

The Measurement, Simulation, and Interpretation of the Lifetime of Cosmic Muons

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Abstract

Using multiple scintillators and a logic array, the lifetime of the muon particle was measured to be $2.176 \pm .037 \mu\text{sec}$. From the muon lifetime and the fundamental constant G_F , values of the decay width Γ_μ and the mass of the muon m_μ were calculated. Slight discrepancies between the measurements and the Standard Model were resolved using known quantum behaviors. The muon decay was also modeled by a Monte Carlo simulation. An unexpected sinusoidal deviation of the decay curve from the predicted exponential curve was observed and possible explanations were given.

1 Introduction

Fueled by violent nuclear reactions, stars spew highly energetic particles called cosmic rays into space [12]. The products of these reactions barrage the Earth and the upper atmosphere. The constituents of these cosmic rays include protons, neutrons, neutrinos, and helium nuclei [3] with energies up to 320 exaelectron volts (EeV) [22]. As the cosmic particles enter the upper atmosphere, some interact with matter in the Earth's atmosphere.

The interaction of a cosmic ray with particles in the upper atmosphere (primarily 9-15 km above Earth's surface) usually produces pions [4], bound states of an up and an anti-down quark[17] (see Appendix A for An Introduction to Particle Physics). With a lifetime of 2.6033×10^{-8} seconds, the pion travels only hundreds of meters at velocities between .9661c and .9778c before decaying into a muon and mu-neutrino [23]: $\pi^\pm \rightarrow \mu^\pm + \nu_\mu$. The muons produced in this reaction descend to the Earth's surface. The ample supply of muons at sea level facilitates the study of these particles [17] (see Appendix B for the History of the Muon). In the experiment, the muon's lifetime will be measured by observing the muon's primary decay mode which occurs for $\approx 100\%$ of decays [17]: $\mu^\pm \rightarrow e^\pm + \bar{\nu}_\mu(\nu_\mu) + \nu_e(\bar{\nu}_e)$.

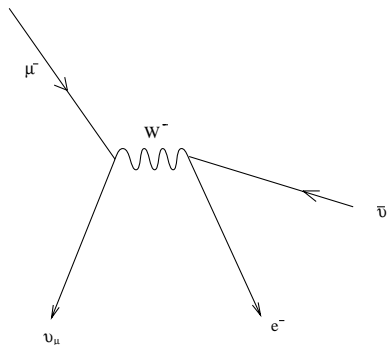


Figure 1: Muon Decay is $\mu^- \rightarrow e^- + \bar{\nu}_\mu + \nu_e$. The antimatter decay is $\mu^+ \rightarrow e^+ + \nu_\mu + \bar{\nu}_e$.

The purpose of this work is to experimentally measure the muon lifetime. To compare these results with the current probabilistic decay model, the decay will be simulated by a Monte Carlo computer simulation. From the muon lifetime and the constant G_F , values of the decay width Γ_μ and the mass of the muon m_μ were calculated. From these values, numerous calculations may be made to describe some fundamental behaviors of matter and energy. Experiments can also be designed with the knowledge of these values.

2 Preliminary Theory

In the ideal experiment, the muon decay would be detected and the lifetime would be recorded through the experiment shown in (Figure 2). In this setup, three scintillators, devices which detect the presence of moving charged particles [1]¹, are stacked on top of each other. The area overlapped by all three devices is the region employed to detect the muon decays.

To observe the muon decay, the muon has to be stopped in the middle scintillator. Once stopped, the muon will decay quickly into an electron and two neutrinos. The neutrinos escape undetected because they have no charge and are therefore invisible to the scintillator[6]. On the other hand, the electron, which speeds out of the scintillators with newly acquired momentum from the decay process, has to exit the bottom of the scintillator stack. The time δt between the time the muon stops and the time it decays will be recorded. After many decays are recorded, the graph of the time until decay versus the number of decays detected in discrete time intervals will be plotted. This histogram will be characterized by an exponential decay curve. From the best fit exponential curve, the lifetime of the cosmic

¹In our experiment, these charged particle are muons that have a charge of -1.

muon can be determined through the time constant τ in the exponential equation: $N = N_0 e^{-\frac{t}{\tau}}$ [14]

3 Experimental Proceedings

3.1 Experimental Parameters

Because the magnitude of cosmic ray flux depends on the angle according to $\cos^2 \theta_{fromzenith}$ for low energy particles ($E_0 < 10$ GeV), the detector array was orientated upward.

The cosmic ray flux is known to be $\approx \frac{180}{m^2 sec}$ at sea level. Based on the geometry of the detector, the expected muon count rate is $\approx \frac{90}{sec}$.

Because electrons and other high energy particles are present in cosmic rays, a filter was constructed to absorb all charged particles in cosmic rays except muons. The filter was ≈ 70 cm of steel stacked vertically above the detector array. It was calculated that the filter would absorb all charged particles with an energy < 4 GeV. This filter reduces the background noise present in the scintillators. The probability of measuring a false muon decay due to random noise inside the electronics and scintillators was calculated to be 1 false decay every 10,000,000 cosmic rays.

3.2 Decay Detection

Scintillators are used to detect charged particles, like muons. Charged particles traveling through the scintillator lose energy ionizing electrons in the surrounding matter [2]. The plastic scintillators used in this experiment absorb $\approx 2 \frac{MeV}{cm}$ [17]. As the excited electrons

return to ground state, photons are given off [24]. For scintillating materials, this light is in the visible range of the spectrum [28]. Other modes of energy loss include Čerenkov radiation, nuclear interactions[10]. These factors are negligible in this experiment[20].

3.3 Signal Processing

The photons from scintillation must be collected for analysis. This collection is executed by the photomultiplier tubes (PMTs). The photons released as the excited electrons return to the ground state reflect off the inside of the scintillating plastic² and eventually collide with the PMT's primary dynode. The photons may also be absorbed in the material itself, thus are not detected by the PMT. Once the photons hit the dynode, the PMT outputs an amplified current pulse. With adjustable gain, the PMTs were calibrated to be accurate and precise enough to detect a single photon. The scintillators used were known to have an efficiency of $\approx 95\%$. The signal is then sent to a discriminator where the rough and jagged signal is processed into a logic pulse.

3.4 Logic Circuit

The signal is then fed into a logic circuit. This logic circuit has two steps; the first step records the time the muon enters the scintillators, and the second records the time when a charged particle exits. The logic circuit basically outputs start and stop signals that other electronics sort through to find two corresponding times that signal a decay event. [28] Normally, incoming cosmic muons are energetic enough to travel through all three scintilators. In this

²Scintillators have total internal reflection [27]

case, the muon is traveling $\approx .97c$ and is detected simultaneously by all three scintillators. No event is recorded. To record a decay, a cosmic muon has to enter the array from above, travel through the first scintillator, and lose enough energy in the system to remain in the middle scintillator long enough to decay. Once the muon is traveling slowly, the scintillator no longer detects the presence of the charged muon [10]. When the scintillator logic circuit encounters a logic pulse from the top and middle scintillators only, the clock is started. As the muon remains in the scintillator, it has a specific probability of decaying.[25] Eventually, the muon will either escape the detector or decay. As discussed in Section 2, the muon decays into an electron and two neutrinos. To record this muon decay, the electron has to exit through the bottom of the array, leaving some of its energy in the middle and bottom scintillators. This signal, from the middle and bottom scintillators but not the top one, will trigger the clock to be stopped. If the muon escapes or the electron exits the array some other direction, the time-to-digital converter (TDC) waits $65 \mu\text{sec}$ to receive a stop signal before it resets the clock. After resetting the clock, the TDC is ready to time the difference between two corresponding start and stop pulses.

The analog timer pulses are converted into a digital signal which is then sent via the General Programming Interface Board (GPIB) to the LabView Program on a computer running Windows 95. The data is ready for analysis.

4 Monte Carlo Simulation

To confirm the accuracy of the data collected, a Monte Carlo computer simulation was conducted of the probabilistic exponential decay curve. The flat probability function (pdf)

generated by the standard computer compiler was transformed to an exponential distribution by the mapping function:

$$y_0 \ln\left(\frac{c-x}{y_0}\right) = y \quad (1)$$

where c is a constant, y_0 is the muon lifetime, x is the original flat pdf, and y is the exponential pdf. With this exponential distribution, the program was run to simulate the random lifetimes of muons. All these random lifetimes were then plotted as an exponential decay curve. This allowed the decay curve from the real data to be cross-referenced for extreme deviations or other unexpected behavior. The full program is listed in Appendix E.

5 Data and Results

Data was collected during over a total of 123 hours. The rate of false starts was $\approx \frac{100}{min}$. The rate of muon decays in the scintillators was $\approx 1 \frac{decay}{min}$. 4526 decay events were recorded. Of those, 24 decay events overflowed the integer memory storage allocation and were invalid data points ³. 4502 good decay events were collected. Figures 3 and 4 show the real decay curve and the simulated decay curve, respectively. The table below compares the results of the two curves. With this data set, the method of plotting and fitting the exponential curve used changed the lifetime calculated from the statistics, thus the methods recommended in [29] were employed.

To determine when the integer overflow occurred and to visualize the distribution of measured lifetimes in the order the decays were observed, a graph of the muon lifetime

³allocation: integer size: 2 bytes range: -32767 to 32767

was plotted against the order the data was taken (see Figure 5). To compare how well the exponential curve fit the data, Figures 6 and 7 show the comparison of the deviation from the exponential fitted predicted curve of the real data and of the simulation.

	Real	Simulation
Fit Equation	$N = 7.104 e^{-.4595 \times 10^{-3}}$	$N = 11.52 e^{-.4636 \times 10^{-3}}$
χ^2	$\frac{24.85}{24}$	$\frac{56.95}{61}$
Life Time	2176 ± 37 ns	2157 ns

6 Discussion

6.1 General Analysis

The scintillators proved to be very efficient. Of 2.35×10^6 expected cosmic ray detections, 2.3×10^6 cosmic rays were detected, an efficiency of 98%. This efficiency improved accuracy and precision in the observed rates and data. The rate of detecting decays was half of the calculated value, but this can be accounted for simply because the logic required the electron to exit the bottom of the detectors. In reality, half of the time, the electron is emitted upwards and thus is not detected. Compensating for this effect, all observed rates were within the error of the calculated values.

At the beginning of the experiment, some data times were stored as negative numbers. The electronics and computer software were checked and it was determined that a software error was causing the error. After the problem was fixed, the data stored correctly.

The exponential curve fit the decay plot very well. The simulation modeled the decay curve accurately. Two slight discrepancies are present in comparing the data, the simulation,

and the Standard Model. First, a sinusoidal wave was seen in the plot of deviation between the data and simulation from the fit line. As the second discrepancy, the life time observed was slightly less than what is predicted by the Standard Model.

6.2 Corrections and Considerations for μ Lifetime

According to the Standard Model, the muon lifetime is calculated to be 2197 ns. In our experiment, the lifetime was experimentally found to be 2176 ± 37 ns. The electronics were examined to find delays that would introduce systematic error⁴. An 8 ns delay was found on the start signal. This delay would cause the lifetime to be too short. The discrepancy is therefore reduced to 11 ns. This discrepancy of the 11 ns may be explained by considering a number of quantum effects which are qualitatively discussed in Appendix E. These effects include muon capture, where the negative muon is captured by a molecule as an electron, and the formation of muonium, where the positive muon captures an electron.

6.3 Significance of the Sinusoidal Deviations

In comparing the two graphs of the deviation from the fitted exponential (data minus fit), a sinusoidal behavior becomes apparent in the real data (Figure 6) where no such form is present in the simulation (Figure 7). This behavior may be attributed to unknown electronics feedback, very improbable random deviation, rare decay processes occurring at specific energies, or muon capture at certain energy values dictated by the scintillating material's molecules. Further investigation is needed.

⁴1 foot of wire takes 1 nanosecond to traverse

7 Discussion of Implications

7.1 Time Dilation and Length Contraction

The existence of muons at sea level illustrates numerous relativistic predictions. With a lifetime of 2176 ns, the muon would be required to travel faster than light to reach the Earth's surface in any detectable quantities. The observation of muons has to be described another way, time dilation and length contraction. Using Lorentz transformations for length contraction and time dilation defined to be: $x_2 = \gamma x_1$ and $t_2 = \gamma t_1$, where γ is a quantity related to how relativistic a particle is, t is time, x is position, and v is velocity [13]. The muon's γ is ≈ 15 , calculated from the equation: $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

Because the muon is traveling at .9661c - .9778c, the distance from the earth to the muon is Lorentz contracted by a factor of 15 from 9 kilometers in the Earth's frame to 600 meters in the muon's frame [15]. This relativistic effect allows the muon to reach the Earth's surface before it decays. From the frame of reference of Earth, the lifetime is time dilated by the same factor γ , making the observations in both frames consistent. The end result is that a significant number of muons travel from the upper atmosphere to the Earth's surface.

7.2 Derivations from the Muon Lifetime

With the decay in the exponential form of $N = N_0 e^{-\frac{t}{\tau}}$, the flux in the upper atmosphere can be calculated from the flux at sea level from $\Phi_{upperatmosphere} = \Phi_{sealevel} e^{\frac{-t}{\tau}}$, where τ is the muon lifetime and $t = \tau$ in calculating the flux present one lifetime before $t_0 = 0$. Thus, the data from this experiment indicates that the flux in the upper atmosphere (9 kilometers)

is $\approx 490 \frac{\mu}{m^2_{sec}}$. This flux can then be used in calculations and simulations in air showers, particles produced when cosmic rays interact with the upper atmosphere.

The muon lifetime and the fundamental Fermi coupling constant G_F can be used to calculate the decay width and the mass of the muon. From the decay width⁵ of

$$\frac{1}{\tau_\mu} = \Gamma_{mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}, \quad (2)$$

values of the decay width Γ_μ , and the mass of the muon m_μ may be calculated [30].

The decay width is calculated to be 4.596×10^5 eV. The mass of the muon is calculated to be $108.7 \frac{MeV}{c^2}$, with an error of 3.5 % from the value from the Standard Model. These values can describe the fundamental behavior of the muon [11].

8 Conclusion

The muon lifetime was experimentally determined to be 2176 ± 37 ns. The Standard Model value of τ_μ is 2197 ns. The differences in the values for the lifetimes were resolved by analyzing cable delays, muon capture, and muonium formation. The values of Γ_μ and m_μ were calculated to be $108.7 \frac{MeV}{c^2}$ with the error 3.5% from the values from the Standard Model and 4.596×10^5 eV, respectively. The sinusoidal deviation of the muon lifetime from the predicted decay curve is left unresolved. In the following appendices, possible areas of future research have been suggested and the muon-muon collider's feasibility has been analyzed.

⁵The decay width is a measure of how tolerant the decay is to variations of the parameters and is used in numerous calculations.

9 Acknowledgments

I would like to thank my mentors, Dr. Melissa Franklin and Dr. Werner Riegler, for providing the opportunity to work under their care and learn about particles physics. I would also like to thank the Center for Excellence in Education, specifically Mrs. DiGennaro and Dr. Saul, for allowing me the great pleasure of attending the Research Science Institute. I would like to thank the two college students who answered all of my questions, Greg Novak and Carter Hall.

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A Introduction to Particle Physics and the Standard Model

Particle physics studies the behavior of the fundamental constituents of the universe. These behaviors often involve observing collisions and decays of the particles being studied[7]. When collisions and decays are studied, the fundamental principle that is always perserved is $E=mc^2$, meaning mass can be converted into energy and vice versa[9]. In decays, the mass of the unstable particle is converted into energy and smaller particles through certain decay pathways. The decays allowed are dictated quantum electrodynamics and quantum chromodynamics [8].

Currently, the Standard Model, a group of theories describing the behavior of matter and energy, consists of three families of particles. Each family has three generations of similiar, yet increasingly heavy particles and their antimatter counterparts. The leptons is the family of particles that this research explores. Leptons are fundamental particles. The leptons are made of three sets of particles, the electron and electron-neutrino, the muon and muon-neutrino, and the tau and tau-neutrino. Each of these particles has an antimatter counterpart that is identical to its partner except for the antimatter has exactly the oppisite charge.

For further reading, see references [4], [6], [11], or [21].

B The Historical Development of the Muon

“The muon was ... signifying the end of the days of innocence...” -Murray Gell-mann, 1957.

“Who ordered that [muon]?!” -I. I. Rabi, 1957

As is evident, the initial discovery of the muon, essentially a heavy electron, was not received very well, because it shattered their theories of matter and energy. Today, the muon is an integral part of almost every field in physics including elementary particle theory, chemistry, and nuclear structure [3].

In 1937, Anderson and Neddermeyer first observed the muon on a highly sensitized photographic plate exposed at the top of a mountain. These scientists collected data at the top of the mountain, because the primary interactions of cosmic rays and the atmosphere occur at high altitudes. At the time, this newly discovered particle was thought to be a meson, a bound state of two quarks, predicted by Yukawa. The particle he predicted was the carrier of the nuclear force since the mass ($\approx 207 \text{ MeV}$) was close to the predicted value of Yukawa’s particle. However, this “mu-meson”, as it was called, was found to be a deeply penetrating particle as it was detected in deep underground mines. Evidently, if the particle was to travel through thousands of feet of matter, it did not interact strongly with matter as the carrier of the nuclear force should. With this result and data from the neutrino production and other decay modes, scientists concluded that a second generation of particle must be accounted for in the theories. Thus, the muon took its position as the heavy electron. As the theories progressed, the Standard Model took form and the three generations emerged. In conclusion, the muon’s discovery led to the development of the current form of the Standard Model.

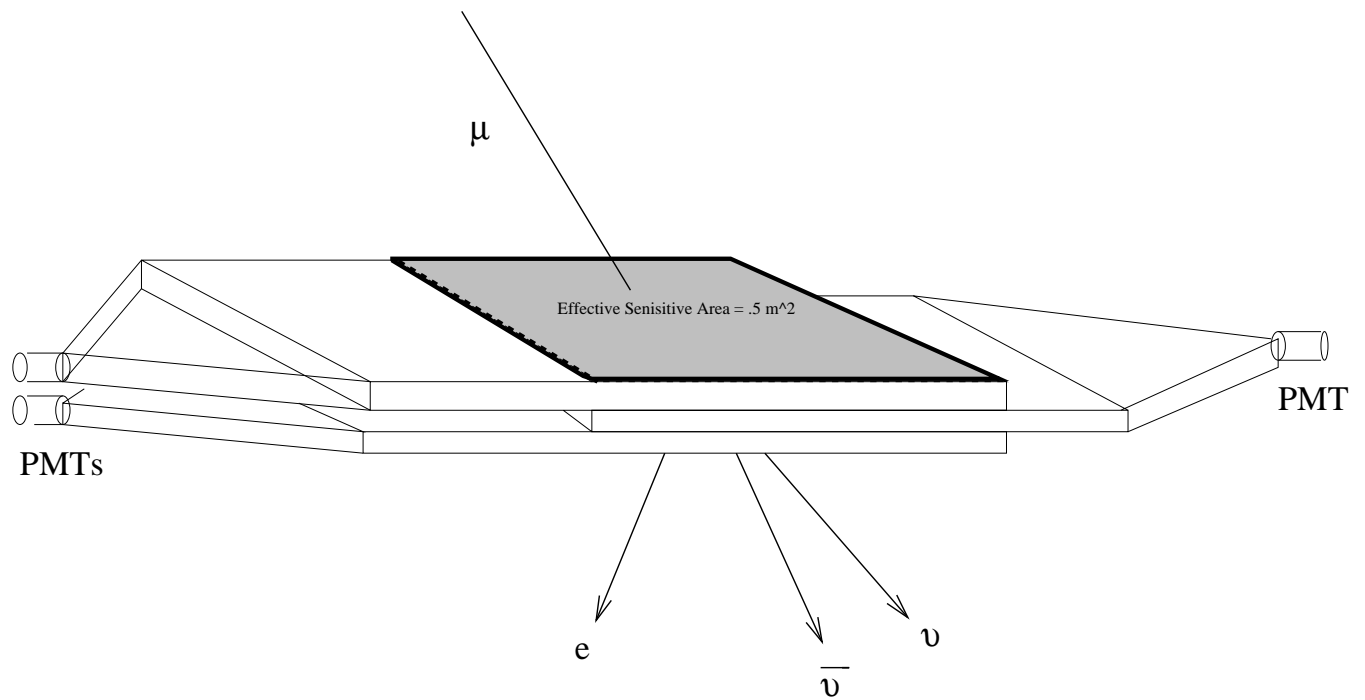


Figure 2: Diagram of Scintillator Array

C Monte Carlo Simulation

```
#include<stdio.h>
#include<math.h>
int main(void)
{
    double data[4526], intermed;
    int index;
    for(index=0, index<4526; index++)
    {
        intermed = rand()/3276.7;
        data[index]=-2197.03 * log((10 - intermed)/2197.03);
        printf('%lf \n', data[index]);
    }
    return 0;
}
```

D Muon Capture and the Formation of Muonium

Another source of error is related with a basic assumption of the experiment, it is assumed that the charge on the muon makes no difference on the lifetime. This assumption is true if the particle is in space. But in the presense of matter, a few quantum effects accent the erroneous assumption. First of all, there is an inherent unbalance of μ^+ and μ^- . Exactly, $\frac{\mu^+}{\mu^-} = 1.14 \pm .04$. The true effect that makes a difference is executed when the muons enter the matter. Once at non-relativistic speeds, the negative muons have a chance of being captured by the atoms in the scintillator. This capture is on the time scale of 10^{-14} sec which makes it a fast enough interaction to effect muon lifetime. By allowing the muon to come to a lower energy state through the interaction: $\mu^- + N \rightarrow N^* + \nu_\mu$, a new pathway effectively shortens the lifetime. This is shown by the lifetime without the absorption mode: $\tau = \frac{1}{\Gamma_{decay}}$ compared to the lifetime with the absorption mode: $\tau = \frac{1}{\Gamma_{decay} + \Gamma_{absorb}}$. The addition term in the denominator shortens the muon lifetime τ .

Finally, the last source of error comes from the formation of muonium. When the free, non-relativistic positive muon captures a free electron, the electron emits photons as it falls into the muon's potential well. This interaction also happens on the time scale of 10^{-14} seconds. Thus, this fast effect can mimic the scintillation of an emitted electron after the decay of the muon at rest that is capturing the electron. This emission could give a premature stop signal in coincidence with a cosmic muon hitting the bottom scintillator, effectively shortening the lifetime.

E Applications

As the Standard Model progresses, new theories are formed and tested by new experiments. This research provides insight into the storage of muons for a muon-muon collider. If such a collider were built, the muons must be kept at relativistic velocities to extend their lifetime to the Earth's frame. Also, no conventional storage ring could stack muons for more effective cross-sections. With the short lifetime, the muons would not be able to circulate in the collider for any extended period. To solve this problem, the means of muon production must be capable of making many muons. In addition to many muons, the beam confinements would have to be very strict to assure enough events are observed. The advantages of a muon-muon collider include a much cleaner interaction at high energies. The muon-muon collider is possible, but new parameters must be met to perform at the appropriate level.

Some areas of future research inspired by this research include the fundamental process of decay being a dissipative effect to lose mass, which may be considered just a disturbance in space-time. Also, numerous fluctuations of the muon behavior can be studied in search for constituents of the muon if it is not fundamental. These fluctuations include fluctuations of the lifetime overtime, over energies, and over different decay modes. Possibly different decay modes have different lifetimes associated. If this is found to be the case, then perhaps the conversion of mass to energy and vice versa actually takes a finite amount of time to complete. Along the same lines, if the decay ratio $\frac{x}{e^{\pm}\nu_{mu}\nu_{e^{\pm}}}$ where x is e^{\pm} , ν_{mu} , $\nu_{e^{\pm}}$ changes between different decays and different decay modes, then again, the muon might be found to be non-fundamental.

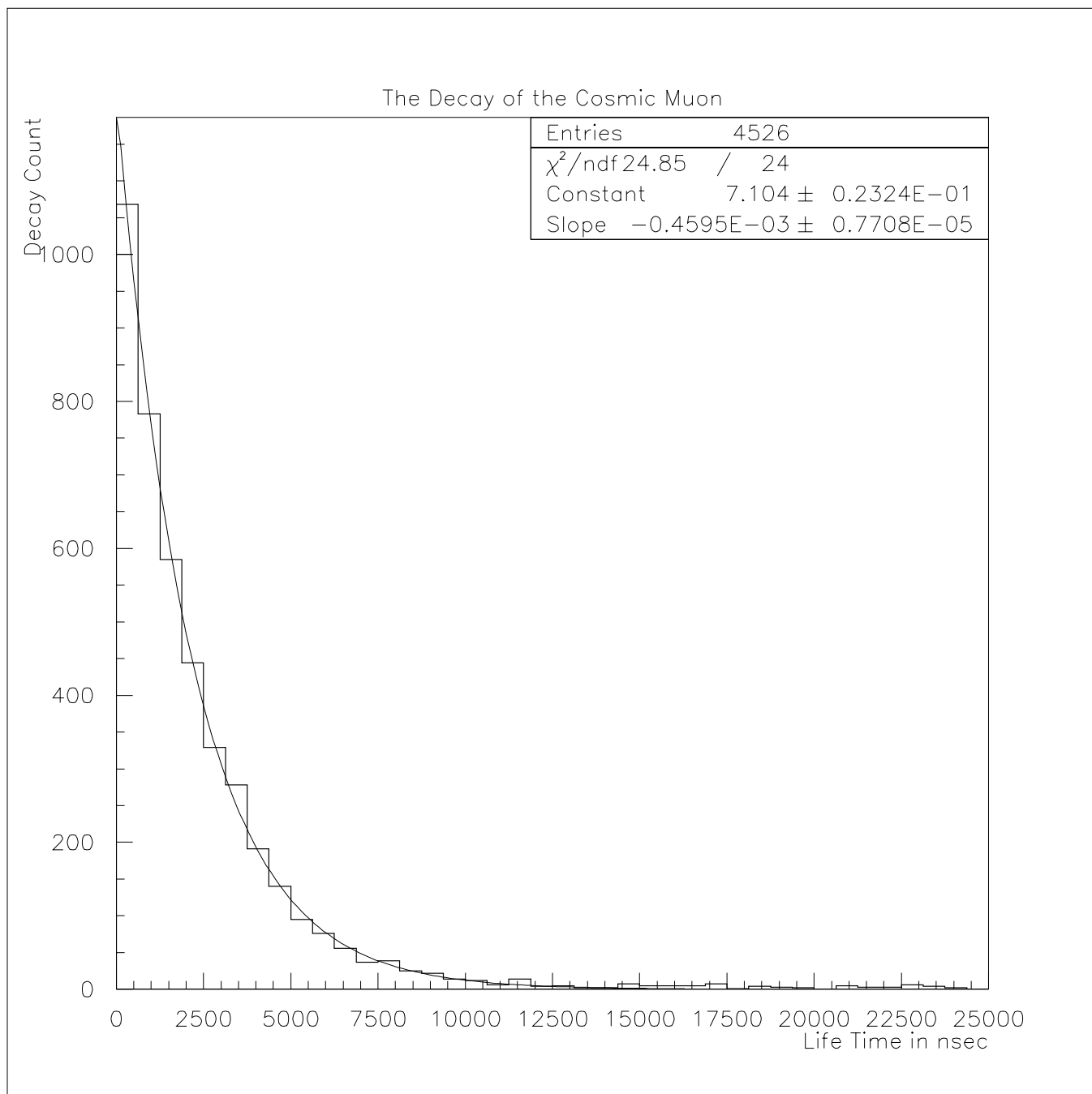


Figure 3: Actual Muon Decay Curve from Data

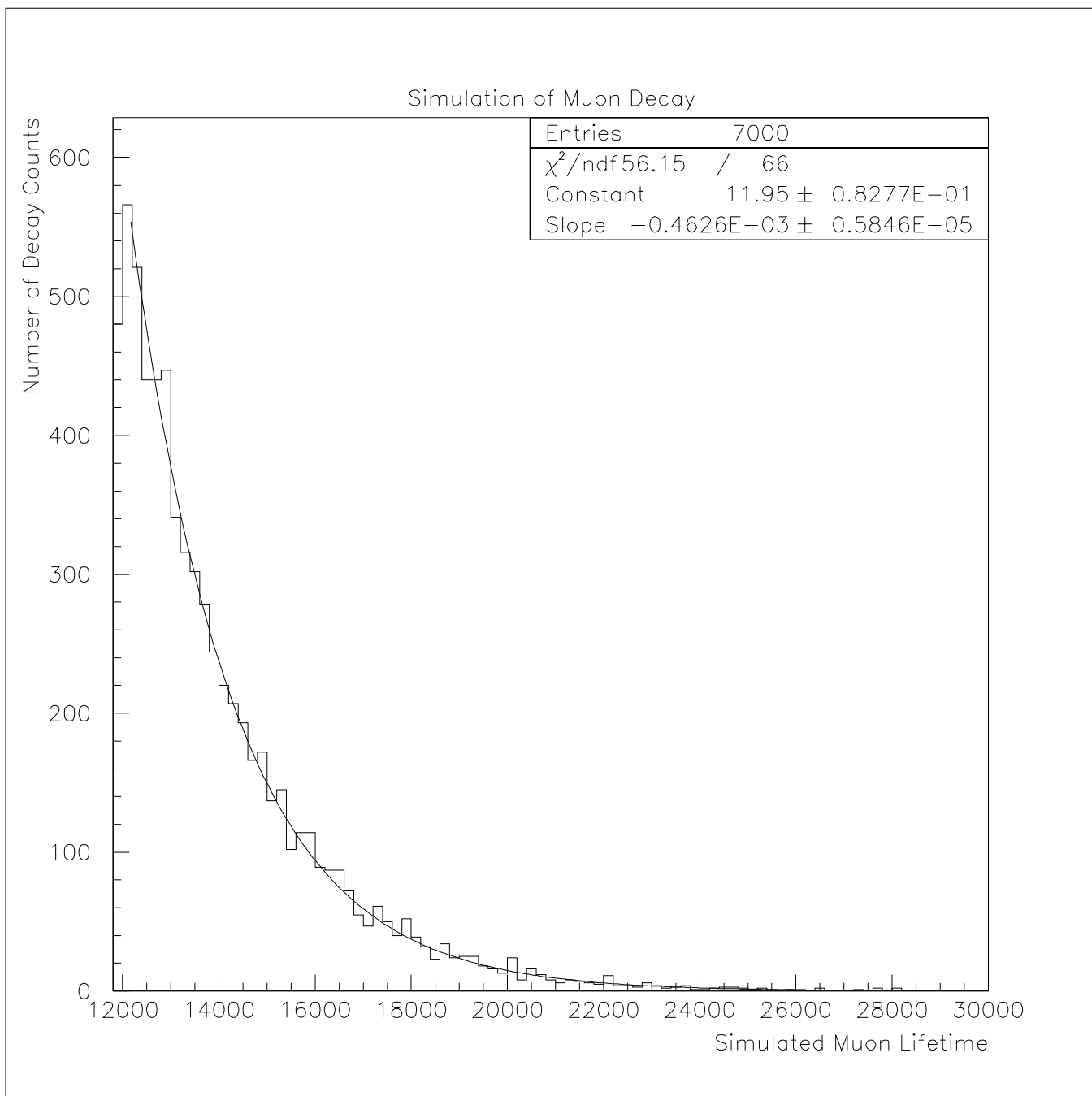


Figure 4: Monte Carlo Simulation of Muon Decay Curve

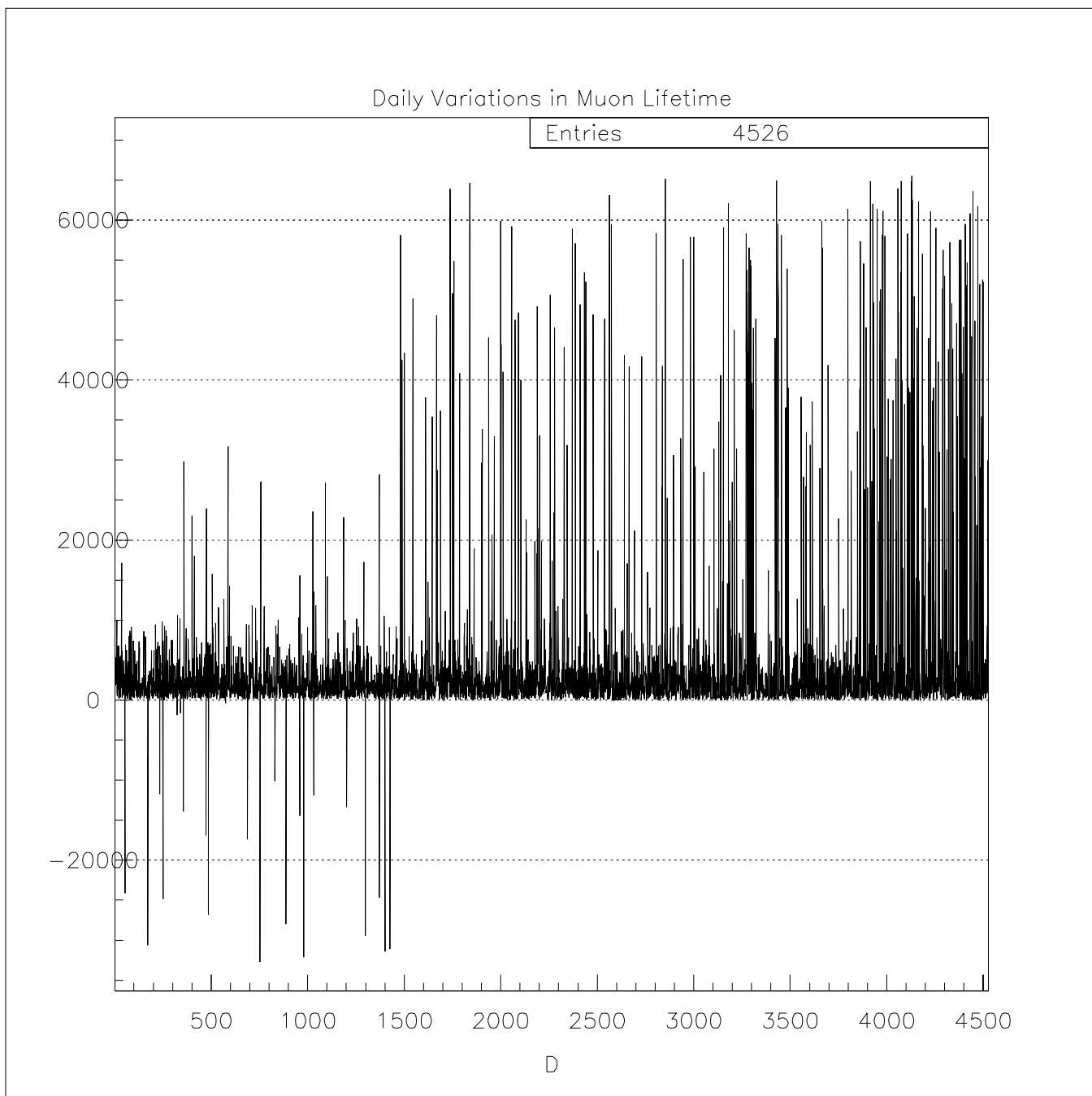


Figure 5: Lifetimes in Order of Detection

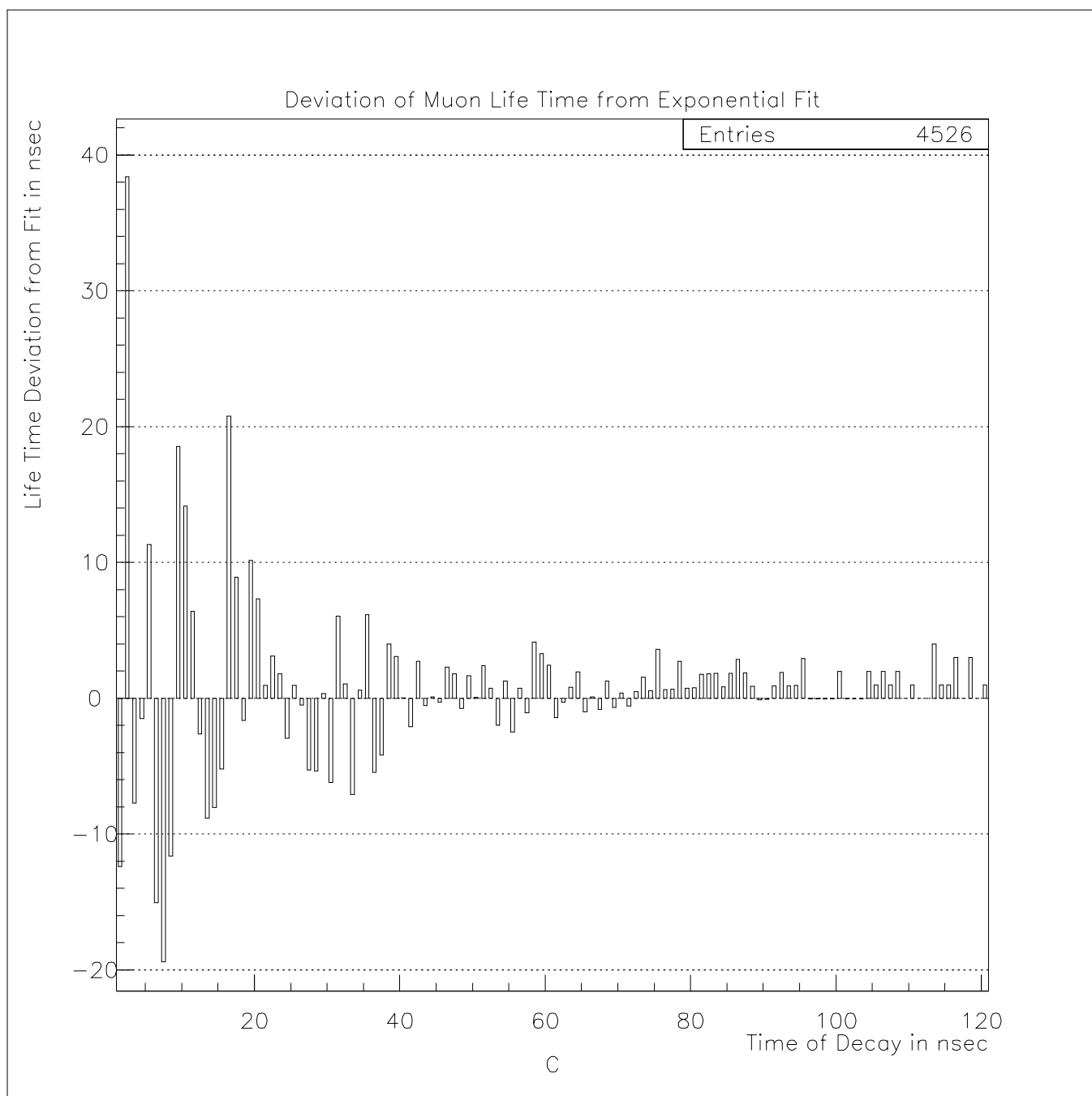


Figure 6: Deviation of Data from Fitted Curve

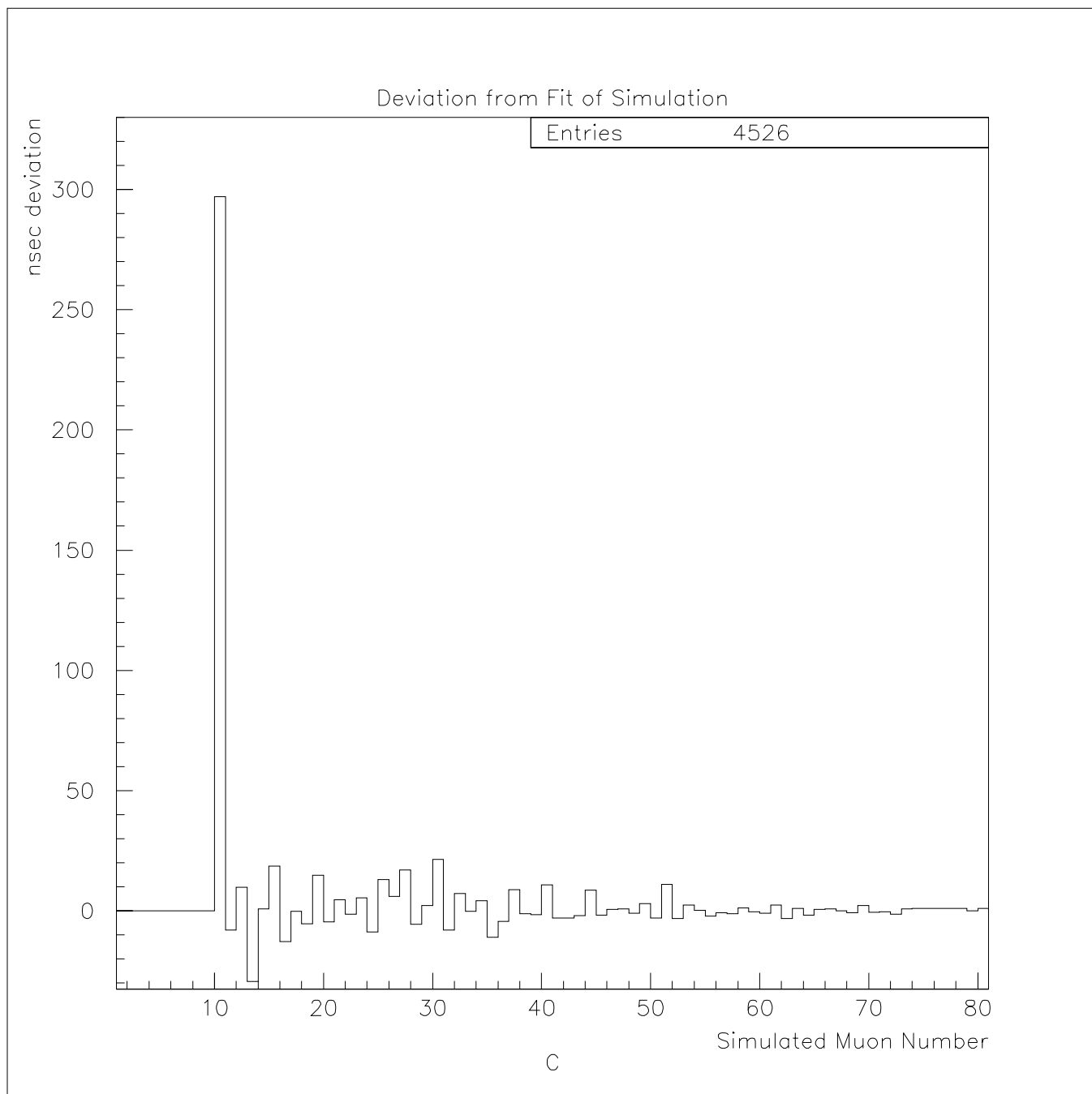


Figure 7: Deviation of Simulation from Fitted Curve