

8.033

Lecture 1

Highland

1 Sept 2011

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Course info & syllabus on stellar

<http://stellar.mit.edu/S/course/8/fa11/8.033>

All important course material - including my lecture notes - listed here.

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Text: My notes! I wish there were a more complete text for you, but there isn't. However, there are some texts that are useful supplements - see web site for suggestions.

Psets: Posted every Thursday, due the following Thursday
Extensions by instructor's discretion - NO credit for a late pset that has not been excused.

Quizzes: Two scheduled, Oct 4 & Nov 10, in class hours, in Walker. Also a comprehensive final, to be scheduled.

Grades: 20% Psets, 20% x 2 Quizzes, 40% final

Stated goal of this course: To understand Einstein's theory of relativity - mostly special, but with a brief introduction to general.

More generally, our goal is to understand how to build a description of the laws of physics in which we build in a symmetry principle from step 1. For S.R., this is "Lorentz symmetry," so this will be our focus. But the idea is far broader than this and is used throughout physics. Master this as a "warm-up" exercise, and you'll be well equipped to tackle most modern descriptions of physics.

Begin with Newton's laws. They ~~also~~ obey a theory of relativity - but not Einstein's! The relativity principle of Newton's laws is called "Galilean relativity" - principles originally laid out by Galileo.

To quantitatively describe this principle, need to introduce some terms:

EVENT: Something that happens somewhere at some time. Key feature: the event's reality is independent of our labels, or ~~is~~ our "reference frame."

REFERENCE FRAME: System for labeling position of an event in space and time.

GEOMETRIC OBJECT: Something with properties that exist independent of the reference frame we use to describe it.

Example of a geometric object: An event.

Less trivial example. A vector in space.

All observers agree that this vector has certain properties (length, orientation), but different observers may describe it differently depending on how they have set up their reference frames. As long as the different reference frames are used consistently, different observers will describe the physics of the system in the same way.

We will endeavor to use geometric objects as much as possible in building a description of the laws of physics. Very important to maintain in your mind a difference between the object (e.g., the vector) and the representation of the object (e.g., the vector's components in some given frame of reference).

Particularly important kind of reference frame: An inertial reference frame. This is a frame in which a body's momentum is constant if no forces act on it: It is an UNACCELERATED ref. frame. (Non-inertial reference frames exist, and require us to introduce "fictitious" forces, like centrifugal or Coriolis forces.)

"Galilean transformation": Mathematical "machine" that relates one inertial reference frame (IRF) to another.

Example: IRF C is used by the class to label objects & events. IRF P is used by the professor to label objects & events.

According to the class, the professor moves with constant speed v in the $+x$ direction. How do we relate their two IRFs?

Consider time first. Is there any difference?

No: In Newtonian physics, all clocks tick at the same rate.

$$t_p = t_c$$

Space differs: An object at fixed position in IRFC
 "falls back" along x in IRFP:

$$X_p = X_c - v t_c$$

(assume they agree
 at $t_c = t_p = 0$)

$$Y_p = Y_c, \quad z_p = z_c$$

This set of 4 equations is a Galilean spacetime
 transformation. It can be neatly written as a
 matrix relation:

$$\begin{bmatrix} t_p \\ x_p \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -v & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t_c \\ x_c \\ y_c \\ z_c \end{bmatrix}$$

-or-

$$\vec{X}_p = G \cdot \vec{X}_c$$

Notational note: over-arrow means "4-vector,"
 i.e. a vector with a time like component.

On blackboard, will use an underline to
 denote a spatial 3-vector, \underline{v} . When typed,
 3-vectors will be put in boldface.)

How do we invert this relation? Easy: If C says P moves with $\underline{v} = v \underline{e}_x$, then P says C moves with $\underline{v} = -v \underline{e}_x$. Hence,

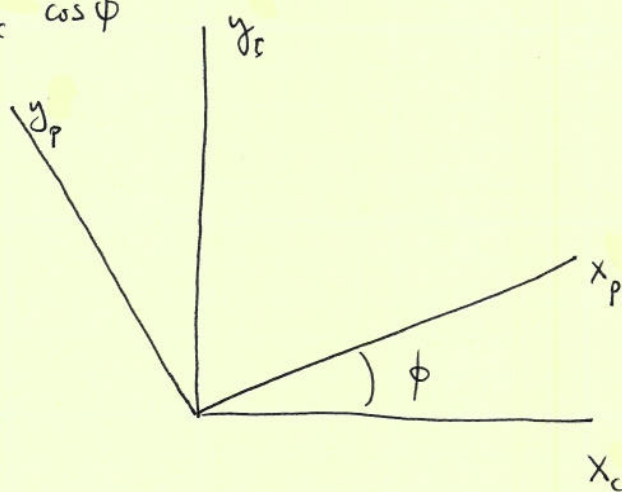
$$\begin{bmatrix} t_c \\ x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ v & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t_p \\ x_p \\ y_p \\ z_p \end{bmatrix}$$

or $\vec{X}_c = G' \cdot \vec{X}_p$

Easy to verify that $G' = G^{-1}$ - Pret $\neq 1$.

Another example: IRFs C & P are at rest with respect to each other, but are rotated about the z-axis:

$$\begin{aligned} t_p &= t_c \\ x_p &= x_c \cos \phi + y_c \sin \phi \\ y_p &= -x_c \sin \phi + y_c \cos \phi \\ z_p &= z_c \end{aligned}$$



The spatial part of this is just a simple rotation, so the Galilean transformation in this case is given by

$$\begin{bmatrix} t_p \\ x_p \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & \sin\phi & 0 \\ 0 & -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t_c \\ x_c \\ y_c \\ z_c \end{bmatrix}$$

or $\vec{x}_p = \underset{\substack{\uparrow \\ \text{"rotation"}}}{G_R} \cdot \vec{x}_c$

Key feature of a Galilean transformation: it leaves inertial ~~reference~~ reference frames inertial. To see, go back to our first example:

$$x_p = x_c - vt_c, \quad y_p = y_c, \quad z_p = z_c$$

Now, look at how velocities transform:

$$u^x = \frac{dx}{dt} \quad u^y = \frac{dy}{dt} \quad u^z = \frac{dz}{dt}$$

-or- $u^i = \frac{dx^i}{dt}$ $x^1 = x$
 $x^2 = y$
 $x^3 = z$

Note: v denotes relative velocity of frames, u denotes velocity within a specified frame.

If the velocity is given by components u^i_c in IRF C, then what are the components in IRF P?

$$u^i_p = \frac{dx^i_p}{dt_p}, \text{ or}$$

$$\begin{aligned} u^x_p &= \frac{d}{dt_p} (x_c - vt_c) \\ &= \frac{d}{dt_c} (x_c - vt_c) \\ &= u^x_c - v \end{aligned}$$

Can similarly show $u^y_p = u^y_c$, $u^z_p = u^z_c$

→ "Normal" velocity transformation law we've all seen in ordinary mechanics!

How about accelerations?

$$a^i_p = \frac{du^i_p}{dt_p}, \text{ or}$$

$$a^x_p = \frac{d}{dt_p} (u^x_c - v) = \frac{d}{dt_c} (u^x_c - v) = a^x_c$$

$$a^y_p = a^y_c, \quad a^z_p = a^z_c$$

→ Accelerations are the same! Galilean transformations do not affect accelerations, or forces.

Particularly important example: How a wave is modified by a Galilean transformation.

Waves in general: Have some "field" F that propagates through space, has the functional form $F = F(x - wt)$

Satisfies the differential equation

$$\frac{\partial^2 F}{\partial t^2} - w^2 \frac{\partial^2 F}{\partial x^2} = 0$$

w is the wave's speed of propagation; the field F depends on the specific kind of wave under consideration - could be the pressure of a sound wave, height of a water wave, magnitude of electrostatic potential, ...

Under a Galilean transformation, velocities are shifted.

We expect the wave speed to shift similarly:

$$\begin{aligned} F &= F(x_c - wt_c) \\ &= F(x_p - (w - v)t_p) \end{aligned}$$

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This in fact is exactly what happens - and you'll prove this by applying Galilean transformation to the wave equation.

Clear, important physics here: There is one special IRF in which the wave speed is w . This is the "rest frame" in the medium that supports the wave. E.g., for a water wave, it is the frame in which the water does not flow.

Food for thought: If we make a wave equation from Maxwell's equations, we predict a unique speed. What is the medium corresponding to this rest frame?