

How black holes get their kicks: Radiation recoil in binary black hole mergers

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Abstract. Gravitational waves from the coalescence of binary black holes carry linear momentum, causing center of mass recoil. This “radiation rocket” has important implications for systems with escape speeds of order the recoil velocity. We describe new recoil calculations using high precision black hole perturbation theory to estimate the magnitude of the recoil for the slow “inspiral” coalescence phase; coupled with a cruder calculation for the final “plunge”, we estimate the total recoil imparted to a merged black hole. We find that velocities of many tens to a few hundred km/sec can be achieved fairly easily. The recoil probably never exceeds about 500 km/sec.

It is very well known that gravitational waves (GWs) carry energy and angular momentum from a binary system, causing decay of the binary’s orbit and eventually driving the system to merge into a single object. Although it has been understood for quite some time (e.g., [1]), it is somewhat less well-appreciated that these waves can carry *linear* momentum from the system as well. The center of mass in this case must recoil in order to enforce global conservation of momentum. If the recoil velocity is comparable to or greater than the escape velocity of the binary’s host structure, there could be important dynamical consequences, such as ejection of the merged black hole remnant.

The recoil arises because the radiation field generated by a binary is typically asymmetric. As a helpful cartoon, consider the following argument due to Alan Wiseman. In an unequal mass binary (Fig. 1), the smaller member, m_1 , moves with a higher speed than the larger member, m_2 . It is thus more effective at “forward beaming” its wave pattern. This means that there is an instantaneous net flux of momentum ejected from the system parallel to the velocity of the smaller body, and a concomitant recoil opposing this.

Over an orbit, the recoil direction continually changes. If the orbit were perfectly circular, this means that there would be no net interesting effect — the binary’s center of mass would run around in a circle, and the *net* recoil would sum to zero. However, when GW emission is strong, the orbit is *not* perfectly circular: Because of the secular, dissipative evolution of the binary’s energy and angular momentum, the black holes slowly spiral towards one another. Since the orbit does not close, the recoil does not sum to zero. The recoil accumulates until the holes merge and settle down to a quiescent state, shutting off the momentum flux and yielding a net, non-zero kick.

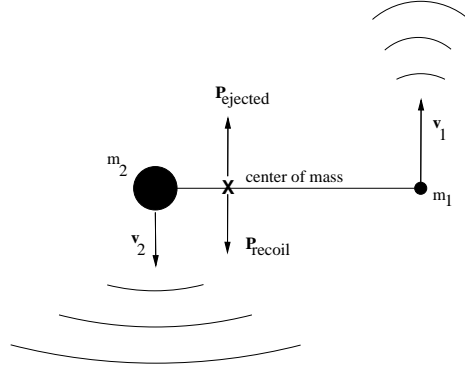


Fig. 1. GW emission from an unequal mass binary. Momentum is ejected parallel to the smaller body’s velocity (v_1). Conservation of momentum requires that the system recoil in the opposite direction.

This recoil is not a wierd property of GWs — it holds for *any* form of radiation¹. This can be brought out by considering a multipolar decomposition. Suppose we build a distribution of charges that has a non-zero electric dipole and quadrupole moment, as in Fig. 2. Suppose further that we spin this charge arrangement about its center point, driving the system to radiate electromagnetic waves. What does this radiation distribution look like from far away?

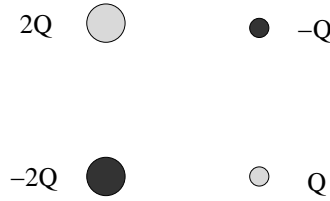


Fig. 2. Charge distribution with non-zero dipole and quadrupole moment. Spinning this distribution about its center point produces radiation carrying non-zero linear momentum due to beating between the dipolar and quadrupolar radiation fields.

The radiation’s *amplitude* has two pieces, dipole and quadrupole:

$$\mathbf{E} = \mathbf{E}^{\text{dip}} + \mathbf{E}^{\text{quad}}, \quad \text{where} \quad (1)$$

$$\mathbf{E}^{\text{dip}} \propto e^{i(\phi - \omega t)}, \quad \mathbf{E}^{\text{quad}} \propto e^{2i(\phi - \omega t)}. \quad (2)$$

Since the intensity $I \propto |\mathbf{E}|^2$, it will contain three pieces:

$$I = I^{\text{dip}} + I^{\text{quad}} + I^{\text{dip-quad}}, \quad (3)$$

where

$$I^{\text{dip}} \propto |\mathbf{E}^{\text{dip}}|^2 \propto \text{constant}; \quad I^{\text{quad}} \propto |\mathbf{E}^{\text{quad}}|^2 \propto \text{constant} \quad (4)$$

¹ Indeed, electromagnetic or neutrino recoil may impact neutron star kicks [2,3].

$$I^{\text{dip-quad}} \propto \text{Re} \left[\mathbf{E}^{\text{dip}} \bar{\mathbf{E}}^{\text{quad}} \right] \propto \cos(\phi - \omega t). \quad (5)$$

The intensity has a preferred direction, which rotates as the charge distribution rotates. The energy from the system is instantaneously beamed in a preferred direction, and so there is a net flux of momentum in that direction as well.

Since the lowest order GWs are quadrupolar, recoil from GW emission must come (at lowest order) from a beating of the mass quadrupole with mass octupole and current quadrupole moments. The mass octupole and current quadrupole vanish for an equal mass binary, demonstrating — in accord with our “forward beaming” intuition — that unequal masses are needed for there to be any recoil. This also demonstrates that GW recoil must be a very small effect, except perhaps in the very late stages of coalescence — the octupole radiation amplitude is smaller than the quadrupole by a factor of order v/c (where v is orbital speed).

The first careful analysis of recoil in binary systems due to GW emission is that of Michael Fitchett [5]. Fitchett’s analysis described the orbital dynamics of the binary using Newtonian gravity and only included the lowest radiative multipoles which contribute to the recoil. His analysis predicted that the recoil of the merged remnant took the form

$$v_{\text{F}} \simeq 1480 \text{ km/sec} \frac{f(q)}{f_{\text{max}}} \left(\frac{2G(m_1 + m_2)/c^2}{R_{\text{term}}} \right)^4, \quad (6)$$

where R_{term} is the orbital separation at which GW emission terminates, $q = m_1/m_2$ is the binary’s mass ratio, and $f(q) = q^2(1 - q)/(1 + q)^5$ is a function whose maximum is at $q \simeq 0.38$, and has the limit $f(q) \simeq q^2$ for $q \ll 1$.

Three features of this formula are particularly noteworthy. First, this result does not depend on total mass — only on the mass *ratio* (bearing in mind that R_{term} scales with total mass M). Thus, this scaling holds for any binary black hole merger — stellar mass mergers through supermassive mergers. Second, the overall scale is quite high. Although there is an important dependence on mass ratio and the termination radius R_{term} is somewhat uncertain, Eq. (6) indicates that kicks of hundreds of km/sec are not difficult to achieve; kicks $\gtrsim 1000$ km/sec are plausible. This is high enough that we might expect black hole ejection following a merger to be common.

Notice, however, that the recoil becomes very strong when the separation of the bodies is small. This is a strong hint that we cannot take Eq. (6) at face value — the strong gravity physics neglected by Ref. [5] is likely to be very important.

A few efforts have improved on Fitchett’s analysis over the years. Fitchett and Detweiler [6] first made a strong field analysis, treating the binary as a Schwarzschild black hole M orbited by a point mass μ . The orbiting body’s influence can then be studied using black hole perturbation theory. Their results suggested that Eq. (6) describes the recoil fairly well. Wiseman [4] analysed the recoil with post-Newtonian theory (roughly, an expansion in $v \sim \sqrt{GM/rc^2}$). He found that Fitchett’s formula tended to systematically overestimate the recoil by $\gtrsim 10\%$. Unfortunately, his results behave somewhat pathologically for $r \lesssim 9GM/c^2$ due to ill behavior of the expansion in the very strong field.

This motivates our analysis of this problem. Our formal setup is quite similar to that of Fitchett and Detweiler: We model the binary as a pointlike body μ moving on the exact, geodesic orbits of a Kerr black hole with mass M and spin parameter a . We compute the GWs emitted from such orbits very accurately using black hole perturbation theory [7–9], and extract the recoil from the wave pattern [10]. The perturbative approach allows us to study the dynamics of the binary’s spacetime with high accuracy. Schematically, we treat this spacetime as that of the Kerr black hole plus a small perturbation:

$$g_{\alpha\beta} = g_{\alpha\beta}^{\text{Kerr}}(M, a) + h_{\alpha\beta}(\mu). \quad (7)$$

By requiring that this spacetime satisfy the Einstein field equations, it can be shown [7] that $h_{\alpha\beta}$ is governed by a wave-like equation. The wave operator automatically captures the most important properties of the strong field physics.

This approach is strictly accurate only when $\mu \ll M$ — the smaller body cannot significantly distort the spacetime if we want the exact Kerr orbits to describe our binary. We believe, though, that we can extrapolate out of this regime with accuracy good enough for most astrophysical purposes — our extrapolation errors are estimated to be at most several tens of percent, at least up to a mass ratio $q \simeq 0.3$, and to scale with the squared mass ratio.

We focus upon circular, equatorial orbits of Kerr black holes. Circularity is surely a good approximation, since eccentricity is rapidly reduced during coalescence. The equatorial assumption is not so good; since the binaries of interest form through captures, we expect no particular alignment between the spin and orbit. We are working to lift this approximation, which requires moderately substantial modifications to our code; early indications are that the inclination does not have a very large effect on the recoil, other than to change the radius at which a transition in the orbital dynamics occurs. Results from the prograde and retrograde equatorial orbits appear to bound the recoil at any spin.

One technical detail of our code is important enough that it requires some explanation. Our calculation expands the perturbation as

$$\Psi_4 = \frac{1}{r} \sum_{lm} Z_{lm} S_{lm}(\theta) e^{im(\phi - \Omega t)}. \quad (8)$$

Ψ_4 represents a curvature perturbation, and serves as a surrogate for $h_{\alpha\beta}$. $S_{lm}(\theta)$ is a spheroidal harmonic (very similar to a spherical harmonic), and Z_{lm} is a complex number found by solving a certain differential equation [7,8]. The most important aspect of this equation for our purposes is the presence of the orbital frequency Ω : We assume that the orbit is well-described by a Fourier expansion. This is only true if the orbit is periodic (or nearly so). This in turn is only true when the separation r of the black holes is greater than the radius of the “last stable orbit” (LSO), $r_{\text{LSO}}(a)$. (For Schwarzschild holes, $a = 0$, $r_{\text{LSO}} = 6GM/c^2$.)

In the regime $r > r_{\text{LSO}}(a)$, dynamically stable orbits exist. GWs slowly evolve the system through a sequence of stable orbits of ever decreasing radius. At any instant in this regime, the orbit is well modeled as periodic with a well-defined

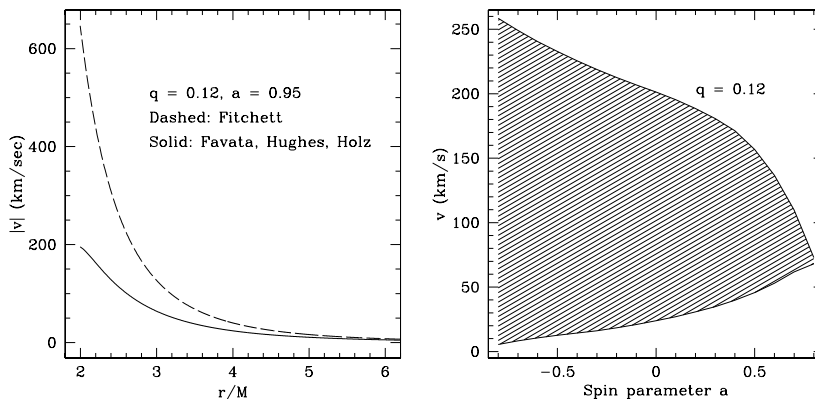


Fig. 3. Left: Fitchett’s accumulated recoil and our results, versus binary separation r . This illustrates the importance of a proper treatment of strong-field physics. Right: Total recoil versus black hole spin a . The upper curve is our overestimate, the lower our underestimate. The span between them shows the importance of the final plunge.

frequency Ω . The recoil is then simply related to the coefficients Z_{lm} and to an overlap integral involving the harmonics $S_{lm}(\theta)$; see Ref. [10] for details.

As we approach and cross $r_{\text{LSO}}(a)$, the orbit becomes a rapid “plunge” in which the small body quickly falls into the event horizon. This final plunge *cannot* be treated using this Fourier expansion, so our calculation breaks down. We are currently developing techniques to model this regime accurately. For the present analysis, we compute a probable overestimate (based on an extrapolation of the inspiral momentum flux beyond its range of validity) and a probable underestimate (based on a low-order flux formula coupled to the plunging motion) in order to provide a reasonable range for the likely recoil.

Our analysis shows that Fitchett’s calculation *consistently overestimates* the recoil velocity, especially as the LSO is approached; see the left-hand panel of Fig. 3. This is due to strong-field physics: A wave packet released near the horizon redshifts as it propagates to large radius, reducing the energy and momentum that it carries. This effect is not present in calculations which neglect curved spacetime physics, as in Ref. [5]. Also, when radiation propagates through a curved spacetime, the anisotropy of the radiation pattern tends to be somewhat reduced due to the phenomenon of *tails* — essentially, the backscatter of the radiation from spacetime curvature itself. Both the redshifting and the anisotropy reduction reduce the recoil relative to Fitchett’s original analysis.

The right-hand panel of Fig. 3 summarizes our results for the total recoil that can be expected in a merger; the curves shown are for $q = 0.12$, but can be rescaled using $f(q)$ [Eq. (6) and subsequent text]. The range shown here shows the uncertainty that results from our inability to model the plunge very well. It is largest for retrograde orbits ($a < 0$) of rapidly rotating black holes because the transition to plunge occurs at relatively large radius there — the smaller body plunges quite a distance before passing through the event horizon. By contrast,

for prograde orbits ($a > 0$), the transition occurs at small radius, so the plunge does not matter quite so much. Our uncertainty is much smaller for those cases.

A very notable feature of this plot is that the recoil, though substantial, *never* exceeds about 500 km/sec, even when the mass ratio is “tuned” to maximize the recoil. On the other hand, it is not difficult for the recoil to reach several tens of km/sec, even for our underestimate. Indeed, when we convolve our over- and underestimates with a distribution of likely mass ratios and spin values (cf. Ref. [11]) we find that recoils of several tens of km/sec are quite easy to achieve; recoils ~ 100 km/sec are likely for an interesting fraction of recoils; and recoils of several hundred km/sec, though possible, are probably rather rare.

In the near future we hope to reduce our error bars, but for now we understand the recoil with sufficient accuracy that these results can be used for many astrophysical applications. The most likely recoil range — several tens to a few hundred km/sec — is particularly interesting: While not large enough to eject black holes from massive galaxies, kicks in this range can lead to ejection in dwarf galaxies and dark matter halos, affect the nuclear density profiles of galaxies with SMBHS, and influence the hierarchical growth of supermassive and intermediate mass black holes [11–17].

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