Recap:

- Laws of Newtonian mechanics respect <u>Galilean</u> relativity
- Laws are particularly simple to formulate and analyze in an "inertial reference frame": A representation in which momentum is conserved if no forces act.
- "Galilean transformations" move us from one IRF to another. Examples:
  - Rotation
  - Offset by constant velocity
  - Change of origin (spatial -or- temporal)

Quick aside: Why not consider uniformly accelerated reference frames?

Key problem: To transform between frames with different accelerations, we would need to introduce forces to make Newton's laws work. Not impossible! - but cumbersome. We will stick with non-accelerated frames for the sake of simplicity.

What if all frames experienced the same acceleration? In this case, we could imagine defining Galilean transformations between those (equally) accelerated frames of reference.

Suggests an interesting question: If everything experiences the same acceleration, does it even mean anything interesting? Perhaps, given that all objects in all frames experience this acceleration, we could define this uniform $\vec{a}$ as a somewhat odd notion of "rest."

Such concepts are key to the formulation of general relativity: Foundation of the principle of equivalence.

Galilean transformations and the wave equation:

$$\frac{\partial^2 F}{\partial t^2} - w^2 \frac{\partial^2 F}{\partial x^2} = 0$$

Solutions of form $F = F(x_c - w t_c)$

Perform Galilean transformation:

$$t_p = t_c$$
$$x_p = x_c - v t_c$$
$$y_p = y_c$$
$$z_p = z_c$$

Equation is modified (details on pset #2), solutions are now of the form

$$F = F\left[x_p - (w - v) t_p \right]$$

Interpretation: Original wave equation was formulated in rest frame of medium that supports the wave.

Change frames: The medium is now in motion, and so the wave has a changed speed as seen in the "new" frame.

Maxwell's Equations:

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{E} = \rho / \epsilon_0 \qquad \underset{\sim}{\nabla} \cdot \underset{\sim}{B} = 0$$

$$\underset{\sim}{\nabla} \times \underset{\sim}{E} = - \partial \underset{\sim}{B} / \partial t \qquad \underset{\sim}{\nabla} \times \underset{\sim}{B} = \mu_0 \underset{\sim}{J} + \mu_0 \epsilon_0 \frac{\partial \underset{\sim}{E}}{\partial t}$$

Set $\rho = 0$, $\underset{\sim}{J} = 0$ - "vacuum" (no charges or currents)

Take curl of the curl equations:

LHSs: $$\underset{\sim}{\nabla} \times (\underset{\sim}{\nabla} \times \underset{\sim}{E}) = \underset{\sim}{\nabla} \cancelto{0}{(\underset{\sim}{\nabla} \cdot \underset{\sim}{E})} - \nabla^2 \underset{\sim}{E}$$

$$\underset{\sim}{\nabla} \times (\underset{\sim}{\nabla} \times \underset{\sim}{B}) = \underset{\sim}{\nabla} \cancelto{0}{(\underset{\sim}{\nabla} \cdot \underset{\sim}{B})} - \nabla^2 \underset{\sim}{B}$$

RHSs: $$\underset{\sim}{\nabla} \times \left(- \frac{\partial \underset{\sim}{B}}{\partial t} \right) = - \frac{\partial}{\partial t} \left(\underset{\sim}{\nabla} \times \underset{\sim}{B} \right) = - \mu_0 \epsilon_0 \frac{\partial^2 \underset{\sim}{E}}{\partial t^2}$$

$$\underset{\sim}{\nabla} \times \left(\mu_0 \epsilon_0 \frac{\partial \underset{\sim}{E}}{\partial t} \right) = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(\underset{\sim}{\nabla} \times \underset{\sim}{E} \right) = - \mu_0 \epsilon_0 \frac{\partial^2 \underset{\sim}{B}}{\partial t^2}$$

So, vacuum Maxwell equations give us wave equations for $\underset{\sim}{E}$ and $\underset{\sim}{B}$:

$$\frac{\partial^2 \underset{\sim}{E}}{\partial t^2} - \frac{1}{\mu_0 \epsilon_0} \nabla^2 \underset{\sim}{E} = 0$$

$$\frac{\partial^2 \underset{\sim}{B}}{\partial t^2} - \frac{1}{\mu_0 \epsilon_0} \nabla^2 \underset{\sim}{B} = 0$$

For simplicity, imagine that $\underset{\sim}{E}$ & $\underset{\sim}{B}$ ~~they~~ only depend on x & t. Then, they simplify further:

$$\frac{\partial^2 \underset{\sim}{E}}{\partial t^2} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \underset{\sim}{E}}{\partial x^2} = 0 \, , \quad \frac{\partial^2 \underset{\sim}{B}}{\partial t^2} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \underset{\sim}{B}}{\partial x^2} = 0$$

Solutions clearly of the form

$$\underset{\sim}{E} = \underset{\sim}{E}(x - ct) \, , \quad \underset{\sim}{B} = \underset{\sim}{B}(x - ct)$$

where $c = 1/\sqrt{\mu_0 \epsilon_0}$ – the speed of light.

QUESTION: What frame did we use for this analysis?

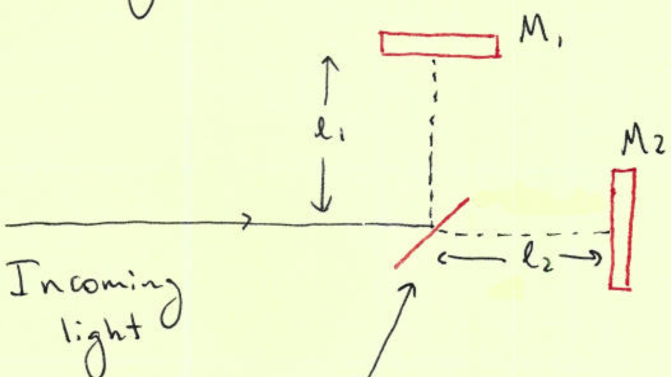
Consensus of late 19TH century: Electromagnetic waves propagate in some underlying medium called the "ether" (or "aether").

Wave equations we just derived are in the ether's rest frame. The speed $c = 1/\sqrt{\mu_0 \epsilon_0}$ describes light <u>in</u> <u>that</u> <u>frame</u>.

Prediction: If we measure light in a lab that moves relative to the ether, we should see a speed that is not c.

Our labs are on Earth: spins on axis, orbits the sun. Even if we are at rest w.r.t. the ether at one moment, we won't be later.

Ingenious experiment designed by Albert Michelson & Edward Morley to test this. Idea: Exploit light's wave nature to build an INTERFEROMETER:



M_1, M_2: Mirrors, reflect all light back to beam splitter.

Beam splitter: $1/2$ of the light transmits, $1/2$ reflects.

What happens when light reflects back to the beam splitter?

Answer depends on the details of the two paths the light takes.

$$t_1 \equiv \text{travel time in arm } 1$$
$$t_2 \equiv \text{" " " } 2$$

$$\Delta t \equiv t_2 - t_1$$

$c\Delta t \equiv$ "optical path difference"

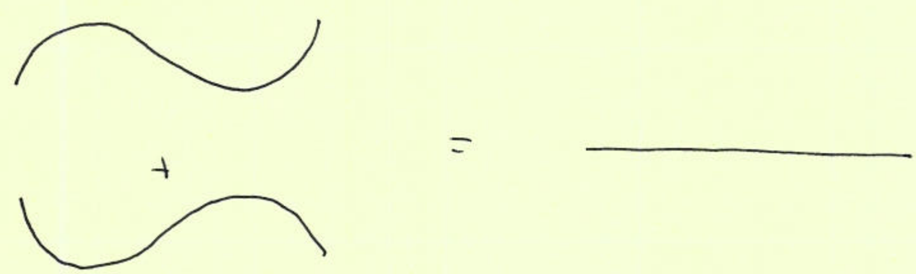
$\frac{c\Delta t}{\lambda} \equiv$ "optical phase difference" (divided by 2π)

If $\frac{c\Delta t}{\lambda} = 0, \pm 1, \pm 2, \pm 3, \ldots$

the light <u>constructively</u> interferes:

If $\frac{c\Delta t}{\lambda} = \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \ldots$

it <u>destructively</u> interferes:

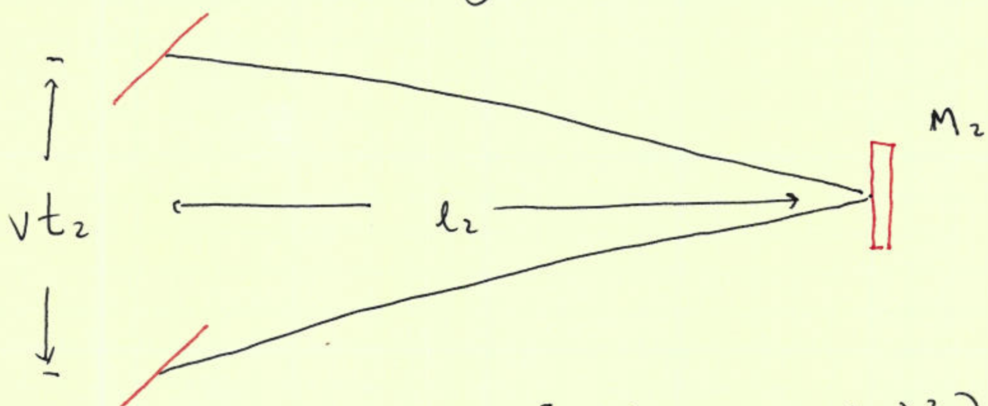


This gives us a <u>very</u> precise way to measure difference in paths down the two arms.

Suppose the apparatus moves parallel to the flow of the ether along arm 1:

$$t_1 = \underbrace{\frac{l_1}{c - v}}_{\text{light moves "upstream"}} + \underbrace{\frac{l_1}{c + v}}_{\text{light moves "downstream"}} = \frac{2 l_1}{c}\left(\frac{1}{1 - v^2/c^2}\right)$$

In arm 2, the light moves across the flow of the ether as seen in the interferometer. Viewed in the ether's rest frame, we have



$$ct_2 = 2\left[l_2^2 + \left(\frac{vt_2}{2}\right)^2\right]^{1/2}$$

Total travel time, multiplied by c

$$\rightarrow \quad t_2 = \frac{2 l_2}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$$

So, $\Delta t = t_2 - t_1$

$$= \frac{2}{c}\left[\frac{l_2}{\sqrt{1 - v^2/c^2}} - \frac{l_1}{1 - v^2/c^2}\right]$$

Useful time to parse and consider some of the numbers.

$v =$ speed of lab with respect to ether

$\lesssim$ orbital speed of Earth

$\simeq 2 \times 10^4$ m/sec

$c = 3 \times 10^8$ m/sec

$\rightarrow \frac{v}{c} \lesssim 10^{-4}$ → Small enough we can use binomial exp. to simplify Δt.

$$\Delta t \simeq \frac{2}{c}\left[l_2\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) - l_1\left(1 + \frac{v^2}{c^2}\right)\right]$$

(Recall: $(1+nx)^\alpha \simeq 1 + \alpha n x$ for $x \ll 1$.)

OK, we do the experiment, measure Δt. Now's the really clever bit: Rotate the interferometer by $90°$. This exchanges the two arms:

$$t_1' = \frac{2l_1}{c}\frac{1}{\sqrt{1 - v^2/c^2}} \simeq \frac{2l_1}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$t_2' = \frac{2l_2}{c}\left(\frac{1}{1 - v^2/c^2}\right) \simeq \frac{2l_2}{c}\left(1 + \frac{v^2}{c^2}\right)$$

$$\Delta t' = t_2' - t_1'$$

$$= \frac{2}{c}\left[l_2\left(1 + \frac{v^2}{c^2}\right) - l_1\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)\right]$$

The really key quantity is the change in the Δts when we rotate: Tells us how the optical path changes due to the modified geometry of the interferometer:

$$\delta t \equiv \Delta t' - \Delta t$$

$$= \frac{2l_2}{c}\left(\frac{v^2}{c^2} - \frac{1}{2}\frac{v^2}{c^2}\right) - \frac{2l_1}{c}\left(-\frac{v^2}{c^2} + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$= \frac{l_1 + l_2}{c} \quad \frac{v^2}{c^2}$$

Optical phase difference for this measurement:

$$\frac{c\,\delta t}{\lambda} = \frac{l_1 + l_2}{\lambda} \quad \frac{v^2}{c^2}$$

For the Michelson-Morley apparatus,

$$l_1 + l_2 \simeq 10 \text{ meters}$$

$$\lambda \simeq 500 \text{ nm} = 5 \times 10^{-7} \text{ m}$$

$$\frac{v^2}{c^2} \simeq 10^{-8} \longrightarrow \frac{c\,\delta t}{l} \simeq 0.2$$

This is HUGE: The original apparatus was sensitive to changes in optical phase of 0.01; 0.2 would be quite easy to see.

It wasn't seen. Experiment repeated at different times of year; different times of day; different locations on Earth. Repeated with more sensitive equipment (can now measure $c\delta t/\lambda \lesssim 10^{-10}$).

$$\boxed{\text{No motion relative to the ether has ever been detected.}}$$

Explanations:

1. The ether is dragged along by the Earth, somehow, so that we are "locally" at rest with it.
2. Maxwell's equations are wrong.
3. The ether squashes moving objects just enough to compensate for the travel time shifts.

→ 4. THERE IS NO ETHER. THERE IS NO SPECIAL REST FRAME FOR LIGHT. LIGHT TRAVELS AT $c = 1/\sqrt{\mu_0\epsilon_0}$ IN <u>ALL</u> INERTIAL REFERENCE FRAMES.