Alliance Formation, Alliance Expansion, and the Core

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This article presents a simple cooperative game theory representation of alliance formation and expansion to counter a conventional threat along the allies' borders. Mutual defense gains, derived from allying, arise from interior borders that no longer require protection. Spatial and locational attributes of the allies are crucial when identifying the gains from mutual defense and the distribution of these gains. The same number of allies can have vastly different cores depending on their spatial configurations. Extensions to the baseline case consider transaction costs, natural defenses, guerrilla warfare, and risk concerns. An application to the NATO alliance indicates that the theory has much to say about which additional Partnership for Peace countries are likely to join NATO after the three Visegrad countries.

The recent admittance of the Czech Republic, Hungary, and Poland into NATO in March 1999 prior to its 50th anniversary raises questions as to how many other Partnership for Peace nations will eventually become members. In particular, will future entrants include the Baltic states (Estonia, Latvia, and Lithuania), Slovakia, Slovenia, Austria, Switzerland, or Romania? Thus far, the debate about expansion has centered on the associated costs, with estimates ranging from $2 billion (NATO 1995) to $125 billion depending on alternative assumptions, time horizons, and expansion cost definitions. Although benefits from expansion are briefly mentioned, the current focus on costs implicitly assumes that expansion benefits always outweigh expansion costs and do not differ among alternative expansion scenarios. Neither of these implicit assumptions is defensible. The expansion issue is related to alliance formation (i.e., an expansion from zero allies), which hinges on whether prospective allies view their membership as providing a net gain after associated costs are covered. These net gains are dependent on alliance size and composition, which can affect benefits (e.g., cost reductions, enhanced deterrence) and costs (e.g., decision making, joint maneuvers).


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The reigning model in the alliance literature is the public good model, originally formulated by Olson and Zeckhauser (1966) and later extended by van Ypersele de Strihou (1967), Sandler and Cauley (1975), Sandler (1977), Murdoch and Sandler (1982), McGuire (1990), and others to impure public goods, joint products, and other considerations. Although this noncooperative game-theoretic model is suited to examining burden sharing, defense demands, and allocative efficiency, the model is ill suited for investigating the distribution of net benefits, alliance formation, expansion, or stability (i.e., is alliance membership stable?). In fact, the pure public representation of alliances implies that all willing nations should be included, since benefits per existing ally remain unchanged whereas cost per ally falls owing to cost sharing when new allies are admitted (Sandler and Hartley 1999).

A club theory formulation of alliances can address membership size based on the thinning of forces and cost-sharing considerations when conventional weapons protect borders (Murdoch and Sandler 1982; Sandler 1977; Sandler and Forbes 1980). Club theory cannot, however, readily investigate alliance stability and expansion while accounting for locational considerations. To address these issues along with alliance formation, we put forward an alternative paradigm based on cooperative game theory. In particular, a mutual defense game is presented for which the motivation for forming an alliance is cost savings as borders become interior and no longer need protecting (Gardner 1995). These interior borders then provide a benefit-based representation of alliance formation and expansion where spatial considerations—ally’s location and size—become crucial. Other factors (e.g., transaction costs, natural defenses, deterrence) are appended to the basic model to increase realism and foster further insights.

The absence of a need to guard interior borders implicitly assumes that nations ally themselves with friendly nations from which an attack will not be threatened. Although there are some notable exceptions to this rule (e.g., Germany and Russia in 1941, Greece and Turkey over Cyprus in 1974), contemporary allies are often drawn from nations sharing economic linkages, democratic principles, and membership in other international organizations (e.g., ANZUS, Japan-U.S. alliance). In a recent study, these factors have been shown to inhibit military conflict (Russett, Oneal, and Davis 1998). Furthermore, Russett, Oneal, and Davis established that alliance membership, economic interdependence, and democratic values help explain membership in international organizations, which serve a problem-solving function. Economic interdependence, for example, implies that conflict between dependent nations would result in significant mutual economic costs. For the Cyprus dispute, the membership of Greece and Turkey in both NATO and the United Nations provided a mechanism for ending the hostilities and separating the forces. Except for the Cyprus incident, NATO has included friendly nations sharing democratic values, economic interests, and membership in a nexus of international organizations (e.g., the European Union, Partnership for Peace, Organization of Security and Cooperation in Europe). A number of NATO’s missions are to promote this economic interdependence and the network of overlapping organizations (Sandler and Hartley 1999, chap. 7). The requirements for Partnership for Peace membership, a precursor to NATO membership, mandate that entrants possess democratic principles and not have territorial disputes with neighbors (Partnership for Peace 1996).
The primary purpose of this study is to present a simple cooperative game theory representation of alliance formation and expansion when allies seek mutual defense from a conventional threat in all directions along their perimeters. This theory represents an alternative paradigm for viewing alliances. In particular, the core solutions, if they exist, are identified for alternative scenarios of mutual defense involving diverse spatial configurations, warfare scenarios, and alliance sizes. Second, the paradigm is applied to illuminate the NATO alliance and the likely outcome of its expansion. Third, the analysis is extended to include numerous additional considerations.

This study demonstrates the importance of spatial and locational considerations when determining the bargaining strength of the allies and the resulting distribution of the net gains from allying. Geographical configurations matter: the same number of allies can have vastly different cooperative equilibria (i.e., cores) depending on how they are arranged. In fact, the classic prediction that the core shrinks when the number of participants increases may not apply for some spatial configurations of allies. The type of threat is also an important factor; nations facing insurgencies offer little if any cost savings from mutual defense and make for bad allies. Transaction cost considerations can have a profound effect on the distribution of alliance gains.

This study contains seven primary sections. The first puts forward theoretical preliminaries and definitions associated with a cooperative game representation. In the second section, a number of three-country mutual defense scenarios are analyzed; in the third section, four-country and five-country alliances are investigated. The effects of deterrence, as derived from a strategic arsenal, are briefly presented in the fourth section. Insights gleaned from the analysis are then applied to NATO in the fifth section. Other considerations are examined in the sixth section, which is followed by concluding remarks.

PRELIMINARIES

A few preliminary definitions and concepts are required to understand the bargaining game of mutual defense. Agreement in bargaining games must be unanimous so that each participant has a veto. This means that an agreement must provide all participants with as much or more than they can obtain on their own. If an agreement is not consummated among bargaining parties, then any dissatisfied party can leave the talks with a disagreement payoff equal to what a potential participant(s) can earn on their (their) own. When parties in a negotiation have unequal disagreement payoffs, the party with the largest such payoff or option is at a bargaining advantage. It can be anticipated that the party will gain relatively more from an agreement, if one is reached. Thus, if a potential ally’s strategic location is responsible for much of the gains from a proposed alliance, then that ally is anticipated to obtain a larger share of these gains from such an alliance.

A coalition is any subset of a population $N = \{1, 2, \ldots, n\}$ of players or countries. Three kinds of coalitions are germane to the model: (1) single-player or singleton coalitions, $\{i\}$, of only player $i$; (2) a grand coalition of all $n$ players, denoted by $N$; and (3) intermediate coalitions, $S$, of 2 to $n-1$ players. An intermediate coalition is a proper subset of the population, so that $S \subset N$. The power set denotes all coalitions or subsets of the relevant population, including the null set. In the case of population $N$, the power set minus the null set contains $2^n-1$ potential coalitions, which can form and are relevant for determining the stability of an agreement among a coalition of players.

The characteristic or coalition function indicates the sum of net payoffs to the set of players in any coalition of the population. For coalition $S$, the characteristic function—denoted by $v(S)$, represents the “security” payoff to the group if it were to leave the negotiations and strike out on its own. Thus, any approved agreement must guarantee a sum of payoffs to participants for all potential coalitions that is at least as good as their coalition values, $v(S)$. Based on the characteristic function, the grand coalition has value $v(N)$ whereas the singleton sets have value $v(\{i\})$ for $i = 1, 2, \ldots, n$.

The net payoffs to any set of $s$ individuals in coalition $S$ from accepting a proposal in a negotiation is vector $u = (u_1, u_2, \ldots, u_s)$, where $u_i$ is the agreement payoff for individual $i$. The utility levels in this vector are assumed to be transferable among people. Transferable utility means that monetary side payments between players are permissible, and the players’ utilities (i.e., payoffs) are essentially linear for the range of possible payoffs (Luce and Raiffa 1957). Allies often engage in side payments to one another (e.g., German and Japanese contributions to Desert Storm, or U.S. transfers of nuclear technology to the United Kingdom and France), which support the transferable utility assumption underlying the analysis. Any positive affine transformation, such that $u_i = au_i + b$ for $a > 0$ for all $i$, does not affect the strategic features of an underlying bargaining game owing to linear invariance (Binmore 1992; Gardner 1995; Luce and Raiffa 1957, 186-87). Such transformations merely change the origin and the unit of measurement of the payoffs. Suppose that a proposal is presented to the grand coalition. If such a proposal is individually rational to accept, then it must satisfy the following inequalities:

$$v(\{i\}) \leq u_i$$

for all $i$ in $N$. That is, each individual must be better off or at least no worse off by its payoff from the proposal relative to going it alone as a singleton set. Individual rationality ensures that there are no regrets from the agreement.

Payoffs associated with an acceptable proposal must fulfill individual rationality and Pareto optimality,

$$v(N) = \sum_{i=1}^{n} u_i,$$

so that the grand coalition offers the population the same payoff as the aggregate payoff of any proposal involving everyone. This Pareto optimality requirement implies group rationality where the entire group is able to match any payoff of the coalition of the whole. Payoff vectors satisfying (1)-(2) are called an imputation.
For purposes here, only those imputations in the core are of interest. A proposal is in the core if the following inequalities are satisfied:

\[ v(S) \leq \sum_{i \in S} u_i \text{ for } S \subseteq N. \]  

Because \( S \) can be the singleton sets as well as the grand coalition, the inequalities in (3) satisfy (1)-(2) and coalition rationality, for which no intermediate coalition in the power set of nonempty sets can block the proposal by doing better when leaving the negotiations. Quite simply, the core ensures that no coalition can form and do better than what it obtains in aggregate payoffs from the agreement. Consequently, no coalition will block the core agreement when the inequalities in (3) are satisfied. A core to a game may be empty, as is the case of the majority rule game for which voters try to divide a fixed prize among themselves. Every majority coalition can be blocked, so that a stable coalition never forms (Gardner 1995, 400-401). The mutual defense game will, however, often have a set of solutions in the core.

MUTUAL DEFENSE: THREE COUNTRIES

Nations are initially assumed to confront a threat to their perimeters or borders that can come from any direction. The focus is on threats involving conventional weapons, since cost savings from sequestrating interior borders applies to these weapons and not to strategic weapons. The impact of strategic weapons on the analysis is taken up after the basic models are presented. A baseline case, taken from Gardner (1995), serves as a benchmark for comparison with myriad novel cases put forward in this study.

The cost of defending a country from an external threat is assumed to depend on the length of its perimeter. Even if a country stations its mobile troops in the interior and deploys them to its borders only when an attack is imminent, the cost of defending the country is still proportional to the perimeter of the area guarded. This follows because the greater a country’s perimeter is, the more forces that must be deployed to the border to provide adequate protection during crises. A country the size of France needs more forces for security than, say, Belgium no matter where the forces are stationed during peaceful times. Thus, the assumption that defense costs against conventional attacks are related to perimeter length is appropriate.

BASELINE CASE

Suppose that three contiguous countries are contemplating a mutual defense pact in which each country is a unit square with four 1-inch borders to protect from an external threat in all directions. Suppose further that the three countries are configured as pact a in Figure 1 so as to form a 3-inch by 1-inch rectangle. The cost of protecting a country or alliance against a conventional attack from abroad is assumed to be proportional to the defended perimeter. In particular, each side of the countries depicted is assumed to cost 1 to defend, so that \( v(\{i\}) = -4 \) is the cost of protecting nation \( i \) on its own. Similarly, \( v(S) \) represents the cost of protecting an alliance of countries in \( S \). For the three-
country region, there are seven nonempty possible coalitions, including the three singleton sets, that could form. To be in the core, the payoff \( u = (u_1, u_2, u_3) \) for the three-country mutual defense pact must satisfy the following inequalities:

\[
\begin{align*}
\nu(i) &= -4 \leq u_i \quad i = 1, 2, 3 \\
\nu(\{1, 2\}) &= -6 \leq u_1 + u_2 \\
\nu(\{2, 3\}) &= -6 \leq u_2 + u_3 \\
\nu(\{1, 3\}) &= -8 \leq u_1 + u_3 \\
\nu(\{1, 2, 3\}) &= -8 = u_1 + u_2 + u_3.
\end{align*}
\]
It is easier to interpret the requirements for the core if the \( u_i \) payoffs are transformed so that the disagreement point has payoffs of \((0, 0, 0)\) as all nations go their separate ways. The positive affine transformation that accomplishes this task, while maintaining the maximum cost savings at 4, is the transformation \( u_i = u_i + 4, i = 1, 2, 3 \). When this transformation is applied to the right-hand inequalities in (4), there is no change to the strategic structure. As a consequence, the core now must satisfy the following requirements:

\[
\begin{align*}
\nu(i) & = 0 \leq u_i, \quad i = 1, 2, 3 \\
\nu(\{1, 2\}) & = 2 \leq u_1 + u_2 \\
\nu(\{2, 3\}) & = 2 \leq u_2 + u_3 \\
\nu(\{1, 3\}) & = 0 \leq u_1 + u_3 \\
\nu(\{1, 2, 3\}) & = 4 = u_1 + u_2 + u_3.
\end{align*}
\]

The coalition function in (5) indicates that alliances containing one or more interior borders (e.g., \( \{1, 2\}, \{2, 3\} \)) achieve cost savings from borders that no longer require protection. Each interior border saves 2 in costs, since an attack can come from either side. It is these cost savings that motivate alliance formation. Alliances containing a middle country have something to bargain over in contrast to an alliance of the two noncontiguous end countries where there are no interior borders. As interior borders grow in number or length, the motivation for allying increases in the mutual defense game.

The core of the mutual defense game can be found graphically in Figure 2 in an equilateral triangle whose three vertices correspond to a single nation obtaining the entire gain from cost sharing in the three-country alliance; that is, \( u_i = 4 \) for \( i = 1, 2, 3 \). Moreover, the three sides of the triangle satisfy \( u_i + u_j = 4, u_j + u_k = 4, \) and \( u_i + u_k = 4 \), where each of the two-country alliances obtains the entire gain. The perpendicular distance of a point in the triangle from any side indicates the payoff of the unlisted ally on that side of the triangle; for example, the payoff to nation 3 is the height from the base where payoffs to nation 1 and 2 sum to 4. Every interior point of the triangle has a non-zero payoff for all three nations, but not every point in the triangle in Figure 2 is in the core. To be in the core, equation system (5) must be satisfied, so that \( 2 \leq u_1 + u_2 \) and \( 2 \leq u_2 + u_3 \) must hold. The line \( u_1 + u_2 = 2 \), parallel to the triangle’s base, denotes the payoffs where nations 1 and 2 obtain an aggregate benefit of 2, whereas nation 3 receives 2. The first inequality rules out all of the cross-hatched area above this line in Figure 2, whereas the second inequality rules out the shaded area to the right of \( u_2 + u_3 = 2 \). The unshaded rhombus with vertices \((0, 2, 2), (2, 0, 2), (0, 4, 0), \) and \((2, 2, 0)\) represents the proposals in the core.

Because the region representing the mutual defense core is convex, any line segment joining two core proposals is itself in the core. The center of mass at \((1, 2, 1)\) in Figure 2 is located at the intersection of the two diagonals of the rhombus, each of which links two of the four “extreme” core proposals where one or two nations receive
no benefits. The center of mass gives a centroidal or median outcome of the core, where the middle ally gains half of the cost savings and the two end allies evenly divide the remaining savings of 2. This centroid has much to recommend it, since it accounts for the relative bargaining strength as reflected in a coalition’s disagreement payoffs. Certainly, the middle country is essential for cost savings, since it is this ally’s relative location vis-à-vis the other two allies that creates interior borders from which savings arise. This simple game representation ignores many complications that add realism, but these are now introduced.

**ALTERNATIVE BORDER LENGTHS: THE INTERIOR ALLY**

We shall first consider cases in which the interior or middle country’s width is smaller than the widths of the end countries. Suppose that the three potential allies are configured along a line as before, but the dimension of the middle country is now .5 inch by 1 inch, whereas the two outer countries are still 1-inch squares (see pact b in Figure 1). Before any transformation of the utility payoffs, the core must satisfy the following equations:

\[
\begin{align*}
\nu(\{1\}) &= -4 \leq u_1 \\
\nu(\{2\}) &= -3 \leq u_2 \\
\nu(\{3\}) &= -4 \leq u_3 \\
\nu(\{1, 2\}) &= -5 \leq u_1 + u_2 \\
\nu(\{2, 3\}) &= -5 \leq u_2 + u_3 \\
\nu(\{1, 3\}) &= -8 \leq u_1 + u_3 \\
\nu(\{1, 2, 3\}) &= -7 = u_1 + u_2 + u_3.
\end{align*}
\]


A positive affine transformation of $u_i$ is then found that maintains the maximum cost savings at 4 while keeping the three disagreement payoffs at 0 or above. The following transformation has the property $u'_i = 4(u_i + 4)/5$. When this transformation is applied to the right-hand inequalities and the single equality of (6), the core must then satisfy

\begin{align*}
    v(1) &= 0 \leq u_i,
    
    v(2) &= 4/5 \leq u_j,
    
    v(3) &= 0 \leq u_k,
    
    v(1, 2) &= 12/5 \leq u_i' + u_j',
    
    v(2, 3) &= 12/5 \leq u_j' + u_k',
    
    v(1, 3) &= 0 \leq u_i' + u_k',
    
    v(1, 2, 3) &= 4 = u_i' + u_j' + u_k' .
\end{align*}

The core is solved graphically in the equilateral triangle in Figure 3 based on the payoff requirements in (7). The rhombus, representing the core of this mutual defense game, is formed by the two lines where $u_i' + u_j = 12/5$ and $u_j + u_k = 12/5$, and the sides of the triangle where $u_i' = 0$ and $u_k' = 0$. The center of mass of the unshaded core is at $(4/5, 12/5, 4/5)$, where the advantage of the interior country has grown at the expense of the end countries as compared with the baseline case, owing to the interior country’s smaller size vis-à-vis its neighbors.

Next, consider the scenario in which the interior ally’s width is just one-quarter of an inch. Based on the procedure just used, the center of mass of the core is $(8/11, 28/11, 28/11)$. 
Once again, the interior country’s advantage at the median position has increased. In the limit, the interior country can shrink to a 1-inch length of border with no width and two directions to defend. By applying the same procedure, we identify the center of mass of the core in this limiting case to be \((2/3, 8/3, 2/3)\), where the interior country’s share is now two thirds of the entire cost savings. Although this last case seems extreme, it is analogous to an interior country in an alliance surrounded by allies so that in effect there are no external exposed borders to guard. This location places it in an extremely strong bargaining position, and the final imputation should reflect this advantage.

When the interior country has a longer width than the two end countries, its bargaining advantage diminishes, insofar as it has a longer perimeter to defend on its own than its two potential allies. The interior nation’s position, if it were to walk away from the negotiations, is less than the other two countries. At the same time, the end countries need the interior country if any cost savings are to arise, and this means that the interior country still maintains some leverage. At which point does the interior country’s disadvantage of a longer width offset its locational advantage so that all three allies share equally in the cost savings at the centroid of the core? Again, consider a three-country configuration, but let the middle country’s dimension be 1.5 inches by 1 inch while the two end countries are 1-inch squares. The following equations must then be satisfied at a core solution:

\[
\begin{align*}
v(\{1\}) &= -4 \leq u_1 \\
v(\{2\}) &= -5 \leq u_2 \\
v(\{3\}) &= -4 \leq u_3 \\
v(\{1, 2\}) &= -7 \leq u_1 + u_2 \\
v(\{1, 3\}) &= -8 \leq u_1 + u_3 \\
v(\{2, 3\}) &= -7 \leq u_2 + u_3 \\
v(\{1, 2, 3\}) &= -9 = u_1 + u_2 + u_3.
\end{align*}
\]

We transform \(u_i\) for \(i = 1, 2, 3\) so that \(u_i' = 2(u_i + 5)/3\) to obtain a nonnegative disagreement point and a maximum cost savings of 4 for the three-ally mutual defense pact. It is these transformed equations (not displayed) that are used to identify the core graphically. In Figure 4, the unshaded rhombus determining the core has vertices at \((2/3, 4/3, 2), (2, 0, 2), (2, 4/3, 2/3),\) and \((2/3, 8/3, 2/3)\), with sides corresponding to the relevant segments of the following binding four equations: \(u_1' + u_2' = 2, u_3' = 2, u_1' + u_2' = 2,\) and \(u_3' = 2/3.\) The center of mass of this rhombus is at \((4/3, 4/3, 4/3)\) for which all allies split the cost savings equally. If the interior country’s width grows beyond 50% longer than that of the end countries, then the median core position favors the two end

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3. The required transformation for utility is \(u_i' = 8(u_i + 4)/11\) for \(i = 1, 2, 3.\)
countries. When, for example, its width is twice that of the end countries, the centroid of the core is at \((3/2, 1, 3/2)\), as the reader can verify.\(^4\)

Next, suppose that the three countries have dimensions of 1 inch by 2 inches so that the interior borders are now longer in the baseline case. This change enhances the possibility of cost savings but leaves unchanged the middle country’s relative bargaining strength. The median position of the core is now \((2, 4, 2)\), where cost savings are 8 because two interior borders of length 2 do not need defending in two directions apiece. Clearly, the spatial configuration of the allies via-à-vis internal and external border lengths plays a crucial role for alliance formation and the distribution of a mutual defense alliance’s gain. The pure public good theory of alliances could not identify this insight because of an absence of force thinning and the resulting savings from interior borders. Moreover, the public good theory focused on allocation and indicated little about the distribution of gains. Although club theory representations acknowledged spatial considerations, the models in the literature never quantified these cost savings or the unique position of interior countries.

**ALTERNATIVE BORDER LENGTHS: THE END ALLIES**

In a three-country alliance, the perimeter of an end country may differ from both the other end country and the interior country. Consider pact \(c\) drawn in Figure 1, where allies 1 and 2 are unit squares but end country 3 is 2 inches by 1 inch. The core to this game is defined, with our procedure, by a rhombus with vertices at \((1, 2, 1)\), \((2, 1, 1)\), \((1, 3, 0)\), and \((2, 2, 0)\) and a center of mass of \((3/2, 2, 1/2)\). In this example, the interior ally maintains its share of 2 because of its unchanged relative position as compared with

\[
4. \text{The relevant transformation for utility is now } u'_j = (u_i + 6)/2.
\]
the baseline case, but the remaining gain of 2 is now unevenly divided between the two outer countries. The short outer ally obtains three times the gain of its longer counterpart at the core’s median position and pulls nearer to the bargaining strength of the interior country. Eventually, an outer country’s width can become so long that the alliance confers cost savings only on countries 1 and 2, so that this outer country will be excluded from the alliance.

However, when an outer country is shorter than the other two allies, it gains a larger share of savings at the core’s median. Suppose that country 3 has a width of .5 inch and a length of 1 inch, whereas the other outer ally and middle ally are unit squares. The center of mass is now (4/5, 8/5, 8/5), where the shorter exterior ally has pulled even with the interior ally, leaving just one fifth of the savings for the other exterior ally.

As another example of a three-country mutual defense pact, consider pact $d$ in Figure 1, where country 3 is not contiguous to countries 1 and 2 and all countries are unit squares. There is now a single interior border between country 1 and country 2. In the associated equilateral triangle (not shown), the core is the base of the triangle consisting of the line $u_i + u_j = 2$, with a center of mass at (1, 1, 0). There is clearly no reason to include a noncontiguous ally; if included, this ally receives nothing at any core position. This case is germane when considering the addition of allies, such as Estonia, to NATO.

**DIRECTIONAL THREAT**

Obviously, mutual defense pacts may confront a threat from some directions and not from others. For the baseline case of pact $a$ in Figure 1, there is little substantial change to the results if the threat is in the direction affecting interior borders. Consider pact $a$, where the attack can come only from the east or west. The cost of defending each country in isolation is 2. Additionally, the cost of defending the two contiguous allies is only 2, whereas the cost of defending the two noncontiguous allies is 4. For the three-ally pact, the cost of defense remains at 2. By transforming the $i$th country’s utility by $u_i = u_i + 2$, we find that the conditions defining the core are unchanged from the baseline line case, so that equation system (5) and Figure 2 apply, with the median of the core still at (1, 2, 1).

If, however, only the east needs guarding in the baseline situation, then the three-country alliance offers a maximum cost savings of 2. The reader can confirm that the median of the core is (1/2, 1, 1/2), where the relative advantage of the interior country remains the same. This last case is most germane to NATO during the flexible-response years of the cold war when conventional weapons were important. It is important to note that the results in terms of relative payoffs are really unaffected when the threat is in the direction of the interior borders. Instead, when the threat is from the north or south, there are no cost savings from interior borders and the core is empty so that there is no advantage to allying when confronted with a conventional war threat.

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5. This equilateral triangle has vertices at (2, 0, 0), (0, 2, 0), and (0, 0, 2).
MUTUAL DEFENSE PACTS: FOUR AND FIVE ALLIES

When the analysis is extended beyond three countries, the spatial configuration has more degrees of freedom and can play an even greater role. This follows because the same number of countries can have different interior borders and, thus, diverse cores depending on their configuration, even when their perimeters are unchanged. In Figure 1, both defense pacts e and f involve four unit-square allies. For pact e, there are two interior countries. This leads to a cost savings from mutual defense of 6 from three interior borders no longer needing protection. For pact f, the four countries form a 2-inch square alliance with four interior borders providing cost savings of 8. The dimension of the four-country problem means that a graphical solution is too cumbersome, and the inequalities and the equality associated with the core must be solved mathematically. For the rectangular pact in e, the median of the core is at (1, 2, 2, 1), where the middle two countries maintain their two-for-one advantage over the exterior countries, analogous to the baseline case. The 2-inch square pact f means that there are no interior countries, since every ally would have two exterior and two interior sides. The centroid of the core is now at (2, 2, 2, 2), with all allies gaining equally.

We consider two alternative five-country mutual defense alliances to identify some essential considerations. For pact g, the five potential allies are 1-inch equilateral triangles that fit together to form a symmetric pentagon with 1-inch sides. This case highlights how the actual configuration of the potential allies can promote alliance expansion. There are now five interior sides with a cost savings of 10 to be shared among the allies. Because there is no country with a positional advantage—each country is adjacent to two allies and has a single exterior side—all countries share equally at the median position of the core located at (2, 2, 2, 2).

Next, suppose that five 1-inch-square countries are configured as pact h in Figure 1, where country 3 has four interior sides and the other four countries have one interior side and three exterior sides. The advantage of country 3 is equal to the other four nations put together, so that the core’s median is at (1, 1, 4, 1, 1). As the number of countries in a mutual defense alliance grows, not only can the maximum cost savings increase, but also the manner of distributing this gain increases in its variety. Pact h is likely to result in a two-bloc negotiation between the interior country and the four exterior countries. The center of mass for the core of pact h illustrates that some countries can be in such a pivotal location that they can demand a relatively large gain even at the core’s median. When an attack can come from any direction, a five-country alliance of unit-square allies is the smallest where a four-sided country can be fully interior. Alliances with more than six members can certainly contain fully interior allies with advantages similar to that of country 3. The analysis can be extended beyond five allies but yields little in the way of new insights.

Typically, the core is anticipated to shrink when the number of participants increases; however, this need not characterize a mutual defense pact. Consider pacts d and f, consisting of three and four nations, respectively. The core for pact d is the straight line connecting (2, 0, 0) and (0, 2, 0), whereas the core for pact f is some three-
dimensional rhombus with \((2, 2, 2, 2)\) at its center of mass. Moreover, the core associated with five-nation pact \(g\) is larger than that associated with four-nation pact \(h\). This realization that the core can grow with the addition of some strategically located allies is yet another reason that spatial considerations matter.

**STRATEGIC WEAPONS AND DETERRENCE**

Thus far, the model applies only to alliances set up to protect borders from a conventional attack. The NATO alliance also relies on strategic weapons that deter an attack—conventional or strategic—through a threat of retaliation. If strategic weapons possess sufficient range that their deployment position is immaterial, then new allies can be protected with no additional costs incurred by the ally with such weapons (Olson and Zeckhauser 1966). The sequestration of interior borders loses any importance for strategic weapons: if there are no retaliatory risk factors, then the core consists of all friendly nations allying within an alliance, solely dependent on strategic weapons. The United States could extend its strategic deterrence to additional allies, leading to no real size restrictions. However, the United States would demand political concessions from its allies to obtain some benefit for taking other countries under its nuclear umbrella (Morrow 1991), especially since retaliatory risks arise. The presence of strategic weapons can, therefore, justify a noncontiguous nuclear ally such as the United States or Great Britain joining a mutual defense pact even though its own borders are not sequestrated. The conventional allies’ motivation to join depends on cost savings from interior borders and the net value (deducting the costs of political concessions) derived from deterrence. The sole determinant behind the formation of the ANZUS alliance, with its three noncontiguous allies, must be based on the trade-off of strategic deterrence from the United States in return for political concessions (e.g., Australia’s participation in the Vietnam War).

**AN APPLICATION TO THE NATO ALLIANCE**

Even in its most elementary characterization, the analysis of mutual defense games can provide insights into NATO burden sharing, formation, and expansion. Exterior frontline allies in NATO (e.g., Germany) are at a bargaining disadvantage compared with more interior European allies (e.g., Belgium, Luxembourg, and the Netherlands). Such exterior allies have assumed relatively greater defense burdens during the cold war. When average benefit shares received are compared with actual defense burden shares paid, West Germany was an overcontributor from 1975 through 1990, except for the height of the Reagan defense buildup in 1985 (Khanna and Sandler 1996, 1997). With the end of the cold war and the lessening of the eastern threat, Germany has displayed relatively large defense cutbacks (Hartley and Sandler 1999).

Noncontiguous NATO members, especially those allies either separated by a large distance or bordering an enemy, are at a decided bargaining disadvantage, leaving them no choice but to assume larger defense burdens relative to other allies. Prime
examples of such allies would include the United States, the United Kingdom, Norway, and Turkey. From the start of the NATO alliance in 1949, the United States has typically carried the largest defense burden in terms of defense spending as a percentage of gross domestic product (GDP). The United Kingdom also shouldered one of the largest defense burdens. Norway and Turkey have assumed a greater defense burden relative to their cohort of NATO allies.\(^6\) Even though Canada has “free rode” on the United States, the long common border between the two allies makes their alliance mutually beneficial despite lopsided burden sharing. Without Canada as an ally, the United States might have to divert many military resources to guard its northern border, which has no natural barriers or defenses.

Next, consider NATO’s formation in 1949 as a counter to Soviet aggression in Eastern Europe and its takeover of satellite states. On December 10, 1948, negotiations began in Washington, D.C., on a North Atlantic Treaty that would tie participants together in a mutual defense alliance.\(^7\) Initial negotiations included representatives from Belgium, France, Luxembourg, the Netherlands, the United Kingdom, Canada, and the United States. The five European countries were best positioned to gain from a mutual defense pact from the creation of interior borders. These seven countries then invited Denmark, Iceland, Italy, Norway, and Portugal to join the negotiations on March 15, 1949. With the exception of Italy, these late participants were not contiguous with the first seven nations pushing for NATO and, therefore, could offer less to the alliance. Their inclusion in the original alliance must then be based on considerations other than mutual defense, for example, deterring Soviet aggression through a threat of strategic weapon retaliation in return for political concessions. Once the 12-nation alliance was established on April 4, 1949, it was just a matter of time to fill in missing pieces—Spain and West Germany—because these countries would create large expanses of interior borders. The addition of Greece and Turkey in 1952 added protection to NATO’s southern flank, but these countries were not contiguous to the original 12 members. In Turkey’s case, other benefits (e.g., a listening post to the Soviet Union) came from its inclusion.

The analysis of mutual defense games can also provide some insights into the NATO expansion process and which nations might ultimately become members following the recent admittance of the three Visegrad nations.\(^8\) The inclusion of East Germany in NATO as part of unified Germany in 1990 is strongly supported by a mutual defense framework, since this entrant moves NATO’s perimeter eastward and produces cost savings from added interior borders. The Baltic nations (Estonia, Latvia, and Lithuania), which seek membership, include only a small inner border with Poland; therefore, their inclusion offers little, if any, cost savings from mutual defense. Given the strong political opposition to their inclusion by Russia and this lack of cost savings, there is little or no gain for the current members from including them. Thus, the Baltic states are very unlikely entrants. The geographical location of Slovakia with


\(^7\) The dates and facts of this paragraph come from Sandler and Hartley (1999, 24-29).

\(^8\) Membership also depends on other factors including Russian opposition, the risk of civil unrest, the absence of border disputes, the state of the country’s military forces, and interoperability of these forces with NATO forces (Sandler and Hartley 1999, chap. 3).
its interior borders with the three entrants and Austria makes Slovakia an attractive candidate for inclusion. If Slovakia is included, there will be only a small exposed border with Ukraine needing defending. Sandwiched between Italy and Hungary, Slovenia is also a likely entrant with only its external border with Croatia needing protection. Romania is, however, a less attractive prospect for NATO membership with its long-exposed borders on all sides except in the west, where it borders Hungary. The geographical position of Romania offers virtually no advantage in terms of cost savings from mutual defense, so that its inclusion must be based on other grounds. Without these other grounds, Romania is an unlikely entrant even though it was among the five considered at the July 1997 Madrid summit.

NATO’s new eastern members may be in favor of including members to their east, since this expansion favors these new members most directly with the acquisition of an interior border. The western allies have, however, much less to gain, and, because entry decisions must be unanimously approved by NATO allies, further eastward expansion is unlikely. Moreover, such eastward expansion is viewed as a threat by Russia, whose strong opposition raises a serious political concern. A more likely scenario involves filling in the missing pieces, that is, Slovakia, Slovenia, and Austria. Other neutral countries—Sweden, Switzerland, and Finland—are unlikely candidates because of their geographical locations and minor cost savings.

OTHER CONSIDERATIONS

The mutual defense game representation can be made yet more realistic by extending the analysis to consider natural defenses, ally-specific risks, guerrilla warfare, and transaction costs. For these extensions, we use the baseline case with three unit-square countries arranged in a row (pact $a$ in Figure 1).

NATURAL DEFENSES AND RISK FACTORS

Suppose that the right-hand ally (country 3) in the baseline model has some lengths of borders with natural defenses (e.g., mountain ranges, cliffs along a coastline). Suppose further that these natural defenses are along one third of its three exterior sides, so that only 3 of its 4 inches of borders require guarding. In this case, its cost of protection is $v(3) = -3$, and we are back to the case of an end country with dimension of .5 inch by 1 inch. The centroid of the core is then $(4/5, 8/5, 8/5)$, as shown earlier, with the end country drawing equal to the middle country. Hence, natural defenses along an exterior border act to shorten an end ally’s borders needing protection and, in so doing, afford the ally a bargaining advantage.

Next, suppose that the natural defenses involve the interior nation (country 2) along its interior boundary with country 1. In particular, suppose that only half of this interior border needs protecting, so that both country 1 and country 2 have to expend only 3.5 in cost to protect their borders if they walk from the alliance. Any advantage to the interior country in natural defenses along its perimeter must also be shared by the contiguous end country. The maximum cost savings for this three-nation defense pact is
reduced to 3 from the baseline case because there are now less exposed inner borders that no longer need protecting. The core is identified by the rhombus with vertices at $(3/8, 9/8, 12/8), (9/8, 3/8, 12/8), (9/8, 15/8, 0),$ and $(3/8, 21/8, 0),$ possessing a center of mass at $(3/4, 6/4, 3/4).$ This centroid has the same relative payoffs between the outer countries and the interior country as the baseline model. Total payments differ because the cost savings is only three quarters that of the baseline case.

Next, suppose that all interior borders of this three-country mutual defense pact have natural defenses, so that only half of either interior border needs defending. Total cost savings are only 2, and the core consists of the rhombus with vertices at $(1/4, 1, 3/4), (3/4, 1/2, 3/4), (3/4, 1, 1/4),$ and $(1/4, 3/2, 1/4).$ The center of mass is now located at $(1/2, 1, 1/2),$ where the relative shares stay the same. Whenever interior borders possess defensive advantages, cost savings shares at the center of mass will remain one for two between the end countries and the interior country. The latter country is unable to improve its bargaining advantage because the natural defense along its border must also be possessed by the neighboring end country. By limiting the maximum cost savings from mutual defense, natural defenses along interior boundaries curtail the benefits and the need for allying. If both interior borders are impenetrable, then there are no cost savings from mutual defense and the nations are not anticipated to ally (i.e., the core is empty). This statement depends on there being no other allying benefit to replace cost savings. Hence, inaccessible mountainous countries in the middle of allied states (i.e., Switzerland) are not needed in NATO. Such natural defenses distinguish the common border of Sweden and Norway from that of the United States and Canada. For Sweden and Norway, their mountainous interior border presents a natural barrier, limiting any cost savings incentive from allying. In contrast, no such natural barrier exists along the shared border of the United States and Canada so significant cost savings from allying arise. Natural defenses have a much different influence when they are along an exterior or interior border.

A related, but different, case concerns borders that are more vulnerable or risky than others. During the cold war, an attack was expected to come along West Germany’s eastern border rather than along Norway’s northern border with the Kola Peninsula of the Soviet Union. The former was more hospitable to Soviet tanks rolling across plains. Consider the three-nation linear array where the eastern border of the end country is twice as risky as all other borders, meaning that country 3’s cost of protection is 2 rather than 1 at this eastern border. This case is analogous to a longer end country and can be shown to have a center of mass for the core at $(4/3, 6/3, 2/3),$ where the interior ally maintains its absolute and relative share of the cost savings, attributable to the interior borders. The remaining gain of 2 is now unevenly divided between the two end countries. West Germany’s complaints that it did more than its fair share during the cold war is not only backed up by statistical analysis (Khanna and Sandler 1996, 1997) but justified given its more vulnerable position vis-à-vis the mutual defense pact. Position and size matter. Since the end of the cold war and the much-reduced eastern threat, Germany has displayed one of the largest relative burden decreases in NATO—from 3.0% of GDP during 1985-1989 to 1.6% in 1997 (Hartley and Sandler 1999). This dra-

9. The transformation of utility is $u_i' = 3(u_i + 4)/4.$
matic change can be explained, in part, by the reduced riskiness of Germany’s eastern border. Now that Germany has become an interior nation with the addition of Poland and the Czech Republic, further such reductions can be anticipated.

The baseline case can also be extended to consider more risky interior borders, which enhance the importance of mutual defense, because the associated cost savings from allying can increase dramatically. Suppose that the two interior borders are twice as difficult to protect, thus requiring double the armaments and troops at twice the cost of guarding an equal expanse of exterior border. Cost savings are now 8 or 4 for each of two interior borders. The center of mass for the core is at (2, 4, 2). If a mutual defense pact can sequester vulnerable perimeters behind more defensive ones, then mutual defense pacts have great gains to offer. This is a consideration that should be driving NATO’s expansion when mutual defense considerations are germane. As such, Sweden has little to offer NATO as a member, but Slovakia has much to offer. Poland has a great deal to offer Germany but little to offer the alliance.

GUERRILLA WARFARE AND INSURGENCIES

The Partnership for Peace program, established on January 10-11, 1994, is intended to promote security in Eastern Europe and prepare some Partnership for Peace members for NATO membership (Partnership for Peace 1996). Partnership for Peace is intended to promote civilian control of armed forces and foster security arrangements between NATO and Partnership for Peace members. Prospective Partnership for Peace members are expected to solve border disputes as well as insurgencies within their borders. Both requirements are essential conditions for NATO membership and sensible from a mutual defense perspective. The existence of border disputes implies more vulnerable borders, whereas insurgencies imply that protection against attack must be countrywide.

Again, consider the baseline model with three 1-inch-square countries in a linear array. Suppose further that defense costs are normalized to equal 1 for guarding a country’s 1-inch-square territory. For guerrilla warfare, interior borders offer no cost savings advantage, since a country must protect everywhere including its side of the border with or without an alliance. Because there are no savings, the core is empty. In fact, an interesting negative externality can develop when each ally attempts to drive the insurgents from its interior and, in so doing, places a neighboring ally at greater risk while overspending (Sandler and Lapan 1988). If, however, all countries drive the guerrillas from their territories into nonallies surrounding the mutual defense pact, then border protection and cost savings are again the essential considerations because nations must keep the insurgents from entering their territory. As such, the previous analysis applies. Thus, the type of warfare is a crucial factor.

TRANSACTION COSTS

As a final extension, transaction costs are added to the model, so that forming and maintaining a mutual defense alliance is costly. In particular, transaction costs can take

10. The vertices of the rhombus for the core are (0, 4, 4), (4, 0, 4), (4, 4, 0), and (0, 8, 0).
the form of decision-making costs, information-gathering costs, interdependency 
costs (from less autonomy), and enforcement costs (Sandler and Hartley 1999, chap. 8). Although there are myriad ways that these transaction costs can be introduced, only a single scenario is investigated.

Suppose that the three-country baseline model applies, where transaction costs are 1 for a two-ally mutual defense pact and 2 for three allies. The following equations must be satisfied for a core, after normalizing utility to obtain a zero disagreement point:

\[
\begin{align*}
\nu(\{1\}) &= -4 \Rightarrow 0 \leq u_i' \\
\nu(\{2\}) &= -4 \Rightarrow 0 \leq u_j' \\
\nu(\{3\}) &= -4 \Rightarrow 0 \leq u_k' \\
\nu(\{1, 2\}) &= -7 \Rightarrow 1 \leq u_i' + u_j' \\
\nu(\{2, 3\}) &= -7 \Rightarrow 1 \leq u_j' + u_k' \\
\nu(\{1, 3\}) &= -9 \Rightarrow -1 \leq u_i' + u_k' \\
\nu(\{1, 2, 3\}) &= -10 \Rightarrow 2 = u_i' + u_j' + u_k'.
\end{align*}
\]

In (9), defense cost includes the unit cost of protecting an exposed border and the relevant transaction costs. For example, an alliance between an interior and an end country has six exposed 1-inch sides to protect and transaction costs of 1 for a total cost of 7. Obviously, an alliance between the two nonadjoining nations is undesirable, which would be true for all noncontiguous countries. The relevant triangle (not shown) for locating the core has vertices at (2, 0, 0), (0, 2, 0), and (0, 0, 2), since maximum cost savings are 2. The center of mass for the core is (1/2, 1, 1/2), with the middle ally maintaining its relative advantage. However, if transaction costs were to increase nonlinearly with alliance size, cost savings would be curtailed and alliance size would be limited.

CONCLUDING REMARKS

The analysis of the core of a mutual defense game represents a new paradigm for studying defense alliances that holds promise for understanding alliance formation, stability, and expansion. Unlike discussions about NATO expansion in the literature, the mutual defense representation is a benefit-based approach founded on cost savings from interior borders. Other benefits can be introduced (e.g., deterrence benefits) and combined with those of cost savings in the framework introduced here. Additional considerations increase the realism of the model and produce further insights.

11. The normalization is the same as that of the baseline case with $u_i' = u_i + 4$ for every $i$. 
A number of conclusions can be drawn. A country’s size, location vis-à-vis potential allies, and border attributes matter for alliance formation and for the distribution of gains if an alliance were to form. Natural defenses along exterior borders or reduced perimeters of an exterior ally can transfer relative gains from the interior allies to the exterior allies. Natural defenses along interior borders, however, do not change the relative distributions of gains and serve to limit the need for a mutual defense pact. Transaction costs and insurgencies curtail gains from mutual defense pacts. The same number of allies can have vastly different gains and imputations of these gains from mutual defense alliances depending on their geographical configuration. As the number of allies increases, this variability of gains and their distributions can also increase. Spatial consideration can allow the core to grow as the number of participants increases.

Three future directions of research should be pursued. First, transaction costs can be introduced in a more interesting and integrated fashion. Second, alternative solution concepts, such as Shapley value or the nucleolus, can be applied (Luce and Raiffa 1957, 246). The Shapley value generalizes the minimax value and gives a computable unique equilibrium. In contrast, the nucleolus is more akin to distributing benefits according to a Rawlsian maximin principle. These alternative solution concepts may be more appropriate for analyzing the alliance expansion decision, which weighs the relative merits of a prospective entrant. Third, the analysis can be extended to investigate multiple alliances.

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12. Yet another solution concept is that of coalition building, since old members do not tend to drop out as new ones enter (Arce 1994).
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