

Recap:

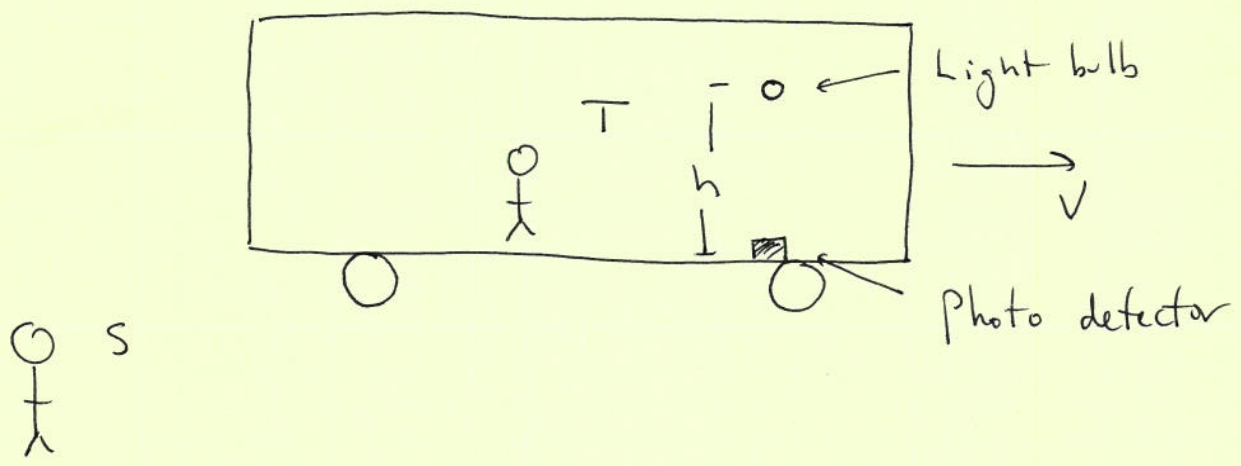
- Galilean relativity predicts that wave speeds - including light - depends on ~~an~~ observer's motion with respect to rest frame of medium supporting the wave ("ether" for light)
- No such effect has been seen for light ... ideas to explain are rather problematic, except:
- Explanation we shall use:

LIGHT TRAVELS AT c IN ALL
INERTIAL REFERENCE FRAMES.

This simple idea has rather amazing consequences

1. Different inertial observers measure different intervals of time between events.
2. Different ^{inertial} observers measure different spatial distances between events.

Consider two observers. One is on a train moving at speed v with respect to a station; the other is at rest with respect to this station.



Imagine a light bulb emits a pulse of light ~~at~~ at some moment. This is event A.

The pulse of light is measured by the photo detector on the train's floor: event B.

What is the time interval between these events?

On the train: Light just moves through vertical distance h ,

$$\Delta t_T = h/c$$

In the station: Observers here see light move through vertical distance h , plus a horizontal distance that depends on how fast the train is going:

$$\Delta t_S = \frac{D}{c} = \frac{\sqrt{h^2 + (v\Delta t_S)^2}}{c}$$

$$\rightarrow \Delta t_S = \frac{h/c}{\sqrt{1 - v^2/c^2}} \equiv \gamma \Delta t_T$$

$$\begin{aligned} \text{where } \gamma &\equiv \frac{1}{\sqrt{1 - v^2/c^2}} \\ &= (1 - \beta^2)^{-1/2} \end{aligned}$$

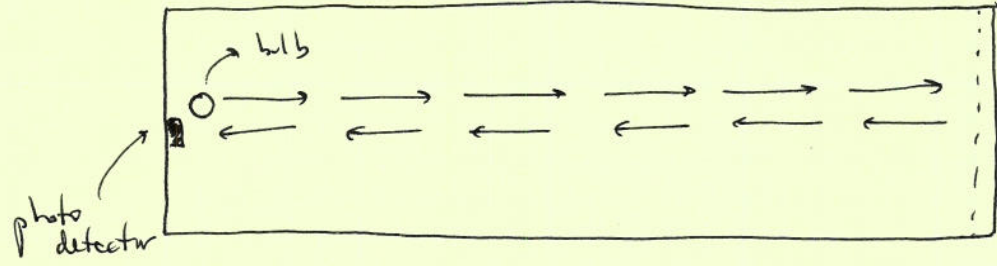
The time interval as seen from the station is larger than the interval seen from the train ($\gamma \geq 1$):

MOVING CLOCKS
RUN SLOW

"Time Dilation"

NOTE: We assumed that the height h is the same for both observers. Hold that thought!

Imagine we now flash the light so that it goes to the front of the car, bounces off a mirror, then returns to a photosensor:



Time the round trip, use the measured time to infer the length of the train car.

On train: $\Delta x_T = c \Delta t_T / 2$

On station: Break calc into two pieces, 1 for emission \rightarrow mirror, 2 for mirror \rightarrow photo detector.

Time for these legs:

$$\Delta t_{s,1} = (\Delta x_s + v \Delta t_{s,1}) / c$$

$$\Delta t_{s,2} = (\Delta x_s - v \Delta t_{s,2}) / c$$

Note asymmetry: Light is "chasing" the mirror on piece 1, but heading toward advancing photo detector on piece 2.

Isolate the travel times and add them to get the total travel time:

$$\Delta t_{s,1} = \frac{\Delta x_s}{c - v}$$

$$\Delta t_{s,2} = \frac{\Delta x_s}{c + v}$$

$$\Delta t_s = \frac{2 \Delta x_s / c}{1 - v^2 / c^2}$$

$$= 2 \gamma^2 \Delta x_s / c$$

We've already seen that the "moving clock" runs slow as measured by observers on the station, so we have

$$\Delta t_s = \gamma \Delta t_T$$

$$= 2 \gamma \Delta x_T / c$$

Equate the two expressions for Δt_s :

$$\Delta x_s = \frac{\Delta x_T}{\gamma}$$

MOVING RULERS ARE SHORTENED

"Length contraction"

Are directions perpendicular to the relative velocity affected?

No: If so, we would get inconsistent physics - different events in different frames of reference.

Example: Train going 90% of c toward a tunnel.

$h_{Train} = 3.5 \text{ m}$ $h_{Tunnel} = 4 \text{ m}$

If length contraction affected height, we'd have a problem.

TUNNEL REST FRAME: $h_{obs}^{Train} = \frac{3.5 \text{ m}}{\gamma} = 1.53 \text{ m}$

TRAIN REST FRAME: $h_{obs}^{Tunnel} = \frac{4 \text{ m}}{\gamma} = 1.74 \text{ m}$

One frame: Train fits.

Other frame: Tremendous crash.

→ NOT CONSISTENT!

In relativity, all observers agree on the events, even if they describe them with different labels. Perpendicular length contraction does not work.

(Our assumption in the time dilation calc was OK.)

Note what we have done here: We have allowed our notions of time intervals and space intervals to get messed up by our demand that all observers measure the same speed of light waves.

This is a good thing! We are using an invariant, the speed of light, as our tool for measuring times and distances. Invariants are your friends: You always know that all observers agree on their value, so exploit them as much as possible.

The idea that "time" and "space" are getting messed up is a consequence of how the Galilean transformation generalizes when we insist that all observers agree on the speed of light.

Galileo: Transforming between different inertial reference frames mixes space (my "left" is a mixture of "left" and "forward" for you), or makes space vary with time (my "forward" increases uniformly with time according to you). Time is invariant.

Lorentz: Transforming between different inertial reference frames mixes time and space however is necessary to leave the speed of light invariant.

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Derivation of Lorentz transformation

Consider two observers:

One is at rest with a lab, assigns events coordinates (t, x, y, z)

Other moves at constant velocity $\underline{v} = v \hat{x}$ according to lab-fixed observers, assigns events coordinates (t', x', y', z') .

Let us assume the following general relation between these two systems:

$$\begin{aligned} t' &= A t + B x & y' &= y \\ x' &= D t + F x & z' &= z \end{aligned}$$

- Points of note:
1. Motion is along x , so y & z are not affected.
 2. Linear transformation. If there were non linear terms (e.g., a t^2 term), the transformation would not be inertial.

Now, solve for A, B, D, F by matching important quantities in the two systems and imposing constancy of c .

1. Match origin of "moving" frame ~~to~~ ($x' = 0$) to correct set of events in lab frame ($x = vt$):

$$x' = Dt + Fx$$

$$\rightarrow 0 = Dt + Fvt$$

$$D = -Fv$$

2. Match origin of "lab" frame ($x = 0$) to correct set of events in "moving" frame: Lab moves with ~~the~~ $v_{\text{lab}} = -v\hat{x}$ as seen by lab frame, so $x = 0 \leftrightarrow x' = -vt'$:

$$x' = -F(vt - x) \quad (\text{general rule plus results of step 1})$$

$$-vt' = -Fvt \rightarrow t' = Ft$$

Plug in the t' transformation rule:

$$t' = At + \cancel{Bx} \quad (\text{considering } x=0)$$

$$Ft = At \rightarrow F = A$$

We now have 2 of the 4 unknown coefficients pinned down. The transformation has been reduced to

$$t' = At + Bx$$

$$x' = -Avt + Ax \\ = A(x - vt)$$

To get the next two, consider how light propagates:

First, imagine a light pulse emitted at $t=t'=0$ (frames match origins at $t=0$) along the x -axis.

As that light pulse propagates, its trajectory is given by

$$x = ct$$

in the "lab" frame, and

$$x' = ct'$$

in the "moving" frame. Let's exploit this:

$$x' = ct'$$

$$A(x - vt) = c(A + Bx)$$

Substitute transformation

$$A(c - v)t = c(A + Bc)t$$

substitute $x = ct$

$$-Avt = Bc^2t \rightarrow$$

$$B = -\frac{Av}{c^2}$$

Transformation is now

$$t' = A(t - xv/c^2)$$

$$y' = y$$

$$x' = A(x - vt)$$

$$z' = z$$

Next, shoot a pulse along the y -axis:

In "lab" frame, we have $y = ct$

In "moving" frame, the pulse moves diagonally along x' and y' , with

$$(x')^2 + (y')^2 = c^2 (t')^2$$

$$A^2 (x - vt)^2 + y^2 = c^2 A^2 (t - vx/c^2)^2$$

$$A^2 v^2 t^2 + c^2 t^2 = c^2 A^2 t^2$$

(Plug in $x=0$,
 $y=ct$)

$$\rightarrow \boxed{A^2 = \frac{1}{1 - v^2/c^2} = \gamma^2}$$

So:

$$t' = \gamma (t - xv/c^2)$$

$$x' = \gamma (-vt + x)$$

$$y' = y \quad z' = z$$

Symmetric form: Put $\beta = v/c$, use ct rather than t (and ct' instead of t') so dimensions are the same:

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$