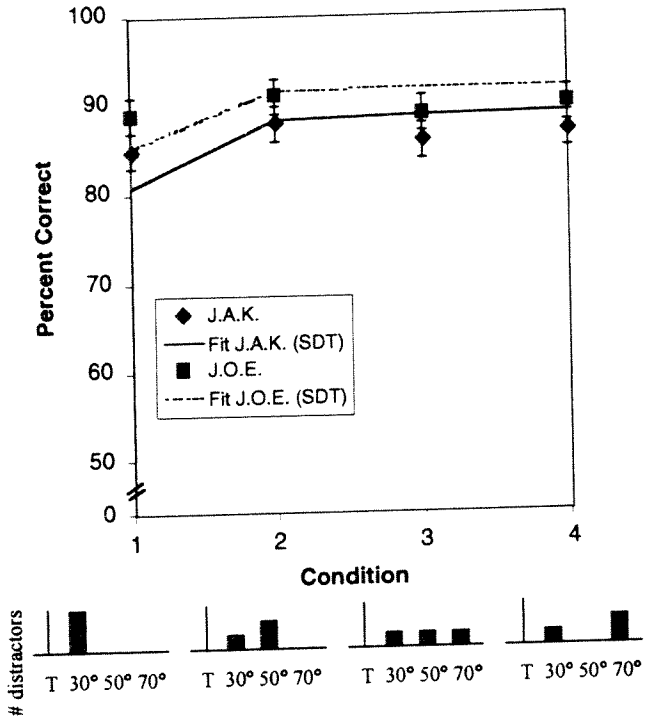

Figure 5. Results of Experiment 2 for observers J.A.K. and J.O.E., along with best-fit predictions from the basic signal-detection theory (SDT) model (solid line: J.A.K.; dashed line: J.O.E.). Distributions of the distractors for the four conditions are shown below the x-axis. The target (T) was always a horizontal (0°) line.

a different internal noise parameter for horizontal target Conditions 1 and 2 than for oblique target Conditions 3 and 4 because internal noise might have been different when observing a horizontal line than when observing one at 45°. This allowed improved quality of fit for the basic SDT model. The internal noise parameters for Conditions 1 and 2 were 8.4°, 9.7°, 9.7°, and 10.6°, and for Conditions 3 and 4 were 10.3°, 10.2°, 10.1°, and 10.2°, for observers R.E.R., M.K., B.D.W., and C.R.B., respectively.

For all 4 observers, the basic SDT model gave a very poor fit to the results of the oblique target conditions. For the 45° target, there was a large effect of heterogeneity and the basic SDT model was unable to replicate this. With this target, observers performed, on average, at 92% correct in the homogeneous condition and at 55% correct in the symmetric flanking condition, as opposed to 80% correct and 69% correct, respectively, as predicted by the model. The basic SDT model predicts a much smaller difference between the two oblique conditions than the data show. For R.E.R., the model also gave a poor fit to the horizontal target conditions. For M.K., the model fit fell outside of the error bars for the data for these conditions, but the difference was small. For the other 2 observers, data for the horizontal target conditions was well fit by the SDT model predictions. Overall, the fit of the basic SDT model was not significantly correlated with the data of Experiment 3 (r = .46, $\upsilon = 8$, $p > .05$).

Sutter, dela Cruz, and Sheft (2000) showed that a larger internal noise variance for oblique lines than for vertical lines can explain the asymmetry that it is easier search for an oblique target among vertical distractors than it is to search for a vertical target among oblique distractors. One might ask whether the results of Experiment 3 might also be explained by a different internal noise for observations of oblique lines than for horizontal lines.

To test this hypothesis, I assumed, as before, that observation noise was distributed according to a wrapped normal distribution and parametrized by a mean and standard deviation. I modeled the data with a three-parameter model, with parameters σ_0, σ_{45}, $\sigma_{20,25}$, which are the noise standard deviations when observing a horizontal line, a line at 45°, and a line at either 20° or 25°, respectively.

This model is not significantly correlated with the data (r = .19, $\upsilon = 4$, $p > .05$). In addition, the best fit yielded nonintuitive values for the three parameters, with average values across subjects of σ_0 = 13.8°, σ_{45} = 21.7°, and $\sigma_{20,25}$ = 8.35°. These numbers imply that humans are more accurate at observing a 20° or 25° line than at observing a horizontal line. Explaining the results of this experiment is not merely a matter of allowing different internal noise for the different line orientations.

Discussion

The results of Experiment 3 were somewhat mixed. For 3 out of 4 observers, the basic SDT model provided a reasonable fit of the data when the target was a horizontal line, yet for all 4 observers, the model provided a very poor fit to the data when the target was a 45° line. As noted above, the increased effect of heterogeneity for an oblique target is not simply a matter of different observation noise for the different line orientations. Instead, there seems to be an issue of ease of representation of the target given the distribution of distractors. The strength of the effect of distractor heterogeneity depends on the identity of the target. Furthermore, observers reported that in the symmetric flanking oblique condition, even with unlimited viewing time, it was difficult to determine whether or not a particular line was at 45° without comparing it to others in the display to ensure that it had neither the shallowest nor the steepest slope. Issues of target representation are further discussed in the General Discussion section of this article.

Modifications to the Basic SDT Model

The results of modeling the heterogeneous search experiments presented in article with the basic SDT model remain ambiguous. The model provided a poor fit to data of Experiment 1. For Experiments 2 and 3, the conditions were designed to be closer to the optimal conditions for making predictions using the SDT model. In Experiment 2, the model still produced predictions that differed both qualitatively and quantitatively from the results, but the difference was subtle. In Experiment 3, the basic SDT model made reasonable predictions in the two horizontal target conditions for 3 of the observers. For the other observer, the model fit the data less well for those two conditions. For all observers, the model very poorly fit the data for the oblique target conditions. Overall, however, the basic SDT model was significantly correlated with the data (r = .68, $\upsilon = 20$, $p < .01$).

In all of these cases, the pattern of the data seems qualitatively to match Duncan and Humphreys's (1989) general rule and the saliency rule mentioned in the *Discussion* of Experiment 2 (and in Rosenholtz, 1999). The saliency rule predicts easier search when

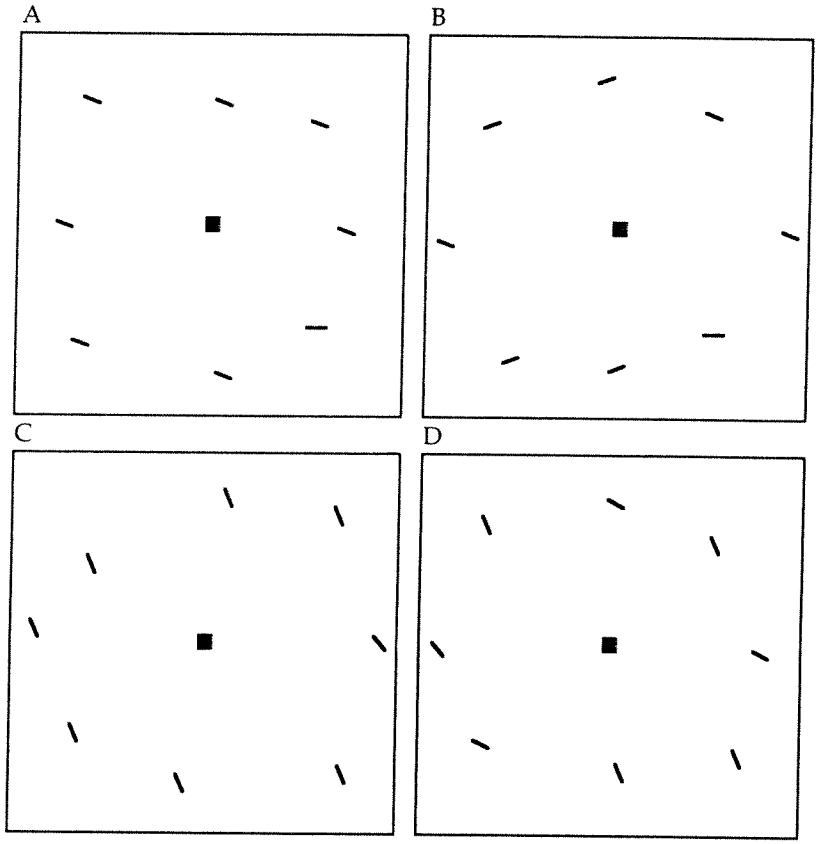

Figure 6. Scale drawings representative of stimuli for Experiment 3. A: Homogeneous horizontal condition; B: symmetric flanking horizontal condition; C: homogeneous oblique condition; D: symmetric flanking oblique condition.

the difference between the target orientation and the mean distractor orientation is large relative to the standard deviation of the distractors. This rule can also be thought of as a version of Duncan and Humphreys's (1989) general rule, which relates search ease to ill-defined target–distractor and distractor–distractor similarities. In this section of the article, I fit the data with models that mimic the qualitative predictions of the saliency rule and compare their fits to that of the basic SDT model.

The Best-Normal Model

The signal-detection theory model of visual search allows one to make predictions of percent correct performance given the distribution of target and distractor observations. In this article, an alternate model is tested, the *best-normal* model, which operates much like the basic SDT model. To understand the alternate model, it is best to have some understanding of the basic SDT model. SDT models make certain assumptions about the distribution of observations of the target and distractors. The only difference between the basic SDT model and the best-normal model is in the assumed distribution of the distractor observations. The basic SDT model assumes that the distractor observations are drawn from a distribution that equals the true, discrete distribution of distractors (i.e., the distribution from which the distractor orientations are drawn when generating the stimuli) plus added noise. The best-normal model assumes that distractor observations are instead drawn from the wrapped-normal distribution that best fits the true distribution of distractor orientations plus added noise. One might think of this as if the visual system was unable, under certain conditions such as short display times, to represent complex distributions and instead represented them using only their mean and variance.

The best-normal model, as a literal description of what happens in the visual system, is somewhat odd. It requires not only the highly plausible error that the observer misjudges the distribution of the distractors—perhaps because of an inability to represent complex distributions—but also the less plausible error that this misjudgment of the distribution affects observations of the distractors but not of the target. This model is more intended as one way of making quantitative predictions that qualitatively mimic the observation that search becomes more difficult as the variability of the distractors increases relative to the difference between the target and the mean of the distractors. I explored whether a model with this property can better fit the data from my experiments.

Figure 8 shows the results of Experiment 1, along with predictions of the best-normal model. This model has one parameter, the internal noise standard deviation. The best-fit values of this parameter were 7.1° and 14.2° for R.E.R. and B.L.B., respectively. The best-normal model fits the data better, both quantitatively (r =

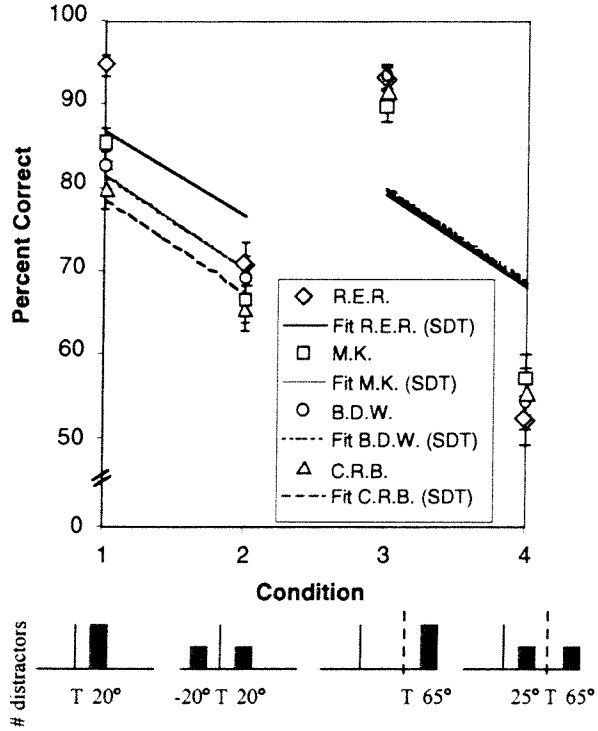

Figure 7. Results of Experiment 3 for observers R.E.R., M.K., B.D.W., and C.R.B., along with best-fit predictions from the basic signal-detection theory (SDT) model. Distributions of the distractors for the four conditions are shown below the x-axis. The target (T) was at 0° for the first two conditions and 45° for the last two conditions.

.89, $\upsilon = 6$, $p < .01$) and qualitatively, than the basic SDT model; however, the fit is still somewhat poor, especially for Condition 4.

In the initial discussion of the results of Experiment 1, it seemed that the observers gave inconsistent performance on Condition 1 relative to Condition 2, and it was unclear as to in which condition one should expect better performance. For the best-normal model, different values of the internal noise parameter yield qualitatively different curves. The model can predict both results like those of R.E.R., in which performance is very similar in Conditions 1 and 2, and results like those of B.L.B., in which performance is better in Condition 2 than in the homogeneous Condition 1. The qualitative predictions of the model depend on internal noise because the saliency of the target in the two conditions depends on the internal noise. In Condition 2, the target feature value lies farther from the mean of the distractors than it does in Condition 1; one might expect this to make search easier. However, the distractors in Condition 2 also have higher variability; one might expect this to make search more difficult. Which of these two effects dominates depends on the internal noise.

Figure 9 shows the results of Experiment 2, along with predictions of the best-normal model. The best-fit values of the internal noise parameter are 14.1° and 12.4° for observers J.A.K. and J.O.E., respectively. Strong conclusions are difficult to draw from the data of this experiment, but the fit of the model is reasonable, both qualitatively and quantitatively. As with the basic SDT model, the best-normal model predicts larger differences between the conditions than are in the data and the error in the fit is larger than the variability in the data; therefore, the r^2 measures break down.

Figure 10 shows the results of Experiment 3, along with predictions of the best-normal model. As with the fit of the basic SDT model to these data, the fit of the best-normal model allowed a different internal noise for the 0° target conditions than for the 45° target conditions. The internal noise parameters for Conditions 1 and 2 were 6.3°, 8.7°, 9.3°, and 10.1°, and for Conditions 3 and 4 they were 7.8°, 8.2°, 7.6°, and 7.9°, for R.E.R., M.K., B.D.W., and C.R.B., respectively. This model yields a nearly perfect fit to the data of 3 observers for the 0° target conditions, and an only slightly poorer fit to B.D.W.'s data for these conditions. The model yields a noticeably better fit to the 45° conditions than the fit of the basic SDT model; however, the fit is still poor, and it fails to predict the near-chance performance in the symmetric flanking condition. The fit of the best-normal model is significantly correlated with the data of Experiment 3 ($r = .81$, $\upsilon = 8$, $p < .05$). Overall, the correlation of the best-normal model with the data of the three experiments was significant and performs better both quantitatively and qualitatively than the basic SDT model ($r = .87$, $\upsilon = 20$, $p < .01$, as compared with $r = .68$ for the basic SDT model).

The Relative Coding-With-Reference Model

Recently, Palmer, Verghese, and Pavel (2000) have suggested another modification to the basic SDT model, related to the feature coding theory of Nothdurft (1991, 1992, 1993). In this model, referred to in this article as the *relative coding* model, observers

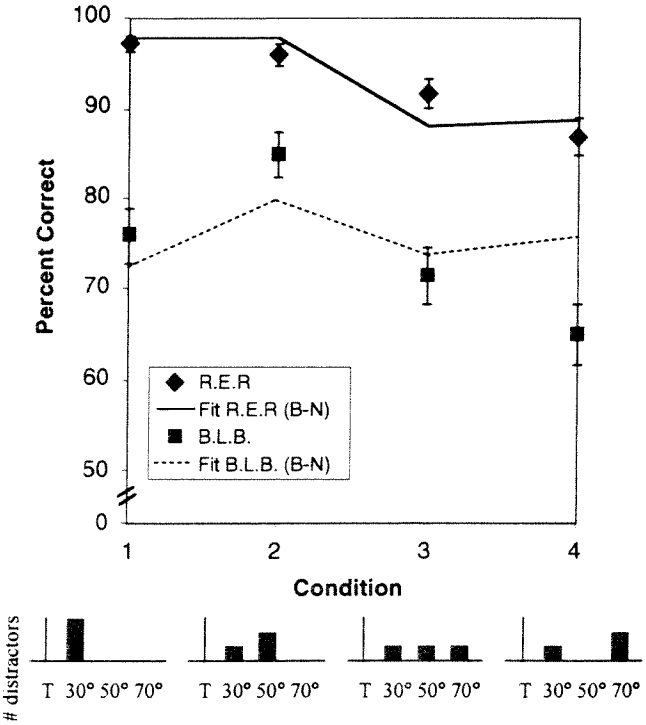

Figure 8. Results of Experiment 1 and best-fit predictions of the best-normal (B-N) model. Distributions of the distractors for the four conditions are shown below the x-axis. T = target.

make an observation for each display element, of the difference between the features of that element and the features of some randomly chosen other element of the display. The observer makes—for the purposes of the visual search task—a measurement of each element relative to some other display element, but no absolute measurement of the element features.

On the basis of these measurements of the differences between elements, the observer in, for example, a 2AFC task, makes a decision about which display contains the target. In Palmer et al. (2000), observers use a maximum-of-differences decision rule. In this experiment, a more general ideal observer decision rule was used. To a first approximation, using this rule amounts to conducting basic SDT as if the observer has observed differences between elements rather than the elements themselves. (There are important yet subtle differences between the implementations of the basic SDT model and the relative coding model because in the latter, some distractors might be compared with a target, whereas others, in a display with no target, would only be compared with other distractors; these subtleties are not discussed in this article.)

The relative coding model performs roughly the same as the basic SDT model. It is significantly correlated with the data overall ($r = .64$, $v = 20$, $p < .01$). As with the basic SDT model, the relative coding model performs worst on the data of Experiment 3, for which it is not significantly correlated with the data ($r = .41$, $v = 8$, $p > .05$).

However, when the relative coding model is applied to a set size of one, then this one element cannot be compared to any other elements in the display because there are no other elements in the display. Obviously, researchers need a modification to the model to allow it to function under these conditions. I suggest it be modified so that sometimes display elements are compared with other elements of the display, and some fraction of the time they are compared with a reference, in memory, of the desired target. Incorporating this notion into the relative coding model greatly improves its performance, and this new model is called the *relative coding-with-reference* model (RCref). The best fit to my data was a model in which the observer compared a display element with the reference target 30% of the time.[2] This model yielded a significant correlation with the data ($r = .87$, $v = 19$, $p < .01$), comparable to the best-normal model. Figures 11, 12, and 13 show the fit of this model to the data.

A number of modifications could be made to further refine and test the RCref model. I used the same internal noise for comparison with an element in the display as I did for comparison with a reference in memory, and the internal noise is likely to differ in the two cases. Furthermore, the internal noise for the reference in memory would likely vary with the identity of the target.

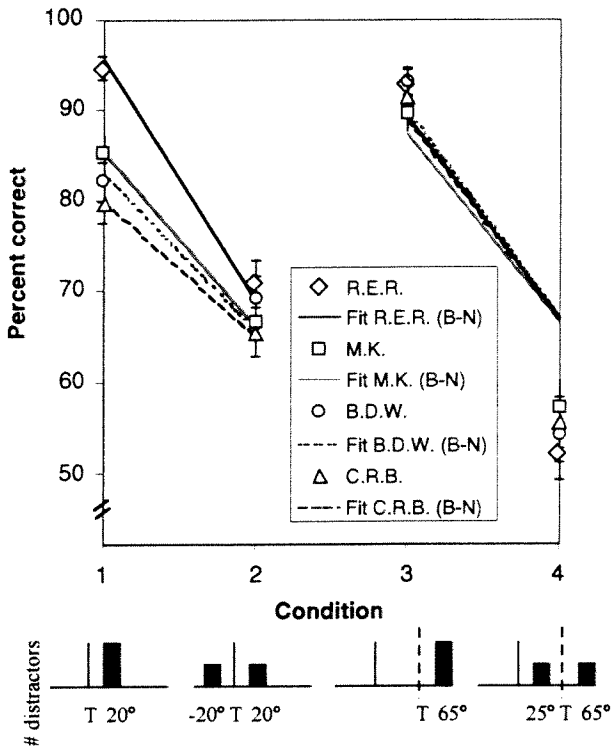

Figure 10. Results of Experiment 3 and best-fit predictions of the best-normal (B-N) model. Distributions of the distractors for the four conditions are shown below the x-axis. T = target.

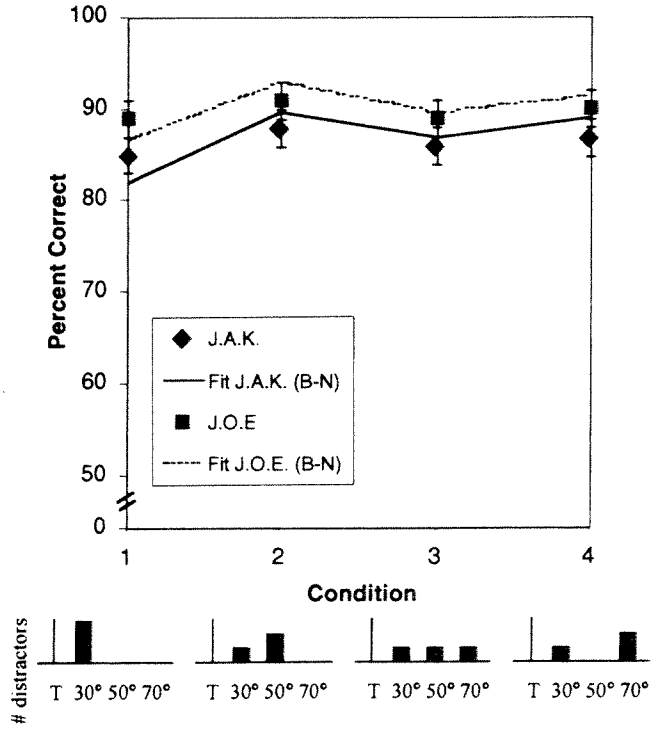

Figure 9. Results of Experiment 2 and best-fit predictions of the best-normal (B-N) model. Distributions of the distractors for the four conditions are shown below the x-axis. T = target.

[2] However, it is not possible from my data to determine whether observers always compare with the reference the same percentage of the time, or whether that percentage varies with the number of elements in the display. Predictions for Experiment 1—the only experiment with set size greater than eight—are highly insensitive to the percentage of comparisons with the reference.

General Discussion

This article has investigated orientation search among heterogeneous distractors with conditions in which heterogeneity was added by making some of the distractors less like the target (Experiments 1 and 2), and with conditions in which the distractor orientations symmetrically flanked the target orientation (Experiment 3). In the dense displays of Experiment 1, search was significantly more difficult for the two most heterogeneous conditions. The basic SDT model did not predict this result. When I used the sparse displays of Experiment 2, there were no significant differences between the four conditions, and results of modeling the data were inconclusive. In Experiment 3, the basic SDT model gave a poor fit to the data when the target was a 45° line, though it gave a considerably better fit when the target was a horizontal line.

Further work remains to determine whether the results presented in this article are typical for search on the basis of features other than orientation. There is some evidence in motion search (Driver, McLeod, & Dienes, 1992) that increasing the variability of the distractors by making some of them less like the target impairs visual search, as in Experiment 1.

Many models of visual search are not sufficiently developed for testing on the data presented in this article. My data require models able to predict percent correct performance as a function of the number of elements in the display—the results of Experiments 1 and 2 demonstrate that heterogeneity effects depend greatly on the number or crowding of elements in the display. Furthermore, for Experiment 3, many models fit the data qualitatively, but so far, none of them provide a good enough fit quantitatively—for modeling the results of this experiment, a model is required that can make quantitative predictions of percent correct performance.

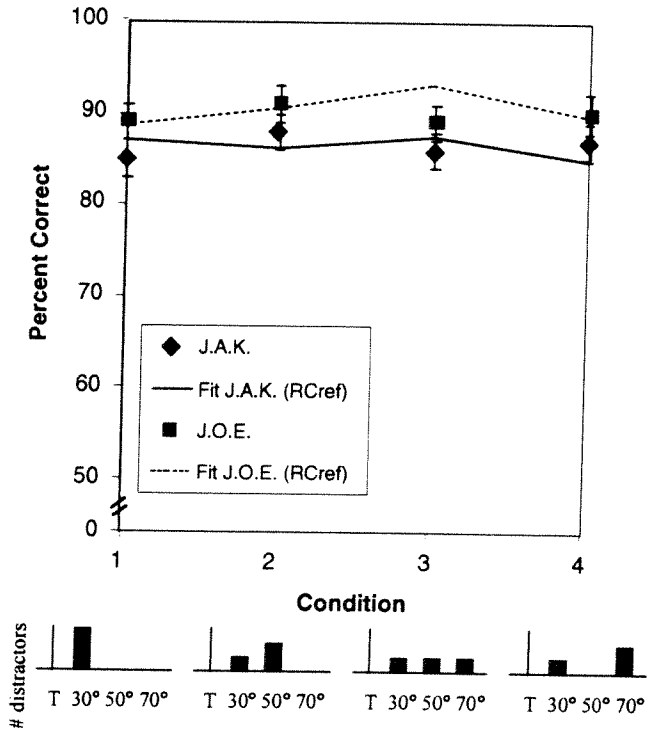

Figure 12. Results of Experiment 2 and best-fit predictions of the relative coding-with-reference (RCref) model. Distributions of the distractors for the four conditions are shown below the x-axis. T = target.

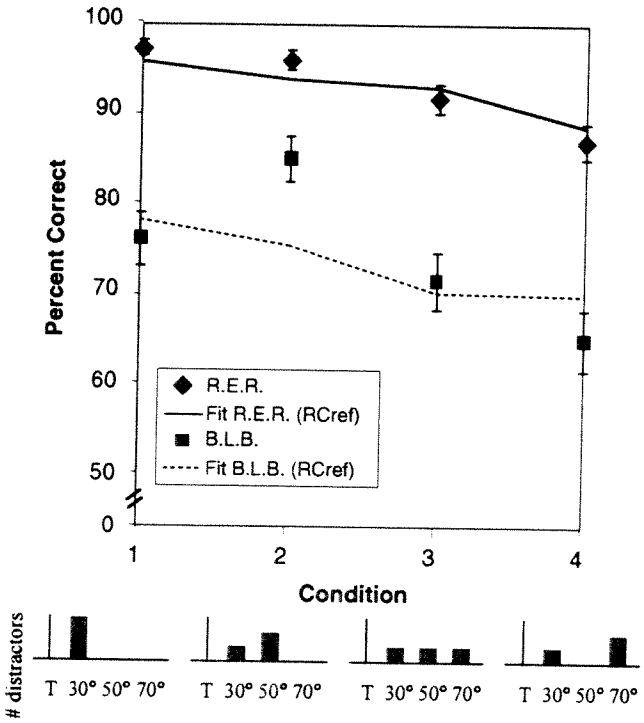

Figure 11. Results of Experiment 1 and best-fit predictions of the relative coding-with-reference (RCref) model. Distributions of the distractors for the four conditions are shown below the x-axis. T = target.

Modifications to the basic SDT model are sufficiently developed to test on our data, and several such modifications fared somewhat better than the basic SDT model. Figure 14 shows the data for all three experiments, which is plotted versus the best-fit predictions of the basic SDT model. Figure 15 shows the data plotted versus the predictions of the best-normal model. Figure 16 shows the data plotted versus the predictions of the RCref model. If a model was to fit the data perfectly, the points should all lie along the diagonal line shown. For all three models, the points are reasonably well clustered along the diagonal, with the worst deviations for conditions in which observers performed particularly poorly. The points are more tightly clustered around the diagonal for the best-normal model and relative coding model; the correlation between data and model was $r = .68$ for the basic SDT model, $r = .87$ for the best-normal model, and $r = .87$ for the relative coding model. For all three models, the correlation with the data is significant (r^2 test, $p < .01$).

For quality of fit, it makes little difference whether one considers all three experiments or only the latter two experiments using sparse displays. For Experiments 2 and 3, $r = .66$ for the basic SDT model, $r = .86$ for the best-normal model, and $r = .86$ for the relative coding model. This suggests that a model of visual search may not necessarily need an additional component to deal with crowding in displays—perhaps the different qualitative effects of heterogeneity may be modeled merely by an increase in set size.

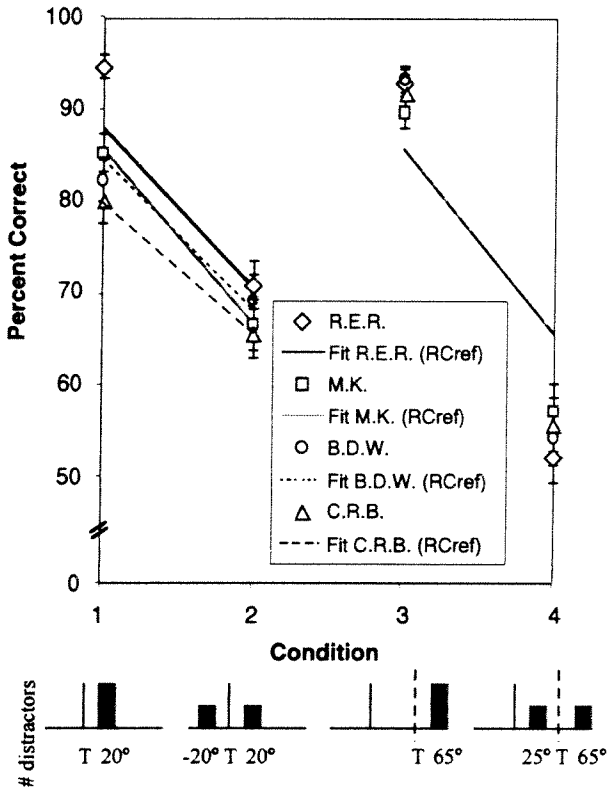

Figure 13. Results of Experiment 3 and best-fit predictions of the relative coding-with-reference (RCref) model. Distributions of the distractors for the four conditions are shown below the *x*-axis. T = target.

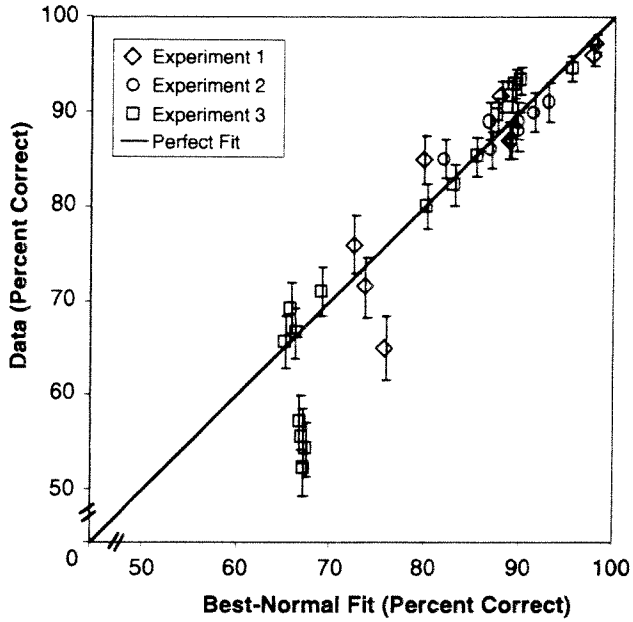

Figure 15. Data versus best fit of the best-normal model. For a perfect fit, all points would lie along the diagonal.

The two modifications to the basic SDT model, the best-normal and RCref models, have two things in common. First, they both, though in different ways, mimic the saliency rule, or, more generally, Duncan and Humphreys's (1989) general rule. The intuition behind the RCref model is that it predicts easier search when the difference between the target and the distractors is large relative to differences among the distractors. Second, both the RCref model and the best-normal model use less of the available information

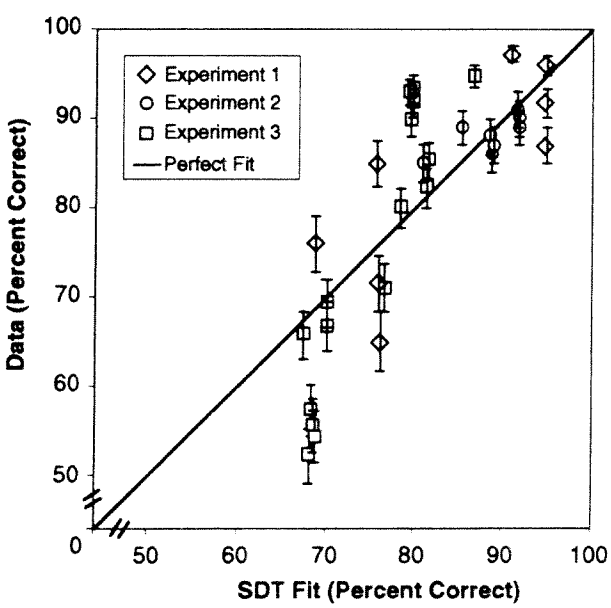

Figure 14. Data versus best fit of the signal-detection theory (SDT) model. For a perfect fit, all points would lie along the diagonal.

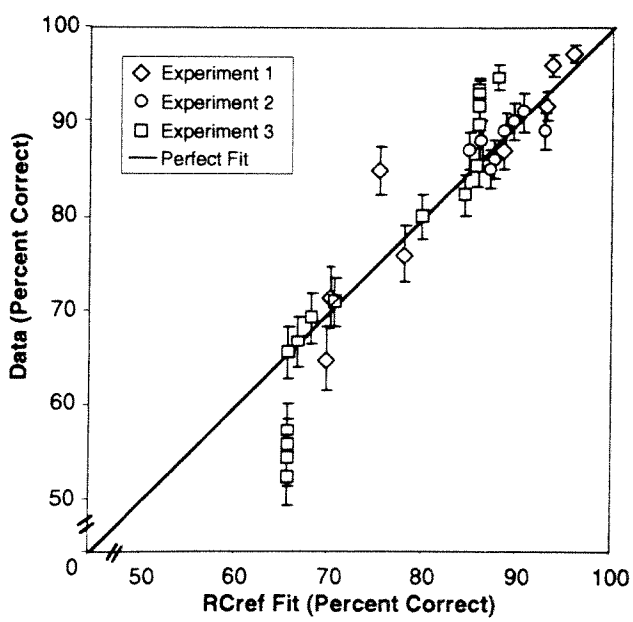

Figure 16. Data versus best fit of the relative coding-with-reference (RCref) model. For a perfect fit, all points would lie along the diagonal.

than the basic SDT model. In the case of the best-normal model, the true and complex distribution of distractors is ignored and only their mean and variance is used. The RCref model ignores the actual feature values of the target and distractors in lieu of the differences between them. This is less information than the basic SDT model, because one can construct the differences from the feature values, but one cannot reconstruct the feature values from the differences. The success of the best-normal and RCref models over the basic SDT model suggests two things. First, visual search is better modeled by a system that does not make full use of the information available in the stimuli. Second, thus far, models seem to perform better when they mimic a rule that search becomes easier when the difference between the target and distractors is large compared with the differences between the distractors.

As shown by the results of Experiment 3, the identity of the target had a large effect on the observed effects of heterogeneity. This suggests that a key component of a visual search model should address the difficulty of representing the target. The ease with which one can represent a target depends not only on the features of the target, but also on the distribution of distractors—merely allowing different internal noise for observations of different orientations did not explain the results of Experiment 3. For the representation of the target to be useful for a search task, that representation must distinguish the target from the distractors.

By this reasoning, it should be easier to represent the target if it is in some sense significantly different from the distractors. This may be why the saliency rule seems to qualitatively describe search performance in many instances (Rosenholtz, 1999) and why both the best-normal and RCref models—which mimic this rule—give improved performance over the basic SDT model. The saliency, which is roughly the difference between the target and the mean of the distractors relative to the standard deviation of the distractors, is a measure of the significance of the difference between the target and the mean of the distractors. Thus, such a measure might be related to the difficulty of representing the target such that it is distinguishable from the distractors.

The ease of representation would likely depend on other issues as well. For example, on one hand, it may be very difficult to represent a 45° target among distractors that are also oblique when representation is not aided by a target that significantly differs from the distractors. This may explain the near-chance performance in the symmetric flanking oblique condition. On the other hand, though a 0° target also may not be significantly different from flanking distractors at ±20°, it may be inherently easier to represent a horizontal target, thus leading to better performance in the symmetric flanking horizontal condition.

A model of visual search should include a model of both the ease of representing the target and the effect that a poor representation of the target has on the observer's ability to find that target. SDT models of visual search provide a framework for modeling the effect of a poor representation on search performance. One could model representational uncertainty as positional uncertainty in feature space—essentially, as an increase in observation noise. However, current implementations of SDT models at worst use the same observation noise for all conditions and at best allow the observation noise to vary with feature value. Perhaps further exploration of what determines the ease of representing a target would allow SDT models of visual search to better explain search among heterogeneous distractors.

References

Bennett, P. J., & Jaye, P. D. (1995). Letter localization, not discrimination, is constrained by attention. *Canadian Journal of Experimental Psychology, 49,* 460–504.

Brainard, D. H. (1997a). The Psychophysics Toolbox. *Spatial Vision, 10,* 443–446.

Brainard, D. H. (1997b). The Psychophysics Toolbox (Version 2.0) [Computer software]. Available from the World Wide Web: http://color.psych.ucsb.edu/psychtoolbox/download.html

Driver, J., McLeod, P., & Dienes, Z. (1992). Are direction and speed coded independently by the visual system? Evidence from visual search. *Spatial Vision, 6,* 133–147.

Duncan, J., & Humphreys, G. W. (1989). Visual search and stimulus similarity. *Psychological Review, 96,* 433–458.

Eckstein, M. P., Thomas, J. P., & Shimozaki, S. S. (2000). A signal detection model predicts the effects of set size on visual search accuracy for feature, conjunction, triple conjunction, and disjunction displays. *Perception & Psychophysics, 62,* 425–451.

Green, D. M., & Swets, J. A. (1966). *Signal detection theory and psychophysics.* New York: Krieger.

Mardia, K. V. (1972). *Statistics of directional data.* London: Academic Press.

MATLAB (Version 5.2) [Computer software]. (1993). Natick, MA: The MathWorks.

Melsa, J. L., & Cohn, D. L. (1978). *Decision and estimation theory.* New York: McGraw-Hill.

Morgan, M. J., Ward, R. M., & Castet, E. (1998). Visual search for a tilted target: Tests of spatial uncertainty models. *Quarterly Journal of Experimental Psychology: Human Experimental Psychology, 51(A), 347–370.*

Nothdurft, H. C. (1991). Texture segmentation and pop-out from orientation contrast. *Vision Research, 31,* 1073–1078.

Nothdurft, H. C. (1992). Feature analysis and the role of similarity in preattentive vision. *Perception & Psychophysics, 52,* 355–375.

Nothdurft, H. C. (1993). The role of features in preattentive vision: Comparison of orientation, motion, and color cues. *Vision Research, 33,* 1937–1958.

Palmer, J. (1994). Set-size effects in visual search: The effect of attention is independent of the stimulus for simple tasks. *Vision Research, 34,* 1703–1721.

Palmer, J., Ames, C. T., & Lindsey, D. T. (1993). Measuring the effect of attention on simple visual search. *Journal of Experimental Psychology: Human Perception and Performance, 19,* 108–130.

Palmer, J., & McLean, J. (1995, November). *Imperfect, unlimited-capacity, parallel search yields large set-size effects.* Paper presented at the meeting of the Society of Mathematical Psychology, Irvine, CA.

Palmer, J., Verghese, P., & Pavel, M. (2000). The psychophysics of visual search. *Vision Research, 40,* 1227–1268.

Rosenholtz, R. (1999). A simple saliency model predicts a number of motion popout phenomena. *Vision Research, 39,* 3157–3163.

Shaw, M. L. (1980). Identifying attentional and decision-making components in information processing. In R. S. Nickerson (Ed.), *Attention and performance VIII* (pp. 277–296). Hillsdale, NJ: Erlbaum.

Sutter, A., dela Cruz, R., & Sheft, S. (2000). Noisy, independent processing of features in visual search explains search asymmetries. *Investigative Ophthalmology & Visual Science, 41*(Suppl.), 423.

Verghese, P., & Nakayama, K. (1994). Stimulus discriminability in visual search. *Vision Research, 34,* 2453–2467.

Verghese, P., & Stone, L. S. (1995). Combining speed information across space. *Vision Research, 32,* 983–995.

Wolfe, J. M. (1994). Guided search 2.0: A revised model of visual search. *Psychonomic Bulletin and Review, 1,* 202–238.

Wolfe, J. M., Friedman-Hill, S. R., Stewart, M. I., & O'Connell, K. M. (1992). The role of categorization in visual search for orientation. *Journal of Experimental Psychology: Human Perception and Performance, 18,* 34–49.

Appendix

Making Predictions for Signal-Detection Theory

The *signal-detection theory* model in this article is a model of visual search in which the observer makes a decision about the presence or absence of a target on the basis of an ideal observer criterion. This is equivalent to using a minimum probability of error criterion or a maximum a posteriori criterion (see, e.g., Green & Swets, 1966; Melsa & Cohn, 1978). When each display element is equally likely to be the target, the decision criterion is also equivalent to a maximum likelihood decision. The model referred to as the *basic signal-detection theory* (basic SDT) model assumes that the observer makes noisy, independent observations of each of the target and distractor elements in the display. All predictions of the basic SDT model presented in the body of the article assume that the noise is additive and independently and identically distributed according to a wrapped-normal distribution (Mardia, 1972). Below, the general theory is developed for the case of a 2AFC task for arbitrary distributions of target, distractors, and noise. This theory is applicable for search that is based on features that wrap around, like orientation, as well as search for standard features like luminance and size. The development of this theory closely follows the development of standard signal-detection theory, except that this theory makes fewer early assumptions about the distributions of target and distractor observations. The development of the theory for a yes–no task proceeds along similar lines but with a different criterion for the decision (see Palmer et al., 1993, for development of a yes–no decision criterion under more restrictive assumptions about the distributions of target and distractor observations).

In the basic SDT model, the observer makes observations of the features of each of n elements in the display. If the vector of n feature estimates is assumed to be $\vec{\mathbf{z}}$, and m_i is the ground-truth message that the ith element is the target (i.e., if the target is in Location 3, then m_3 is true, and all other m_j are false), then the ideal observer criterion states that an observer should pick element i as the target if

$$p(m_i|\vec{\mathbf{z}}) > p(m_j|\vec{\mathbf{z}}) \text{ for all } j \neq i. \tag{A1}$$

Using Bayes's rule, this can be written as

$$p(\vec{\mathbf{z}}|m_i)P(m_i)/P(\vec{\mathbf{z}}) > p(\vec{\mathbf{z}}|m_j)P(m_j)/P(\vec{\mathbf{z}}) \quad \forall\, j \neq i; \tag{A2}$$

and, assuming $P(m_i) = P(m_j)$ for all i and j (each element of the display is equally likely to be the target) and simplifying, the criterion becomes

$$p(\vec{\mathbf{z}}|m_i) > p(\vec{\mathbf{z}}|m_j) \quad \forall\, j \neq i. \tag{A3}$$

The n observations are independent, thus

$$p(\vec{\mathbf{z}}|m_i) = p(Z_1 = z_1|m_i)p(Z_2 = z_2|m_i)\cdots p(Z_n = z_n|m_i), \tag{A4}$$

where $p(Z_j = z_j|m_i)$ is the probability that the observation for the jth element in the display has value z_j, given that the ith element is the target.

For $j \neq i$, $p(Z_j = z_j|m_i)$ is just the probability of drawing value z_j from a distribution of distractor observations. All distractor observations are assumed to be drawn from the same distribution, f, independent of whichever element is being observed. (Multiple distractor types are accommodated by making this distribution multimodal; see Figure A1. This accommodation of multiple distractor types represents a minor, though straightforward, deviation from standard development of signal-detection theory models.) Similarly, $p(Z_i = z_i|m_i)$ is the probability of drawing value z_i from the distribution of target observations, g. Then the criterion becomes

$$f(z_1)f(z_2)\cdots g(z_i)\cdots f(z_j)\cdots f(z_n) > f(z_1)f(z_2)\cdots f(z_i)\cdots g(z_j)\cdots f(z_n) \quad \forall\, j \neq i. \tag{A5}$$

Canceling terms that appear on both sides of the inequality (assuming f and g are strictly positive functions, as is the case for all modeling in this article), the criterion becomes

$$g(z_i)f(z_j) > f(z_i)g(z_j) \quad \forall\, j \neq i. \tag{A6}$$

This equation specifies a decision region, $Z_{\text{decision}} \subset \Re^2$. For points (z_i, z_j) within that decision region, it is more likely that the ith element is a target and the jth element is a distractor than the ith element is a distractor and the jth element is a target. For $(z_i, z_j) \in Z_{\text{decision}}$, the ith element with observation z_i "wins" in the pairwise comparison with an element with observation z_j. This parallels closely the usual application of detection theory to modeling visual search, except that as more general distributions of distractors are allowed, more general decision regions need to be considered in the development that follows.

Equation A6 means that an observer can make the maximum-likelihood decision about which element is the target by making a number of simple comparisons between pairs of observations. The maximum-likelihood decision involves solving a number of two-dimensional problems as opposed to n-dimensional problems. An ideal observer decides that the ith element is the target if, for each $j \neq i$, the point (z_i, z_j) falls within the decision region Z_{decision}. In other words, an ideal observer decides that the ith element is the target if it "wins" in all paired comparisons with other observations.

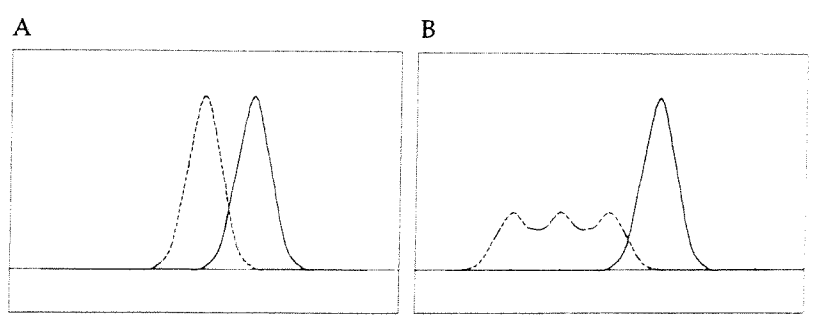

Figure A1. Distributions of target and distractor observations for homogeneous distractors (A) and heterogeneous distractors (B; three equally likely distractor types, as in Experiments 1 and 2). In all cases, the observation noise is Gaussian. Solid curves represent distributions of target observations; dashed curves represent distributions of distractor observations.

(*Appendix continues*)

The decision region is a function of only f and g, the distributions of the target and distractor observations; it is not a function of i or j (the elements that are observed). So, the same decision region is used for each of the paired comparisons. Three sample decision regions are shown in Figure A2. For homogeneous distractors, with additive, normally distributed noise in the observations of both target and distractors, $f \sim N(\mu_D, \sigma)$ and $g \sim N(\mu_T, \sigma)$. In this case, when $\mu_T > \mu_D$, the decision region Z_{decision} has been shown to be equal to $\{(z_i, z_j) : z_i > z_j\}$. This decision region, shown in Figure A2a, has led to the common *maximum rule* for visual search. For a 2AFC task, this rule says that the optimal decision is to pick the display that elicits the largest observation as the one containing the target (Green & Swets, 1966).

For the experiments in this article, the task is an orientation search task. Unlike features with linear distributions, such as line length or luminance, orientation distributions wrap around. Furthermore, in Experiment 3, there is a target at 0° with distractors at ±20°. Clearly, without using a different internal representation of the observations, the maximum rule is not the optimal rule for modeling our experiments. The more general ideal observer criterion given in Equation A6 must be used. This is done numerically, rather than analytically, as described below.

First, the decision region must be found numerically. For this, some range of possible pairs of observations (z_i, z_j) is selected. For a distribution of line segment orientation, it makes sense to sample within the range $z_i, z_j \in [-90°, 90°)$, because an orientation of $-90° - \Delta$ is the same as an orientation of $90° - \Delta$. The two-dimensional image $T_i(z_i, z_j) = g(z_i)f(z_j)$ is then calculated for each pair of values (z_i, z_j). The transpose of this image, $T_j(z_i, z_j) \equiv T_i(z_j, z_i)$, gives $f(z_i)g(z_j)$. From Equation A6, the decision region Z_{decision} is equal to $\{(z_i, z_j) : T_i(z_i, z_j) > T_j(z_i, z_j)\}$.

Figure A2a depicted a typical decision region for homogeneous distractors with normally distributed noise in both target and distractor observations. Figure A2b shows what such a decision region looks like once wraparound of angles is accounted for (and the noise is distributed according to a wrapped-normal distribution). Figure A2c shows the decision region for the case of a 0° target, distractors at ±20°, with observation noise distributed according to a wrapped-normal distribution. This corresponds to a *minimum absolute value* rule, in which one chooses the stimulus that yields the observation closest to 0° as the one containing the target. One can analytically show that this rule is optimal for this condition.

The next step is to use the decision region to make predictions of percent correct response on a 2AFC task. This requires some additional notation, as follows. If there is a one-dimensional vertical slice through the decision region Z_{decision}, as shown in Figure A3, then this slice is a function of the horizontal position, x, of the slice and consists of points $S(x) = \{y : (x, y) \in Z_{\text{decision}}\}$. For a given value of x, $F(x) \equiv \int_{S(x)} f(y)dy$ gives the probability of drawing y from the distribution of distractor observations, such that (x, y) falls within the decision region. In other words, given observation x, this formula represents the probability of getting a distractor observation, y, such that it is more likely that x came from a target and y from a distractor than vice versa (i.e., x wins in the pairwise comparison with y). In a similar manner, then $G(x) \equiv \int_{S(x)} g(y)dy$, the probability of drawing y from the distribution of target observations, such that it is more likely that x came from a target and y from a distractor than vice versa (again, x wins).

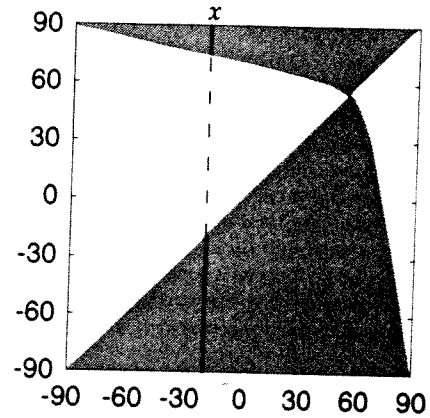

Figure A3. A one-dimensional slice through the decision region, shown for the particular case of observations of orientation and the decision region of Graph B of Figure A2. The solid line specifies, for observation x, the observations, y, for which x is more likely to be the target and y the distractor than vice versa.

In the 2AFC task, there are n items in each of two displays, and exactly one of those displays contains a target element. All of the other elements are distractors. Without loss of generality, if the target appears in the first display, then the ideal observer answers incorrectly if it decides that one of the n distractors in the second display is the target. This happens when that distractor observation wins in pairwise contests with all of the other observations from both displays. If that distractor yields observation z, then the probability of drawing distractor observation z, that distractor winning in pairwise contests with all other distractors, and that distractor winning in a contest with the target is

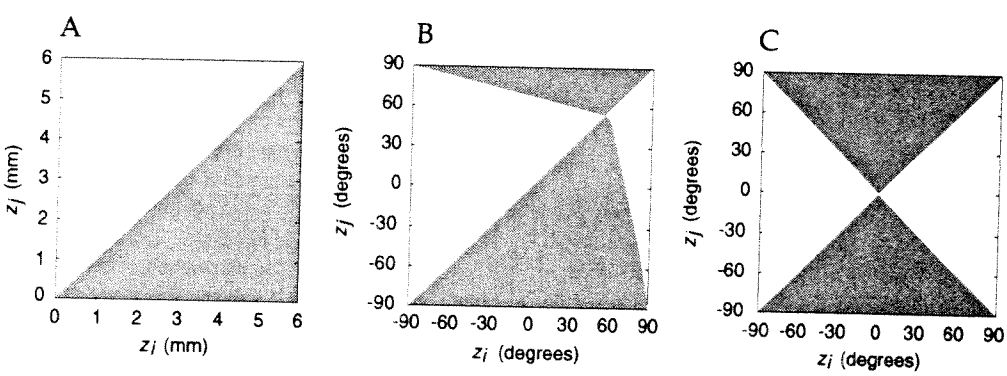

Figure A2. Example decision regions. A: Homogeneous distractors, observations (e.g., of length, as shown here) distributed according to a linear statistical distribution, $\mu_T > \mu_D$. This decision region leads to the common max rule; B: Typical decision region for observations (e.g., of orientation as shown here) with a directional distribution when wraparound of angles is accounted for; C: Target = 0°, distractors at ±20°. The shaded portion of each plot indicates the decision region in which observation z_i more likely came from the target and z_j from a distractor than vice versa.

$$f(z)F(z)^{2n-2}G(z). \tag{A7}$$

The probability of mistaking one of the distractors for the target is the integral of this quantity over all possible values of the observation z. The ideal observer makes an error if any of the n distractors in the second display is mistaken for the target. Thus, the probability of error is

$$n \int f(z)F(z)^{2n-2}G(z)dz \tag{A8}$$

and the probability of a correct response is

$$1 - n \int f(z)F(z)^{2n-2}G(z)dz \tag{A9}$$

In the particular case in which $f \sim N(\mu_D, \sigma)$, $g \sim N(\mu_T, \sigma)$, and $\mu_T > \mu_D$, the decision region is as depicted in Figure A2a, and $F(z)$ and $G(z)$ are just the cumulative distribution functions for distributions $f(z)$ and $g(z)$, respectively. Thus, in this case, Equation A9 reduces to the usual equation for probability of a correct response (used by, e.g., Palmer et al., 1993).

Below, is the MATLAB (1993) code that makes SDT predictions for percent correct performance on an orientation search task, given functions $f(z)$ and $g(z)$, which represent the distributions of target and distractor observations, respectively. (I included the program comments to clarify some aspects of MATLAB notation and functions—the code itself is only eight lines long.)

```
% To solve this problem numerically, we must select a
% set of possible pairs of observations, (Z1, Z2) .
% Z1 & Z2 are matrices of equal size. Each row of Z1 and
% each column of Z2 is:
%     (-90, -89, -88, . . . . , 88, 89, 90) .

[Z1, Z2] = meshgrid(-90:90, -90:90);

% We also need the 1-D vector of possible observations.
% z is the vector (-90, -89, -88, . . . . , 88, 89, 90).

z = -90:90;

% Find the decision region.
% Note that "x.*y" indicates pointwise multiplica-
% tion of matrices x and y (each element (i,j) of
% matrix x is multiplied by element (i,j) of matrix y,
% producing a new matrix of the same size as x and y).

Ti = g(Z1) .* f(Z2);
Tj = Ti'; % Tj is the transpose of Ti.
Zdecision = (Ti > Tj);

% Zdecision is a binary matrix, "1" indicating points
% in the decision region, "0" points not in the
% decision region.

% For each value of x, integrate f and g over (x,y) in
% the decision region.
% Note that "sum" computes the sum within each column
% of a matrix, not the sum over the entire matrix.
% Thus F and G are vectors.

F = sum(f(Z1) .* Zdecision);
G = sum(g(Z1) .* Zdecision);

% Predicted percent correct performance, from Eqn. A9:

Percent_correct = 1 - n*sum(f(z) .* F^(2n-2) .* G);
```

Received December 16, 1997
Revision received August 31, 2000
Accepted November 22, 2000 ■