Recap:

- Large-scale structure of the universe described by spacetime

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{d\bar{r}^2}{1-k\bar{r}^2} + \bar{r}^2 d\theta^2 + \bar{r}^2 \sin^2\theta \, d\phi^2\right]$$

Spatially uniform, consistent with our universe on the largest scales we can observe.

- Parameter $k = +1, 0, -1$ determines geometry of $t =$ constant "slices." Can be "closed" ($k = +1$; max separation between events exists), "open" ($k = -1$; separation is unbounded), or "flat" ($k = 0$; geometry of slices is Euclidean).

- Function $a(t)$ controls scale of spatial distances. Directly imprinted on the energy of radiation propagating across the universe:

$$\frac{E_{measured}}{E_{emit}} = \frac{a(t_{emit})}{a(t_{meas})} \equiv \frac{1}{1+z}$$

Note: If $a(t)$ changes, it <u>doesn't</u> mean that all objects expand (or contract) according to the behavior of a.

Common misperception is that as a changes, everything expands or contracts in synch with it ... in which case we wouldn't notice the change.

However, expansion / contraction as $a(t)$ evolves only affects <u>COMOVING</u> points: Points which are not experiencing forces which push them off the comoving geodesic. This means that only things which are not bound together will expand or contract.

So - very distant galaxies move relative to us according to the dictates of $a(t)$. But the galaxies themselves do not expand because they are bound objects. Neither do stars, planets, people.

Remember: The Robertson-Walker metric describes things only on very large spatial scales. Won't describe the physics of smaller scales!

How do we constrain $a(t)$ & the parameter k?

Turn to Einstein field equation:

$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

Compute for R-W spacetime

Take to be a perfect fluid - appropriate given focus on longest spatial scales.

Einstein field equation tells us

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho}{3c^2} - \frac{kc^2}{a^2}$$

Discovered by Alexander Friedman in 1922, known as the "Friedman equation." An FRW spacetime whose $a(t)$ & k are related to ρ by this is an "FRW" cosmology.

Before discussing the physics of this, it is useful to introduce some terminology:

$$\frac{\dot{a}}{a} \equiv H \rightarrow \text{"Hubble" expansion parameter}$$

$$H_0 = H(t = \text{now}) \equiv \text{Hubble constant}$$

$$= 70\,(\text{km/sec})/\text{Mpc}$$

Note: H has dimensions of $(\text{time})^{-1}$.

Another useful definition: $\rho_{crit} = \dfrac{3H^2c^2}{8\pi G}$ "Critical density"

And one more: $\Omega = \rho / \rho_{crit}$ → density normalized to "critical" value.

Using this, can rearrange Friedmann:

$$1 = \frac{8\pi G}{3H^2c^2}\rho - \frac{kc^2}{a^2H^2}$$

or

$$\boxed{\Omega - 1 = \frac{kc^2}{a^2H^2}}$$

Now, can see significance of ρ_{crit}:

$\rho > \rho_{crit}$: $\Omega > 1$ → k positive, universe is closed.

$\rho < \rho_{crit}$: $\Omega < 1$ → k negative, universe is open.

$\rho = \rho_{crit}$: $\Omega = 0$ → $k = 0$, universe is spatially flat.

On pset #10, you'll see interesting dynamical consequences:

$\rho > \rho_{crit}$: Universe expands, reaches a maximum scale, then recollapses.

$\rho < \rho_{crit}$: Universe expands forever.

$\rho = \rho_{crit}$: Universe expands forever, but with ever decreasing speed.

How do we know which of these options is right? We need to know how the universe behaves for a couple of representative examples of what goes into it.

Here are some illustrative limiting cases. Take a $k=0$ universe, and fill it with dust. In this limit, the total number of dust particles is fixed, but their density changes as $a(t)$ changes:

$$\rho_D(t) = \rho_D(\text{now}) \left[\frac{a(\text{now})}{a(t)} \right]^3$$

$$= \rho_0 \frac{a_0^3}{a(t)^3}$$

Friedman equation in this form:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} \rho_0 \frac{a_0^3}{a^3}$$

This equation has a power law solution:

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^n$$

Plug in, find $n = 2/3$ needed to satisfy Friedman:

$$\boxed{\begin{array}{c} a(t) \propto t^{2/3} \\ \text{for a "matter dominated"} \\ \text{cosmology.} \end{array}}$$

Notice that this solution grows with time: a universe that expands. If you run it backwards, it implies $a=0$ at some time in the past. This means all spatial locations are smashed into a zero-size point. Rolling forward, spacetime itself comes into existence as we evolve from that time.

The birth of all space is known as the "Big Bang" Notice it is not an explosion into space - it is an explosion <u>OF</u> space. There wasn't any "there" to explode into until the Big Bang happened!

Another example: Universe filled with "radiation." Imagine that the number of photons is fixed, but their density varies as a^{-3}. In addition, each photon has an energy that itself varies as a^{-1} - redshift effect!

$$\rho_R(t) = \rho_0 \left(\frac{a_0}{a(t)}\right)^4$$

Plug this into Friedman, again get a power law:

$$\boxed{a(t) \propto t^{1/2}}$$

in a "radiation dominated" cosmology

One last example: "vacuum energy." This arises in quantum field theory as an energy associated with the ground state of quantum fields. Key property: it must be invariant with respect to Lorentz transformations in a freely falling frame:

$$T^{\mu\nu} \propto \eta^{\mu\nu} \quad \text{in FFF}$$

This means it looks like a perfect fluid, but one with "negative pressure": $P_\Lambda = -\rho_\Lambda$

We can deduce how this evolves by enforcing the rule $\sum_\nu \nabla_\nu T^{\mu\nu} = 0 \rightarrow$ find $\rho_\Lambda = \underline{\text{constant}}$. Very odd, but arises as a consequence of the fact that it is associated with the vacuum itself.

Plug into Friedman: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho_\Lambda}{3c^2}$

$$\dot{a} = \pm a \sqrt{\frac{8\pi G \rho_\Lambda}{3c^2}}$$

$$\rightarrow \boxed{a(t) \propto \exp\left[\pm\sqrt{\frac{8\pi G \rho_\Lambda}{3c^2}}\, t\right]}$$

Exponential expansion or contraction! Expansion dominates, since the contracting solution crushes away its own relevance to the universe.

These are idealized limits. However, they demonstrate that different contributions to the universe give us different rates of expansion. By measuring the rate of expansion at many different times, we can infer what the universe is made of.

(Note, we generally expect a mixture of ingredients, for which the exact power laws don't apply. But, these limits give an asymptotic form; and, the general case is easy to solve numerically.)

So, we just need to measure a at a few values of t and see what we get. Our tools:

1. Distance. Since light's speed is known, this tells us the time at which it was emitted. Get time/distance, plus redshift: You know $a(t)$!

   - "Standard rulers": Sources whose size is known by some physics. Compare apparent size to physical size, infer distance.
   - "Standard candles": Sources whose intrinsic brightness is known. Compare apparent and intrinsic brightness, infer distance.

2. Mass: look at clumps of matter (galaxy clusters in particular), gives us an independent handle on "normal matter" contribution to universe's make up.

Result: $H_0 = (70.1 \pm 1.3)\ (\text{km/sec})/\text{Mpc}$

$$\Omega_M = \rho_M/\rho_{crit} = 0.279 \pm 0.014$$

$$\Omega_\Lambda = \rho_\Lambda/\rho_{crit} = 0.721 \pm 0.015$$

$$\Omega_{TOTAL} = \Omega_M + \Omega_\Lambda = 1.0052 \pm 0.0064$$

→ Consistent with $k = 0$, ie spatial flatness.

These are highly disturbing results. 3 mysteries:

1: why is $k = 0$? One can show that if

$$\Omega - 1 = \epsilon, \text{ then } \epsilon \text{ \underline{grows} with time!}$$

Ie, the deviations from spatial flatness should be magnified.

We have a plausible solution to this mystery: Make the universe vacuum energy dominated at an early phase! In this case, ϵ shrinks: We are dynamically driven to a spatially flat configuration.

Theory of INFLATION: States that the universe was in a "false vacuum" at early times and was exponentially inflated for about 10^{-30} seconds.

Key formulation presented by Alan Guth in 1980; details tweaked, but big picture holding up. People now trying to find direct evidence of this early false vacuum state.

2: What is the matter that contributes to Ω_M?

Problem: If we add up all the matter that produces light - stuff we know from the standard model of particle physics - we get

$$\Omega_M = 0.0462 \pm 0.0015$$

The remaining $\Delta\Omega_M = 0.233$ is something "dark" that we've never detected directly. We see its gravitational influence, but have never unambiguously seen it in our detectors.

3: What is Ω_Λ??

The appearance of vacuum energy today is a complete surprise. If we try to calculate its value using standard model physics, we get a number that is too big by a factor of 10^{120}! If we include plausible modifications (supersymmetry) we reduce this to 10^{60}.

Punchline: We don't have a clue.