Focus of previous lecture: Spacetime and gravity of "compact" bodies, ie things that can be totally localized to some region. Reproduces Newton for $GM/rc^2 \ll 1$ and $v/c \ll 1$. Introduces new effects when speeds are high (bending of light, precession of orbits) and when GM/rc^2 is not small (unstable orbits, event horizons).

One feature of all these spacetimes: They are "asymptotically flat." so $ds^2 \to -c^2dt^2 + dx^2 + dy^2 + dz^2$ as we get really far away.

Is this correct?

Ask Einstein field equations : $G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$

Flat spacetime, $g_{\mu\nu} = \eta_{\mu\nu}$ is a solution ... but only if $T^{\mu\nu} = 0$.

Does $T^{\mu\nu} = 0$ describe our universe? Ie, is it totally empty as far as we can see?

No! Looking at in the universe, we see our galaxy, other galaxies, clusters of galaxies, light, neutrinos, clouds of gas....
Indeed, on the largest scales, the universe appears to be a uniform fog of matter & radiation - the "cosmic microwave background", or CMB.

How do we deal with this? First, think about how matter and energy are distributed in the universe.

What we see is that things are very uniform on the largest scales - variation in CMB is only at 10^{-5} level after correcting for Doppler of our motion relative to the CMB rest frame. Things become clumpier as we move to smaller scales - tendency of gravity to make matter more dense.

Punchline: As long as we restrict ourselves to scales larger than about $10\,\text{Mpc} = 10 \times 10^6$ parsecs, the universe is pretty uniform spatially. (Note: $1\,\text{pc} \simeq 3.26$ light year.)

We will use this to describe matter as a perfect fluid - same idea as describing a gas of individual atoms as a fluid, only works if you focus on scales where the matter's granularity has no effect.

Although the universe is uniform in space, it is NOT uniform in time. Light travels at a finite speed, so large distances are seen at earlier times. We see a universe that was much denser at earlier times: significant evolution with time!

So, we want a spacetime that is uniform in space, but not time. The most spatially symmetric spacetime we know of has

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{d\bar{r}^2}{1-k\bar{r}^2} + \bar{r}^2 d\theta^2 + \bar{r}^2 \sin^2\theta \, d\phi^2\right]$$

This is a "Robertson-Walker" spacetime. Interesting features:

$a(t)$: "Scale factor." This controls the physical scale associated with distance between two objects. If $k=0$, then the distance between $(\bar{r}_1, \theta, \phi)$ and $(\bar{r}_2, \theta, \phi)$ is

$$L = a(t)\left[\bar{r}_2 - \bar{r}_1\right].$$

Note: $a(t)$ has dimension length.

$\bar{r}$: A dimensionless radial coordinate. It cannot have dimension length since $a(t)$ does.

For $k=0$, $a(t)\,\bar{r}$ is essentially our "normal" spherical distance.

k: The "Spatial curvature" parameter. Its value is $+1$, 0, or -1.

For $k = 1$, we have

$$\frac{d\bar{r}}{\sqrt{1 - \bar{r}^2}} \equiv d\chi$$

$$\rightarrow \quad \bar{r} = \sin \chi .$$

The value of $\bar{r}$ is bounded: we can never exceed $\bar{r} = 1$. This describes a "closed universe" - the physical separation between objects has a maximum at each moment in time.

For $k = -1$,

$$\frac{d\bar{r}}{\sqrt{1 + \bar{r}^2}} \equiv d\chi \quad \longrightarrow \quad \bar{r} = \sinh \chi$$

"Open universe": Physical separation between objects is totally unbounded.

For $k = 0$, space has a "flat" Euclidean geometry: $\left(\frac{d\text{"distance"}}{a(t)}\right)^2 = d\bar{r}^2 + \bar{r}^2 d\theta^2 + \bar{r}^2 \sin^2\theta \, d\phi^2$

Often called a "flat universe," although this is only <u>SPATIALLY</u> flat - space<u>TIME</u> is still curved. Can be a bit confusing and unclear.

The value of k and the behavior of $a(t)$ must be determined from the Einstein field equations, which we'll do soon. As a setup, it is useful to examine how light and matter behave in this spacetime.

Begin by asking what happens to observers at rest:

$$u^t = c, \quad u^{\bar{r}} = u^\theta = u^\phi = 0.$$

Examine geodesics: Find that they remain fixed at coordinate $(\bar{r}, \theta, \phi)$.

However, as they are fixed there, the proper separation of different observers changes at $a(t)$ evolves. These observers "comove" as the universe's geometry changes - they are "comoving" observers.

Next, examine light - Crucial, since we use it to measure and understand our universe.

For simplicity, focus on $k=0$. (Calculation can be generalized to $k = \pm 1$, but it is messy.)

Imagine that light is emitted at some time t_e, and received ~~###~~ by an observer at some later time t_R. It's enough to consider light that moves radially, so we'll put $\vec{p} \doteq (p^t, p^{\bar{r}}, 0, 0)$.