Recap:

- Focus on Schwarzschild spacetime,

$$ds^2 = -c^2\left(1 - \frac{2GM}{rc^2}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2$$

- Not so weird for $r \gg 2GM/c^2$, but very odd near there!
  - Clocks stop
  - Light stops
  - Infalling observers don't notice anything special.

- Key difficulty is in communicating with this location.

- Motion in this spacetime of a material body given by

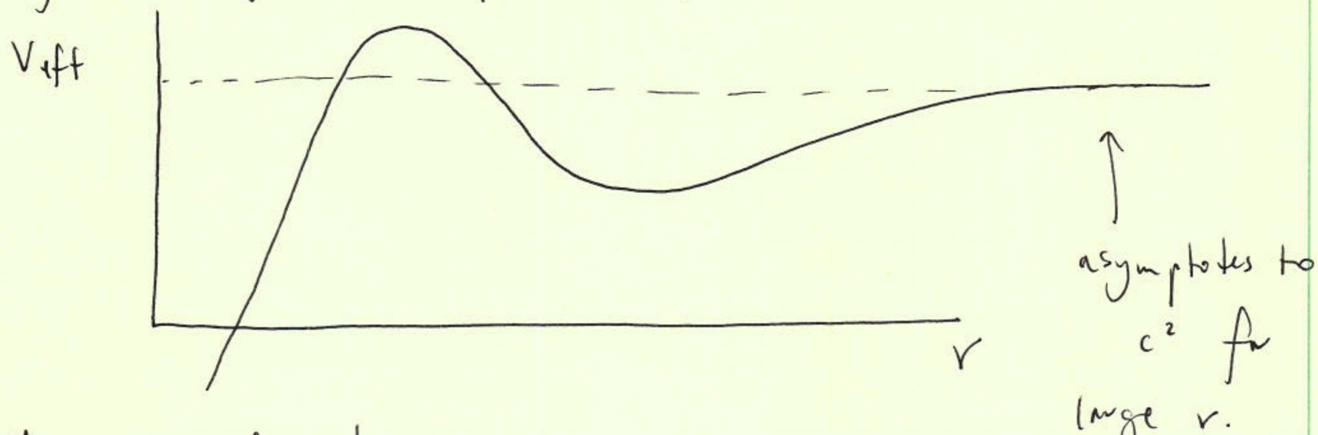
$$\left(\frac{dr}{d\tau}\right)^2 = \frac{\hat{E}^2}{c^2} - \left(1 - \frac{2GM}{c^2 r}\right)\left(c^2 + \frac{\hat{L}^2}{r^2}\right)$$

$\hat{E}$, $\hat{L}$ are constant on any trajectory.

Non-zero angular momentum: Details on pset #9.

Key results:

Given some value of $\hat{L}$, examine V_{eff} as a function of r. Typical shape:



Minimum of this curve is at

$$\frac{\partial V}{\partial r} = 0 \rightarrow \hat{L} = \pm \sqrt{\frac{GM r_{orb}}{1 - 3GM/r_{orb}c^2}}$$

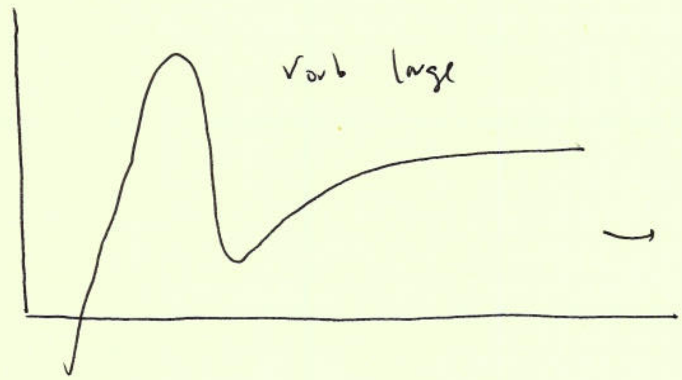
If an orbit sits at this value of r,

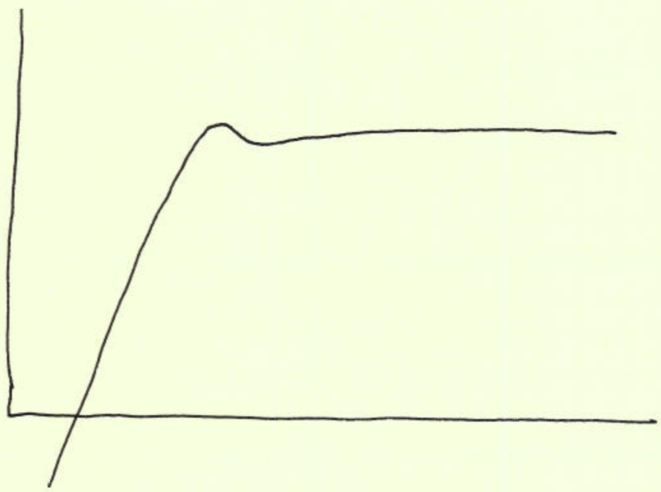
$$\left(\frac{dr}{d\tau}\right)^2 = \frac{\hat{E}^2}{c^2} - V_{eff} = 0$$

$$\rightarrow \hat{E} = c^2 \left(\frac{1 - 2GM/r_{orb}c^2}{\sqrt{1 - 3GM/r_{orb}c^2}} \right)$$

For large r_{orb}, this reduces to exactly the energy of a particle bound by Newtonian gravity to a mass M.

New feature: Consider smaller and smaller r_{orb}. You'll see that the depth of the minimum in V_{eff} gets shallower and shallower.





Eventually, minimum and maximum land on top of each other: Minimum turns into a point of inflection.

$$\frac{\partial^2 V}{\partial r^2} = 0 \rightarrow \quad r_{orb} = \frac{6GM}{c^2}$$

NO STABLE ORBITS EXIST for $r < \frac{6GM}{c^2}$.

Totally non-Newtonian behavior - hallmark of gravity in the very strong field.

Why are these objects interesting / important?

→ They are the inevitable outcome of the collapse of dense matter. (More accurately, the Kerr solution's spinning spacetime.)

- Make any object sufficiently compact. Eventually, pressure will be unable to resist the compression of gravity.

- Object then "collapses": Becomes more compact and dense until all of its matter fits inside an event horizon

Once it reaches this point, it settles down to the Kerr solution on a timescale

$$\Delta t \sim (\text{several}) \times GM/c^3$$

Note: $GM_{sun}/c^3 = 5 \times 10^{-6}$ seconds. Very fast.

A key test of relativity: Do massive and compact objects in fact settle down to this state?

Answer points to yes ... but it's really hard to test.