# Erratum: Measuring coalescing massive binary black holes with gravitational waves: The impact of spin-induced precession [Phys. Rev. D 74, 122001 (2006)]

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Since the publication of this paper, we found a rather silly error in the code used for this analysis. We use the variable $\cos\theta$ to describe an event's location on the sky and to specify its orbital angular momentum vector. In several places, we need to convert to $\sin\theta$, which we do using the rule $\sin\theta = \sqrt{1 - \cos^2\theta}$. In two instances (out of eight), we failed to square the $\cos\theta$ under the square root. This error impacted both the code that includes black hole precession physics and the "no precession" code.

In fixing this error, we find that our qualitative results are unchanged. However, almost all of our results are quantitatively impacted at least slightly. Below we show the corrected versions of Tables I, II, III, IV, V, and VI, which give the median errors in intrinsic signal parameters (masses and spins; Tables I, II, and III) and extrinsic signal parameters (sky position and distance; Tables IV, V, and VI) for a range of masses and redshifts.

TABLE I. Median errors in intrinsic quantities for $10^4$ binaries of various masses at $z = 1$, including comparisons with the "no precession" case where possible. We have omitted the errors in chirp mass and reduced mass for equal mass binaries because that parametrization of the waveform fails the Gaussian approximation at those points.

| $m_1(M_\odot)$ | $m_2(M_\odot)$ | $\Delta m_1/m_1$ | $\Delta m_2/m_2$ | $\Delta\chi_1$ | $\Delta\chi_2$ | $\Delta\mathcal{M}/\mathcal{M}$ (no precession) | $\Delta\mathcal{M}/\mathcal{M}$ (precession) | $\Delta\mu/\mu$ (no precession) | $\Delta\mu/\mu$ (precession) |
|---|---|---|---|---|---|---|---|---|---|
| $10^5$ | $10^5$ | 0.000 783 | 0.000 782 | 0.004 15 | 0.004 14 | — | — | — | — |
| $3\times10^5$ | $10^5$ | 0.000 667 | 0.000 541 | 0.001 57 | 0.003 06 | $5.92\times10^{-5}$ | $5.51\times10^{-6}$ | 0.0114 | 0.000 239 |
| $3\times10^5$ | $3\times10^5$ | 0.001 09 | 0.001 09 | 0.005 39 | 0.005 36 | — | — | — | — |
| $10^6$ | $10^5$ | 0.000 629 | 0.000 440 | 0.001 02 | 0.004 40 | 0.000 156 | $1.18\times10^{-5}$ | 0.0180 | 0.000 343 |
| $10^6$ | $3\times10^5$ | 0.001 11 | 0.000 882 | 0.002 56 | 0.004 99 | 0.000 170 | $1.19\times10^{-5}$ | 0.0274 | 0.000 423 |
| $10^6$ | $10^6$ | 0.001 95 | 0.001 95 | 0.009 02 | 0.008 97 | — | — | — | — |
| $3\times10^6$ | $3\times10^5$ | 0.000 988 | 0.000 691 | 0.001 37 | 0.005 63 | 0.000 583 | $2.53\times10^{-5}$ | 0.0550 | 0.000 539 |
| $3\times10^6$ | $10^6$ | 0.002 38 | 0.001 92 | 0.003 80 | 0.006 74 | 0.001 17 | $4.19\times10^{-5}$ | 0.135 | 0.000 849 |
| $3\times10^6$ | $3\times10^6$ | 0.005 84 | 0.005 82 | 0.0271 | 0.0275 | — | — | — | — |
| $10^7$ | $10^6$ | 0.002 39 | 0.001 77 | 0.002 33 | 0.0122 | 0.007 70 | 0.000 174 | 0.469 | 0.001 40 |
| $10^7$ | $3\times10^6$ | 0.008 14 | 0.006 71 | 0.008 29 | 0.0159 | 0.008 51 | 0.000 436 | 0.607 | 0.003 32 |
| $10^7$ | $10^7$ | 0.0804 | 0.0802 | 0.492 | 0.493 | — | — | — | — |

TABLE II. Median errors in intrinsic quantities for $10^4$ binaries of various masses at $z = 3$.

| $m_1(M_\odot)$ | $m_2(M_\odot)$ | $\Delta m_1/m_1$ | $\Delta m_2/m_2$ | $\Delta\chi_1$ | $\Delta\chi_2$ | $\Delta\mathcal{M}/\mathcal{M}$ (no precession) | $\Delta\mathcal{M}/\mathcal{M}$ (precession) | $\Delta\mu/\mu$ (no precession) | $\Delta\mu/\mu$ (precession) |
|---|---|---|---|---|---|---|---|---|---|
| $10^5$ | $10^5$ | 0.003 62 | 0.003 62 | 0.0187 | 0.0185 | — | — | — | — |
| $3\times10^5$ | $10^5$ | 0.003 63 | 0.002 94 | 0.008 79 | 0.0171 | 0.000 406 | $3.31\times10^{-5}$ | 0.0715 | 0.001 30 |
| $3\times10^5$ | $3\times10^5$ | 0.005 69 | 0.005 69 | 0.0271 | 0.0269 | — | — | — | — |
| $10^6$ | $10^5$ | 0.003 30 | 0.002 31 | 0.004 98 | 0.0208 | 0.001 20 | $7.09\times10^{-5}$ | 0.128 | 0.001 80 |
| $10^6$ | $3\times10^5$ | 0.006 48 | 0.005 17 | 0.0120 | 0.0229 | 0.001 74 | $9.17\times10^{-5}$ | 0.228 | 0.002 48 |
| $10^6$ | $10^6$ | 0.0138 | 0.0139 | 0.0627 | 0.0630 | — | — | — | — |
| $3\times10^6$ | $3\times10^5$ | 0.005 69 | 0.004 02 | 0.006 64 | 0.0287 | 0.006 33 | 0.000 241 | 0.456 | 0.003 14 |
| $3\times10^6$ | $10^6$ | 0.0181 | 0.0148 | 0.0223 | 0.0386 | 0.007 08 | 0.000 554 | 0.596 | 0.006 58 |
| $3\times10^6$ | $3\times10^6$ | 0.0744 | 0.0737 | 0.412 | 0.415 | — | — | — | — |
| $10^7$ | $10^6$ | 0.0301 | 0.0283 | 0.0256 | 0.177 | 0.0189 | 0.005 06 | 0.690 | 0.0231 |
| $10^7$ | $3\times10^6$ | 0.434 | 0.359 | 0.282 | 0.448 | 0.0182 | 0.0428 | 0.643 | 0.180 |
| $10^7$ | $10^7$ | 12.1 | 12.0 | 62.2 | 61.5 | — | — | — | — |

 

TABLE III. Median errors in intrinsic quantities for $10^4$ binaries of various masses at $z = 5$. The results for the highest masses are meaningless—the parameters are completely undetermined.

| $m_1(M_\odot)$ | $m_2(M_\odot)$ | $\Delta m_1/m_1$ | $\Delta m_2/m_2$ | $\Delta\chi_1$ | $\Delta\chi_2$ | $\Delta\mathcal{M}/\mathcal{M}$ (no precession) | $\Delta\mathcal{M}/\mathcal{M}$ (precession) | $\Delta\mu/\mu$ (no precession) | $\Delta\mu/\mu$ (precession) |
|---|---|---|---|---|---|---|---|---|---|
| $10^5$ | $10^5$ | 0.007 91 | 0.007 92 | 0.0392 | 0.0389 | — | — | — | — |
| $3\times10^5$ | $10^5$ | 0.008 11 | 0.006 58 | 0.0193 | 0.0359 | 0.001 03 | $8.00\times10^{-5}$ | 0.172 | 0.002 90 |
| $3\times10^5$ | $3\times10^5$ | 0.0134 | 0.0134 | 0.0615 | 0.0616 | — | — | — | — |
| $10^6$ | $10^5$ | 0.007 18 | 0.005 02 | 0.009 93 | 0.0409 | 0.003 26 | 0.000 184 | 0.305 | 0.003 91 |
| $10^6$ | $3\times10^5$ | 0.0156 | 0.0124 | 0.0249 | 0.0460 | 0.004 27 | 0.000 289 | 0.469 | 0.005 96 |
| $10^6$ | $10^6$ | 0.0424 | 0.0423 | 0.197 | 0.200 | — | — | — | — |
| $3\times10^6$ | $3\times10^5$ | 0.0161 | 0.0117 | 0.0158 | 0.0808 | 0.0115 | 0.00103 | 0.643 | 0.009 22 |
| $3\times10^6$ | $10^6$ | 0.0576 | 0.0475 | 0.0606 | 0.107 | 0.0108 | 0.002 65 | 0.635 | 0.0214 |
| $3\times10^6$ | $3\times10^6$ | 0.396 | 0.391 | 2.43 | 2.44 | — | — | — | — |
| $10^7$ | $10^6$ | 0.279 | 0.282 | 0.208 | 1.41 | 0.0374 | 0.0640 | 0.704 | 0.232 |
| $10^7$ | $3\times10^6$ | 10.1 | 8.41 | 6.10 | 7.61 | 0.106 | 1.11 | 0.769 | 4.28 |
| $10^7$ | $10^7$ | 2280 | 2290 | 10 300 | 9900 | — | — | — | — |

TABLE IV. Median errors in extrinsic quantities for $10^4$ binaries of various masses at $z = 1$, including comparisons with the "no precession" case. Note that the given major axis and minor axis are the medians for each data set and do not correspond to the same binary. However, they still represent an average sky position error ellipse in the following sense: $\sqrt{\pi ab}$, calculated using the median values of $2a$ and $2b$, differs in most cases by less than 10% from the median value of $\sqrt{\Delta\Omega_N}$ calculated from the covariance matrix and (4.13) (except at more extreme mass ratios—when $m_1/m_2 = 10$, the difference can be 25%).

| $m_1(M_\odot)$ | $m_2(M_\odot)$ | $2a$ (arcmin) (no precession) | $2a$ (arcmin) (precession) | $2b$ (arcmin) (no precession) | $2b$ (arcmin) (precession) | $\Delta D_L/D_L$ (no precession) | $\Delta D_L/D_L$ (precession) |
|---|---|---|---|---|---|---|---|
| $10^5$ | $10^5$ | 133 | 27.3 | 84.7 | 13.3 | 0.0193 | 0.003 98 |
| $3\times10^5$ | $10^5$ | 115 | 16.9 | 72.6 | 7.33 | 0.0165 | 0.002 40 |
| $3\times10^5$ | $3\times10^5$ | 101 | 23.3 | 62.8 | 11.8 | 0.0143 | 0.003 57 |
| $10^6$ | $10^5$ | 105 | 27.2 | 65.1 | 6.62 | 0.0149 | 0.003 20 |
| $10^6$ | $3\times10^5$ | 93.1 | 31.3 | 57.5 | 13.2 | 0.0132 | 0.003 93 |
| $10^6$ | $10^6$ | 90.1 | 40.2 | 54.1 | 21.9 | 0.0125 | 0.005 60 |
| $3\times10^6$ | $3\times10^5$ | 95.0 | 34.1 | 57.3 | 9.20 | 0.0135 | 0.003 76 |
| $3\times10^6$ | $10^6$ | 102 | 32.3 | 56.0 | 14.7 | 0.0135 | 0.004 19 |
| $3\times10^6$ | $3\times10^6$ | 135 | 43.3 | 68.5 | 22.3 | 0.0182 | 0.006 89 |
| $10^7$ | $10^6$ | 149 | 37.6 | 75.2 | 12.2 | 0.0200 | 0.004 57 |
| $10^7$ | $3\times10^6$ | 238 | 42.1 | 119 | 19.0 | 0.0322 | 0.006 10 |
| $10^7$ | $10^7$ | 466 | 81.3 | 232 | 38.6 | 0.0636 | 0.0250 |

TABLE V. Median errors in extrinsic quantities for $10^4$ binaries of various masses at $z = 3$.

| $m_1(M_\odot)$ | $m_2(M_\odot)$ | $2a$ (arcmin) (no precession) | $2a$ (arcmin) (precession) | $2b$ (arcmin) (no precession) | $2b$ (arcmin) (precession) | $\Delta D_L/D_L$ (no precession) | $\Delta D_L/D_L$ (precession) |
|---|---|---|---|---|---|---|---|
| $10^5$ | $10^5$ | 432 | 81.0 | 271 | 40.8 | 0.0617 | 0.0123 |
| $3\times10^5$ | $10^5$ | 389 | 92.5 | 242 | 39.5 | 0.0551 | 0.0126 |
| $3\times10^5$ | $3\times10^5$ | 356 | 142 | 220 | 75.7 | 0.0502 | 0.0201 |
| $10^6$ | $10^5$ | 379 | 141 | 233 | 36.6 | 0.0550 | 0.0155 |
| $10^6$ | $3\times10^5$ | 359 | 129 | 215 | 56.7 | 0.0500 | 0.0161 |
| $10^6$ | $10^6$ | 416 | 158 | 224 | 84.3 | 0.0556 | 0.0237 |
| $3\times10^6$ | $3\times10^5$ | 425 | 132 | 233 | 40.3 | 0.0568 | 0.0153 |
| $3\times10^6$ | $10^6$ | 599 | 142 | 302 | 64.6 | 0.0809 | 0.0193 |
| $3\times10^6$ | $3\times10^6$ | 990 | 224 | 494 | 111 | 0.134 | 0.0422 |
| $10^7$ | $10^6$ | 1320 | 206 | 648 | 78.5 | 0.178 | 0.0293 |
| $10^7$ | $3\times10^6$ | 2380 | 297 | 1180 | 152 | 0.326 | 0.0805 |
| $10^7$ | $10^7$ | 6820 | 2000 | 3390 | 583 | 0.935 | 2.41 |

TABLE VI. Median errors in extrinsic quantities for $10^4$ binaries of various masses at $z = 5$. Again, the results for the highest masses are essentially meaningless—the parameters are completely undetermined.

| $m_1(M_\odot)$ | $m_2(M_\odot)$ | $2a$ (arcmin) (no precession) | $2a$ (arcmin) (precession) | $2b$ (arcmin) (no precession) | $2b$ (arcmin) (precession) | $\Delta D_L/D_L$ (no precession) | $\Delta D_L/D_L$ (precession) |
|---|---|---|---|---|---|---|---|
| $10^5$ | $10^5$ | 729 | 169 | 456 | 85.7 | 0.104 | 0.0260 |
| $3\times10^5$ | $10^5$ | 676 | 217 | 419 | 95.8 | 0.0957 | 0.0284 |
| $3\times10^5$ | $3\times10^5$ | 650 | 295 | 395 | 161 | 0.0917 | 0.0409 |
| $10^6$ | $10^5$ | 686 | 248 | 416 | 66.8 | 0.0983 | 0.0273 |
| $10^6$ | $3\times10^5$ | 716 | 233 | 404 | 101 | 0.0961 | 0.0294 |
| $10^6$ | $10^6$ | 976 | 315 | 497 | 162 | 0.132 | 0.0501 |
| $3\times10^6$ | $3\times10^5$ | 986 | 265 | 507 | 86.4 | 0.133 | 0.0318 |
| $3\times10^6$ | $10^6$ | 1620 | 304 | 810 | 139 | 0.220 | 0.0436 |
| $3\times10^6$ | $3\times10^6$ | 2930 | 538 | 1460 | 260 | 0.400 | 0.140 |
| $10^7$ | $10^6$ | 5080 | 577 | 2480 | 290 | 0.689 | 0.124 |
| $10^7$ | $3\times10^6$ | 10 500 | 1720 | 5130 | 621 | 1.42 | 1.24 |
| $10^7$ | $10^7$ | 75 500 | 180 000 | 35 000 | 29 600 | 10.3 | 377 |

Comparison of these tables to the published versions shows the impact of the coding error on our results. Considering the "precession" results first, we find that the corrected errors in $m_1$ and $m_2$ are typically larger by about 10%–20% than previously. The spin errors follow the same pattern, although in some cases the differences reach ~40%. The errors to the chirp mass $\mathcal{M}$ are typically worsened by only a few percent (and actually improve a bit for larger masses). The reduced mass $\mu$ behaves similarly to $m_1$ and $m_2$, showing an accuracy degradation of 10%–20%.

Turning next to the "no precession" results, we find that the errors in chirp and reduced mass are typically slightly *better* than our previous results by a few percent. Taken together with the fact that correcting this bug often degrades our precession results a bit, we see that the *improvement* in parameter estimation is not always quite as good as we had originally claimed. It is still, however, a significant improvement. For example, precession physics improves the accuracy with which the reduced mass can be determined by one to several orders of magnitude.

Because of the nature of the coding error, errors in the extrinsic parameters were more strongly affected than errors in the intrinsic parameters. With the precession code corrected, the major axis of the sky position error ellipse $2a$ and errors in the luminosity distance $D_L$ increase by almost a factor of 2 in the worst cases. The "no precession" results change by similar factors. So, on average, the improvement due to precession is essentially unaffected by this bug for both $2a$ and $D_L$.

The story is somewhat different for the minor axis of the sky position error ellipse, $2b$. Its accuracy is also degraded, but only by a factor ~1.5 or so (in the worst cases). In *all* cases, we find that this bug has less impact on $2b$ than it does on $2a$. Our corrected precession code thus indicates that the ratio $2a/2b$ is more extreme, approaching 4 in large mass ratio

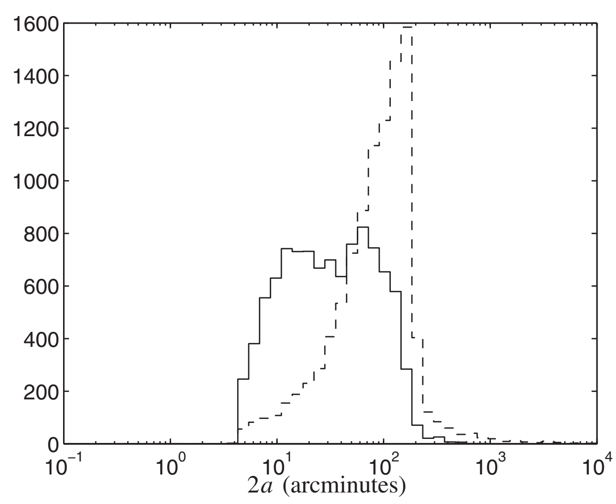


FIG. 7. Distribution of the major axis of the sky position error ellipse, $2a$, for $10^4$ binaries with $m_1 = 10^6 M_\odot$ and $m_2 = 3\times10^5 M_\odot$ at $z = 1$. The dashed line is the precession-free calculation; the solid line includes precession. Sky position, as an extrinsic parameter, is improved somewhat indirectly by precession; therefore, the improvement is less than for the masses.

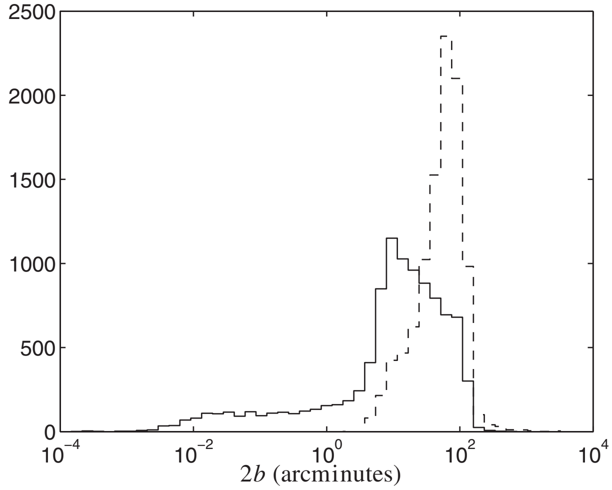


FIG. 8. Distribution of the minor axis of the sky position error ellipse, $2b$, for $10^4$ binaries with $m_1 = 10^6 M_\odot$ and $m_2 = 3 \times 10^5 M_\odot$ at $z = 1$. The dashed line is the precession-free calculation; the solid line includes precession.

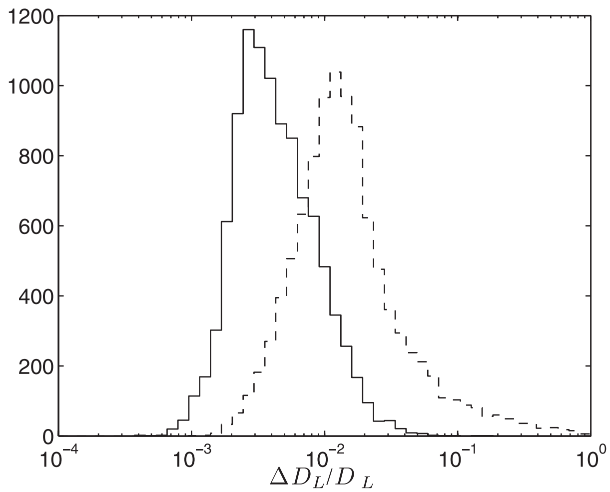


FIG. 9. Distribution of errors in the luminosity distance for $10^4$ binaries with $m_1 = 10^6 M_\odot$ and $m_2 = 3 \times 10^5 M_\odot$ at $z = 1$. The dashed line is the precession-free calculation; the solid line includes precession.

binaries with enhanced precession effects. Interestingly, we find that $2b$ for the ‘‘no precession’’ case is degraded by as much as a factor $\sim 3$. Correcting this bug therefore indicates that the minor axis of the localization ellipse is improved by precession even more than we previously reported.

The changes in the intrinsic parameter accuracy are so slight that the distributions shown in Figs. 2–6 are not noticeably affected. However, the distributions of the extrinsic parameters change significantly enough that we present new versions of Figs. 7–9 below. Despite the various changes in shape, the distribution of $2b$ still contains a long tail of small errors when precession effects are included.

An updated version of our manuscript, including these new results and with small changes in the summary text to account for them, has been posted to arxiv.org.