

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
 DEPARTMENT OF PHYSICS
 8.022 SPRING 2005

LECTURE 23:
 MAGNETIC MATERIALS

23.1 Magnetic fields and stuff

At this stage of 8.022, we have essentially covered *all* of the material that is typically taught in this course. Congratulations — you now have a solid grounding in all the major concepts of electricity and magnetism!

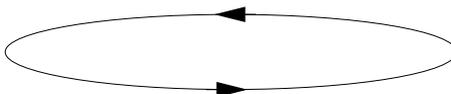
There remains one major, important subject which we have not discussed in depth: the interaction of magnetic fields and materials. At this point, we don't even have a really good understanding of a bar magnet works — an unsatisfying state of affairs, given that this is the way in which we normally encounter magnetism in the real world!

One reason we have avoided covering this subject is that it is not really possible to discuss it properly within getting into a detailed discussion of the quantum mechanical description of matter. The way in which matter responds to magnetic fields is *totally* determined by the quantum mechanical nature of their molecular structure, particularly their electrons.

Nonetheless, we can make significant headway in understanding the interaction of magnetic fields and materials by combining what we have learned so far with a somewhat approximate, qualitative description of how materials respond to magnetic fields. The main concepts we will need are summarized in the following two subsections:

23.1.1 Electron orbitals

The electrons in a molecule exist in *orbits*. *Very roughly*, we can picture an orbit as a simple loop of current:



A loop of current like this of course tends to generate its own magnetic field. In most materials, there are an enormous number of orbits like this, randomly oriented so that they produce no net field. Suppose an external magnetic field is applied to some substance that contains many orbitals like this. The net tendency can be understood in terms of Lenz's law: the orbits "rearrange" themselves in order to *oppose* the change in magnetic flux. This tendency for Lenz's law to work on the microscopic scale ends up *opposing the magnetic field from the material*.

23.1.2 Intrinsic magnetic moment of the electron

One other quantum mechanical property of electrons plays an extremely important role in this discussion: electrons have a built-in, *intrinsic* magnetic moment. Roughly speaking,

this means that each electron *all on its own* acts as a source of magnetic field, producing a dipole-type field very similar to that of current loop.

Because this field is associated with the electron itself, it does not exhibit the Lenz's law type behavior of the field that we see from the orbits. Instead, the most important behavior in this context is the fact that a magnetic moment \vec{m} placed in an external field \vec{B} feels a *torque*:

$$\vec{N} = \vec{m} \times \vec{B}.$$

In this case, the action of this torque tends to line up the electrons' magnetic moments with the external field. In this case, you find that *the magnetic field is augmented within the material*.

23.1.3 Which wins?

To summarize the above discussion, the quantum mechanical nature of electrons in molecules leads to two behaviors:

- Lenz's law on the scale of electron orbitals *opposes* magnetic fields from entering a material.
- Magnetic torque acting on the individual electrons *augments* magnetic fields in a material.

These two behaviors are in complete opposition to one another! However, both occur, and both are important. A natural question to ask is: *Which* of these two behaviors is *more* important?

The answer to this question varies from material to material, depending upon its detailed electronic orbital structure. The first property — Lenz's law on the orbital scale — plays some role in *all* materials. In many cases, this is the end of the story. Such materials are called *diamagnetic*. A diamagnetic material is one whose *magnetization* (to be defined precisely in a moment) opposes an external magnetic field. Even though almost all materials are diamagnetic, the effect is so puny that it is typically very difficult to see. A consequence of this is that a diamagnetic substance will be expelled from a magnetic field.

For some materials, the second property — alignment of electron magnetic moments — wins out. Such materials typically have several electron orbits that contain unpaired electrons; the orbit thus has a net magnetic moment. (In most diamagnetic materials, the orbits are filled with paired electrons, so that the orbit has no net moment¹.) Such materials are called *paramagnetic*. A paramagnetic material has a magnetization that augments an external field. Paramagnetic substances are pulled into a magnetic field. In the vast majority of cases, paramagnetic effects are also so puny that they can barely be seen².

One particular class of materials acts essentially as paramagnetic materials do. However, they respond *so* strongly that they truly belong in a class of their own. These are the *ferromagnetic* materials.

¹Bear in mind that this is a *highly* qualitative discussion!

²There are some interesting exceptions. Oxygen, O₂, turns out to be paramagnetic. However, the random motion of gaseous oxygen wipes out any observable effect. If we cool it to a liquid state, though, the paramagnetism is easily seen.

23.2 Some definitions

A couple of definitions are needed to describe what we are going to discuss. Foremost is the *magnetization* \vec{M} : the magnetic dipole moment per unit volume of a material. Notice that magnetization has the same dimensions as magnetic field:

$$\begin{aligned}\vec{M} &= \frac{\text{magnetic moment}}{\text{volume}} \\ &= \frac{\text{current} \times \text{area/velocity}}{\text{length}^3} \\ &= \frac{\text{current/velocity}}{\text{length}}.\end{aligned}$$

A material with constant magnetization \vec{M} and volume V acts just like a magnetic dipole with $\vec{m} = \vec{M}V$.

The second important definition is a bit of a wierd one: it is a new kind of magnetic field denoted \vec{H} . This \vec{H} is defined in terms of the “normal” magnetic field \vec{B} and the magnetization \vec{M} via

$$\vec{B} = \vec{H} + 4\pi\vec{M}.$$

In other words, when there is no magnetization, the “normal” magnetic field \vec{B} is just this new field \vec{H} . This means that \vec{H} is nothing more than the bit of the magnetic field that arises from “normal” electric currents, as opposed to those that are “bound” in matter.

23.2.1 More details on \vec{H}

We can do some more stuff with \vec{H} , though this is really beyond the scope of what we need for our discussion. If we take the curl of \vec{H} , we get

$$\vec{\nabla} \times \vec{H} = \frac{4\pi}{c} \vec{J}_{\text{free}}$$

where J_{free} is the density of *free* electrical current. The curl of the magnetization defines *bound* currents:

$$\vec{\nabla} \times \vec{M} = \vec{J}_{\text{bound}}/c.$$

Putting these together, we have

$$\begin{aligned}\vec{\nabla} \times \vec{B} &= \vec{\nabla} \times \vec{H} + 4\pi\vec{\nabla} \times \vec{M} \\ &= \frac{4\pi}{c} \vec{J}_{\text{free}} + \frac{4\pi}{c} \vec{J}_{\text{bound}} \\ &= \frac{4\pi}{c} \vec{J}.\end{aligned}$$

In other words, the *total* current density is given by adding the *free* current density (the current due to moving charges that we can imagine controlling) and the *bound* current density (the current that is “bound up” in electron orbits, and is really a part of the material).

23.3 Magnetic susceptibility

A wide variety of materials exhibit *linear magnetization*, meaning that the magnetization \vec{M} depends linearly on the field \vec{H} :

$$\vec{M} = \chi_m \vec{H} .$$

The quantity χ_m is the *magnetic susceptibility*. From this, we have

$$\vec{B} = \vec{H} (1 + 4\pi\chi_m) .$$

If $\chi_m < 0$, the magnetic field is decreased by the material. This corresponds to *diamagnetism*. If $\chi_m > 0$, the magnetic field is increased by the material. This corresponds to *paramagnetism*.

23.4 Ferromagnetism

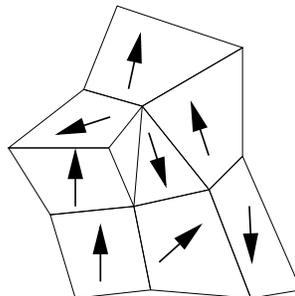
Roughly speaking, ferromagnetism is paramagnetism on steroids. Its key feature is that it is *nonlinear*: the \vec{M} and \vec{H} do *not* have a simple linear relation between one another. This leads to some surprising results. In particular, it means that once the “external” field \vec{H} is turned off, the magnetization \vec{M} remains! This of course is how permanent magnets are made.

The reasons for this nonlinear behavior are surprisingly complicated, and are difficult to explain correctly in a simple manner. (It is ironic that permanent magnets, probably the most familiar manifestation of magnetism, turn out to be so difficult to explain!) It isn’t so hard to describe the most important facets of ferromagnetism qualitatively; the hard part is explaining *why* those qualitative behaviors emerge. That, sadly, is far beyond the scope of 8.022.

23.4.1 Ferromagnetic domains

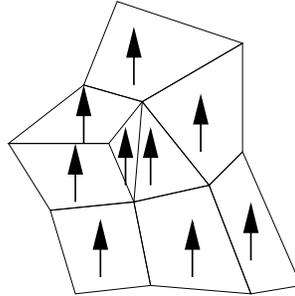
Since ferromagnetism is conceptually a lot like paramagnetism, we expect that it will be explainable in terms of the alignment of many atomic scale magnetic moments. This is indeed the case. However, with ferromagnetism there is an important modification: one finds that the moments of *many* atoms or molecules tend to be aligned in small regions. In “normal” paramagnetic materials, the moments are randomly arranged until an external field comes along and aligns them.

If you were to look at the magnetic moment layout of a typical ferromagnetic materials, you would find something like this:



Enough atomic scale moments are aligned that definite regions or *domains* of magnetization exist. Each domain is fairly small, and each is randomly oriented with respect to its neighbor; the material as a whole has no net magnetization. This is why a random piece of iron (a representative ferromagnetic material) is usually not magnetized.

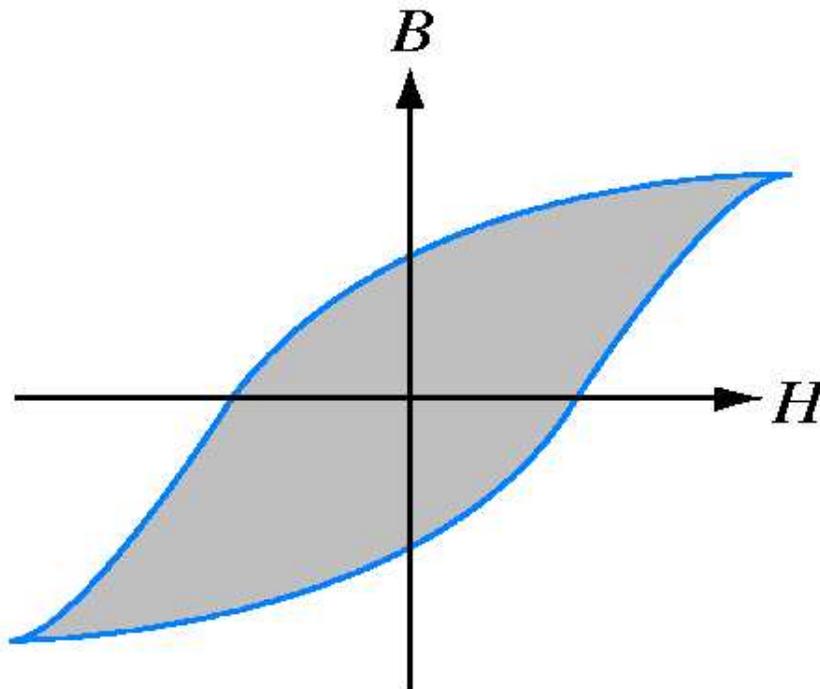
If this material is placed into a strong enough external magnetic field, the domains are re-aligned so that they all point in the same direction:



Once the external magnetic field is removed, it turns out to be energetically favorable for these domains to remain pointing in this direction. The domains are fixed with this magnetization. This is how permanent magnets are made.

23.4.2 Hysteresis

The nonlinear relationship between \vec{H} and \vec{B} means that the magnetic field \vec{B} in a ferromagnetic material has a rather complicated dependence on the “external” applied field \vec{H} . As \vec{H} is varied between a minimum and a maximum, one finds that \vec{B} in the material traces out an interesting shape called a *hysteresis curve*:



Notice that when $H = 0$, $B \neq 0$ — reflecting the fact that the material has been given a permanent magnetization. You may wonder looking at this plot which of the two possible values it will take — the positive B or the negative one? The answer depends on the history of the magnetization. If H was large and positive most recently, B will go to the positive value when H is set to zero. If H was large and negative, B will go to the negative value.

It turns out (though it's a bit beyond the scope of this course to show this) that it takes work to “go around” the hysteresis curve. As we go from $+H$ to $-H$, we have to flip lots of magnetic domains around — it takes energy to do that! The amount of work that is done in going from $+H \rightarrow -H \rightarrow +H$ is proportional to the area of the hysteresis curve. One consequence of this is that solenoids with an iron core can be very *lossy* elements in AC circuits. An AC circuit is continually changing the direction of flow of the current, and hence of the \vec{H} field. If a portion of the circuit contains a ferromagnet, some of the power put into the circuit will be lost by continually changing the orientation of the ferromagnet's domains.

23.4.3 Curie temperature

No discussion of ferromagnetism is complete without mentioning the *Curie temperature*. The Curie temperature, T_C , is a temperature above which a material *very suddenly* ceases to act like a ferromagnetic. If the temperature is above T_C , the random motion of the magnetic moments is so strong that they can *never* become aligned. Just a tiny bit below, though, and they can align! It's somewhat amazing how strong the transition is at this temperature. For iron, $T_C = 770^\circ$ Celsius.