Massachusetts Institute of Technology
Department of Physics
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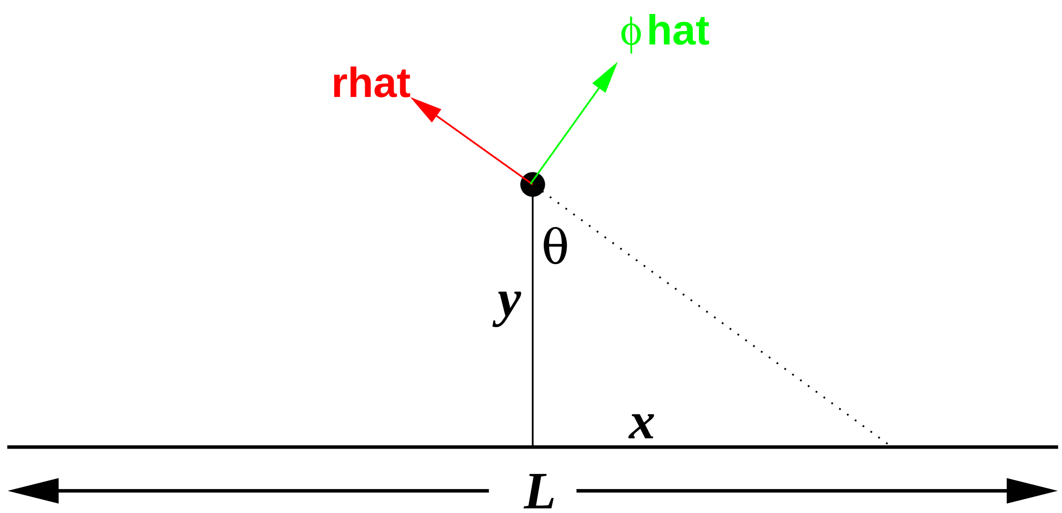
Lecture 10: Addendum
Clarification to the magnetic field of a sheet

10.7 Field of a plane of current, revisited

The geometry of the vectors that are needed to understand the magnetic field produced by a sheet of current is kind of grotty. A *lot* of people were somewhat befuddled at the end of the March 10 lecture. To clarify things, I present here some more detailed versions of the figure that appeared in Lecture 10; hopefully, this will help lay out why the magnetic field is (a) purely horizontal and (b) depends on the *cosine* of the angle in the figure.

Here is the sheet. In this figure, I've squished all the wires down into a nice, thin sheet. Consider the contribution to the magnetic field arising from a "chunk" of current to the right of the midpoint:

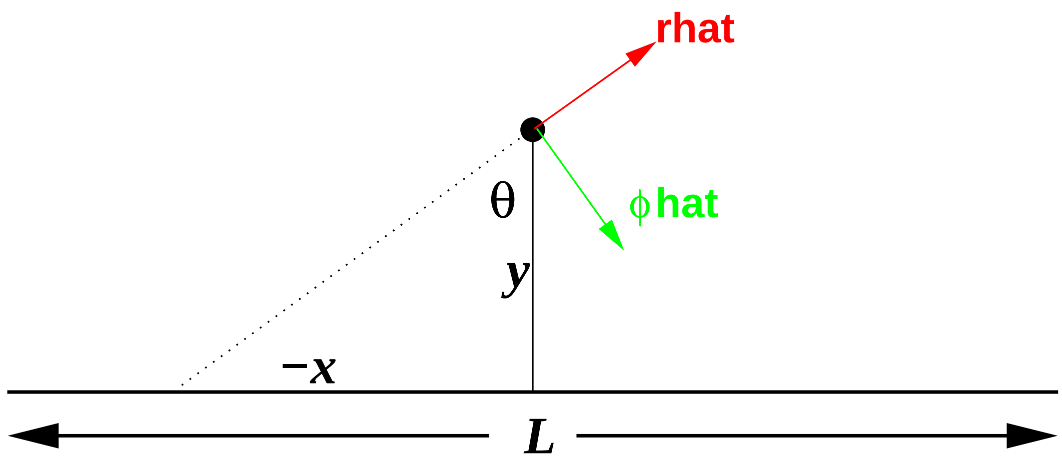


The radial direction from that chunk of current points along the red $\hat{r}$ vector that I've drawn. The magnetic field points along $\hat{\phi}$, which is rotated 90° clockwise (right hand rule) from $\hat{r}$. It is drawn in green. The magnetic field arising from this chunk of current is

$$d\vec{B}_{\rm right} = \frac{2K}{c}\frac{dx}{r}\hat{\phi}_{\rm right}\ ,$$

where $r = \sqrt{x^2 + y^2}$. (Don't forget that the current in this drawing is flowing into the page.)

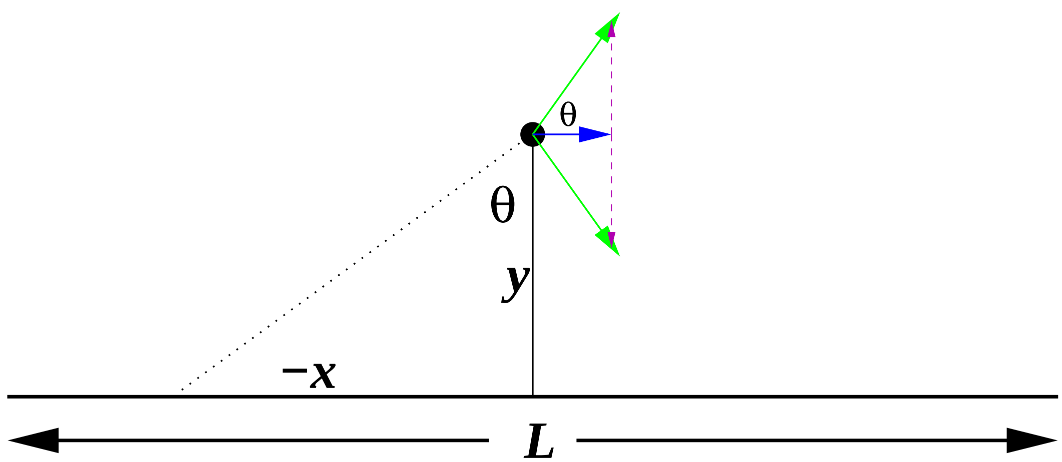
Now consider the field from a chunk to the left of the midpoint:



I've again drawn the radial direction from the chunk of current $\hat{r}$ in red; the magnetic field points along $\hat{\phi}$, rotated 90° clockwise from $\hat{r}$. The magnetic field arising from this chunk of current is

$$d\vec{B}_{\rm left} = \frac{2K}{c}\frac{dx}{r}\hat{\phi}_{\rm left} \ .$$

Now superpose the two contributions: The vertical components (drawn purplish) of the



fields clearly cancel each other out. The horizontal components by contrast reinforce one another. In other words, we can ignore the vertical component of the field and just take the horizontal one.

How do we get the horizontal component? It will clearly be something like $dB_x = |d\vec{B}|$[some trig function](θ); the question is which trig function to use. To get the correct trig function, we need to study the triangles in the figure very carefully — the "little" triangle that is built out of the components of $d\vec{B}$ is congruent with the big one that is built out of x and y. Careful drawing[1] shows that θ belongs where I've drawn it in the figure above.

[1] *Much* more careful than is really possible using thick chalk on a blackboard...

This shows us that the trig function we need is cosine; this in turn tells us that to get the magnetic field we want

$$dB_x = \frac{2K}{c}\frac{dx}{r}\cos\theta\ .$$

Substituting for r and $\cos\theta$ and setting up the integral, we have

$$\begin{aligned}\vec{B} &= \frac{2K}{c}\hat{x}\int_{-L/2}^{L/2}\frac{dx}{\sqrt{x^2+y^2}}\frac{y}{\sqrt{x^2+y^2}}\\ &= \frac{2Ky}{c}\hat{x}\int_{-L/2}^{L/2}\frac{dx}{x^2+y^2}\ .\end{aligned}$$

The rest of the derivation follows from here.

Summary: The key confusing bit is that the horizontal component of the field was given by the cosine of θ (with θ defined as in the drawing). Our usual intuition told us that it should be the sine of θ. However, the intuition we've developed so far is based on the idea that the field direction points along the hypotenuse of the triangle (i.e., along $\hat{r}$). The magnetic field actually points at right angles to the hypotenuse. This extra 90° changes the sine which we might have expected into the cosine that we actually get.

Summary of the summary: magnetic fields can be a pain in the butt.