THE ASTROPHYSICAL JOURNAL, 607:L9–L12, 2004 May 20

CONSEQUENCES OF GRAVITATIONAL RADIATION RECOIL

DAVID MERRITT,[1] MILOŠ MILOSAVLJEVIĆ,[2] MARC FAVATA,[3] SCOTT A. HUGHES,[4] AND DANIEL E. HOLZ[5]
Received 2004 February 8; accepted 2004 April 5; published 2004 April 20

ABSTRACT

Coalescing binary black holes experience an impulsive kick from anisotropic emission of gravitational waves. Recoil velocities are sufficient to eject most coalescing black holes from dwarf galaxies and globular clusters, which may explain the apparent absence of massive black holes in these systems. Ejection from giant elliptical galaxies would be rare, but coalescing black holes are displaced from the center and fall back on a timescale of order the half-mass crossing time. Displacement of the black holes transfers energy to the stars in the nucleus and can convert a steep density cusp into a core. Radiation recoil calls into question models that grow supermassive black holes from hierarchical mergers of stellar-mass precursors.

Subject headings: black hole physics — galaxies: nuclei — gravitation — gravitational waves

1. KICK AMPLITUDE

In a companion paper (Favata, Hughes, & Holz 2004, hereafter Paper I), the amplitude of the recoil velocity resulting from anisotropic emission of gravitational waves during coalescence of a binary black hole (BH) is computed. Here we explore some of the consequences of the kicks. Unless otherwise indicated, notation is the same as in Paper I.

For in-spiral from a circular orbit, the kick velocity is a function of the binary mass ratio $q = m_1/m_2 \leq 1$, the BH spins $\tilde{a}_1$ and $\tilde{a}_2$, and the initial angle ι between the spin of the larger BH and the orbital angular momentum of the binary. Following Paper I, the spin of the smaller BH is ignored. Although Paper I only considers the cases $\iota = 0$ and $\iota = 180$, the recoil for arbitrary inclination is likely to be bounded between these extreme values. Also, the detailed inclination dependence is unimportant in comparison with the large uncertainty already present in the contribution to the recoil from the final plunge and coalescence. We will therefore assume that the restriction to equatorial-prograde/retrograde orbits ($\tilde{a}_2 = [-1, 1]$) considered in Paper I encompasses the characteristic range of recoil velocities.

Figure 2*b* of Paper I shows upper and lower limit estimates of the recoil velocity as a function of the effective spin parameter $\tilde{a}$ for a reduced mass ratio $\eta = \mu/M = q/(1+q)^2 = 0.1$. The *upper limit* for $\eta < 0.1$ is well fitted in the range $-0.9 \leq \tilde{a} \leq 0.8$ by the following fifth-order polynomial:

$$V_{\rm upper} = 465 \text{ km s}^{-1} \frac{f(q)}{f_{\rm max}} (1 - 0.281\tilde{a} - 0.0361\tilde{a}^2 - 0.346\tilde{a}^3 - 0.374\tilde{a}^4 - 0.184\tilde{a}^5). \quad (1)$$

Fitchett's (1983) scaling function $f(q)/f_{\rm max}$, with $f(q) = q^2(1-q)/(1+q)^5$, equals 0.433 for $\eta = 0.1$. The *lower limit* curve of Paper I is well fitted by

$$V_{\rm lower} = 54.4 \text{ km s}^{-1} \frac{f(q)}{f_{\rm max}} (1 + 1.22\tilde{a} + 1.04\tilde{a}^2 + 0.977\tilde{a}^3 - 0.201\tilde{a}^4 - 0.434\tilde{a}^5). \quad (2)$$

We convert these expressions into estimates of the bounds on $V_{\rm kick}$ as follows. First, as discussed in Paper I, there is an ambiguity in how one translates the physical spin parameter $\tilde{a}_2$ of the larger hole into the effective spin parameter $\tilde{a}$ of equations (1) and (2). Here we adopt the Damour (2001) relation $\tilde{a} = (1 + 3q/4)(1+q)^{-2}\tilde{a}_2$. Second, Fitchett's scaling function assumes that both bodies are nonspinning and vanishes when $q = 1$. In fact, when $\tilde{a} \neq 0$, significant recoil would occur even for $q = 1$ as a result of spin-orbit coupling. We can guess the approximate form of a new scaling function by examining the spin-orbit corrections (Kidder 1995) to Fitchett's recoil formula. For equatorial orbits, equation (4) of Paper I suggests that $f(q)$ should be multiplied by the factor $|1 + (7/29)\tilde{a}_2/(1-q)|/|1 + (7/29)\tilde{a}_2/(1-q')|$, where $q' = 0.127$ is the value used in defining $V_{\rm upper}$ and $V_{\rm lower}$ in equations (1) and (2).

Figure 1 plots upper and lower limits to $V_{\rm kick}$ as functions of $\tilde{a}_2$ and q. The average over $\tilde{a}_2$ of the upper limit estimates are ~(138, 444, 154) km s^{-1} for $q = (0.1, 0.4, 0.8)$; Figure 1 suggests a weak dependence on $\tilde{a}_2$. Lower limit estimates are more strongly spin-dependent; the averages over $\tilde{a}_2$ are ~(21.1, 63.6, 24.9) km s^{-1} for the same values of q. For moderately large spins ($\tilde{a}_2 \gtrsim 0.8$) and prograde capture, the lower limit estimates exceed 100 km s^{-1} for $0.2 \lesssim q \lesssim 0.6$.

2. ESCAPE

When $V_{\rm kick} \geq V_{\rm esc} \equiv [2\phi(\boldsymbol{r} = 0)]^{1/2}$, with $\phi(\boldsymbol{r})$ the gravitational potential of the system (galaxy, dark matter halo) hosting the BH, the BH has enough kinetic energy to escape. Figure 2 shows central escape velocities in four types of stellar system that could contain merging BHs: giant elliptical (E), dwarf elliptical (dE), and dwarf spheroidal (dSph) galaxies and globular clusters (GCs). We fit the trend $\log(V_{\rm esc}/1 \text{ km s}^{-1}) = \lambda - \beta M_V$ separately for each class of object. Dwarf elliptical galaxies and GCs each separately establish a relation $L \sim V_{\rm esc}^2$; for GCs, this is compatible with the relation found by Djorgovski et al.

[1] Department of Physics, Rochester Institute of Technology, 84 Lomb Memorial Drive, Rochester, NY 14623; drmsps@ad.rit.edu.
[2] Theoretical Astrophysics, California Institute of Technology, Pasadena, CA 91125; milos@tapir.caltech.edu.
[3] Department of Astronomy, Cornell University, 610 Space Sciences Building, Ithaca, NY 14853-6801; favata@hewitt.astro.cornell.edu.
[4] Department of Physics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139; sahughes@mit.edu.
[5] Center for Cosmological Physics, University of Chicago, Chicago, IL 60637; deholz@cfcp.uchicago.edu.

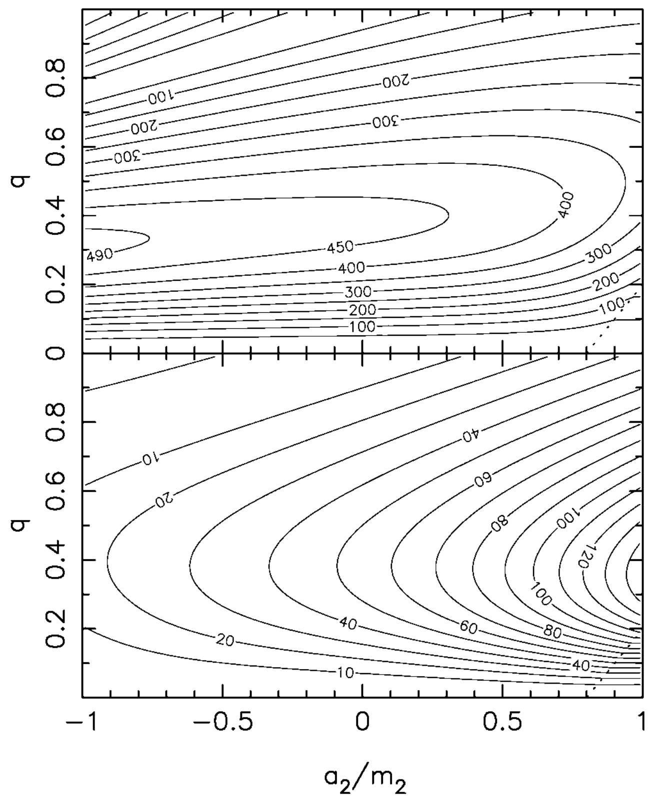

FIG. 1.—Upper limit (*top*) and lower limit (*bottom*) estimates of $V_{\rm kick}$ as functions of mass ratio q and spin of the larger black hole $\tilde{a}_2$. Units are in kilometers per second. Values of $\tilde{a}_2$ and q corresponding to $\tilde{a} > 0.8$ lie in the region to the right of the dotted line. Since eqs. (1) and (2) are not valid for $\tilde{a} > 0.8$, $\tilde{a}$ was replaced by 0.8 in this region.

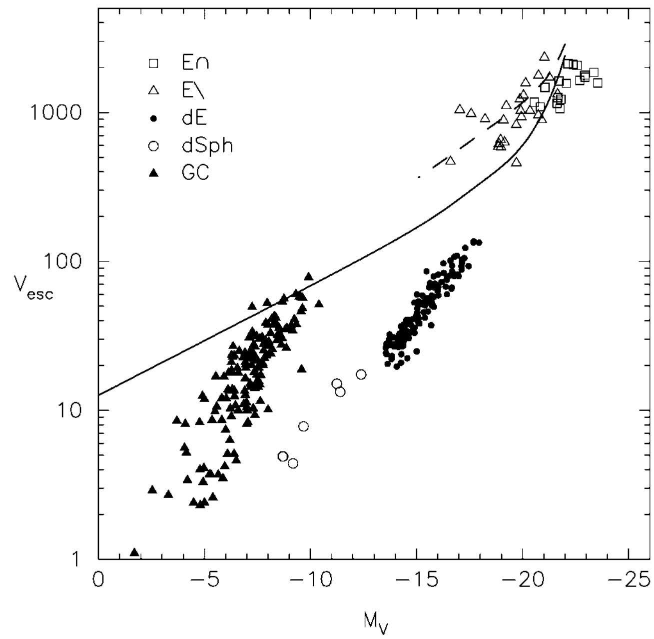

FIG. 2.—Central escape velocities in units of kilometers per second in four types of stellar system that could harbor merging BHs. E galaxy data are from Faber et al. (1997), with separate symbols for core (*open squares*) and power-law (*open triangles*) galaxies; dE data are from Binggeli & Jerjen (1998), with mass-to-light ratios from Mateo (1998). GC and dSph data are from the tabulation of Webbink (1996). The solid line is the mean escape velocity from the DM halos associated with the luminous matter. The dashed line is the escape velocity from the combined luminous+mean DM potentials for E galaxies.

(1997). The E sample is consistent with the Faber-Jackson (1976) relation.

The solid line in Figure 2 shows escape velocities from the dark matter (DM) halos associated with the luminous stellar systems. To relate halo properties to galaxy luminosities, we use the conditional luminosity function $\Phi(L|M)dL$ from the concordance Λ cold dark matter (ΛCDM) model M1 of Yang, Mo, & van den Bosch (2003). The average luminosity L_1 of the brightest ("central") galaxy in the halo of mass $M_{\rm vir}$ is implicitly given by the condition $\int_{L_1}^{\infty} \Phi(L|M_{\rm vir})dL = 1$. Inverting this, we obtain $M_{\rm vir}(L_1)$ and relate this mass to the escape velocity via $V_{\rm esc}^2 = 2cg(c)GM_{\rm vir}/R_{\rm vir}$, where $R_{\rm vir}$ is the virial radius of the halo, c is the concentration of a halo obeying the Navarro, Frenk, & White (1996) profile, and $g(c) = [\ln(1 + c) - c/(1 + c)]^{-1}$ (e.g., Łokas & Mamon 2001). At $z = 0$, the average escape velocity is given by $V_{\rm esc} = 239$ km s$^{-1}(m_{11}/h)^{1/2}$, where $M_{\rm vir} = (10^{11}m_{11})\ M_\odot$ and h is the Hubble parameter, set to 0.7 in Figure 2.

Figure 2 suggests that the consequences of the kicks are strikingly different for the different classes of stellar system that might host BHs. Escape velocities from E galaxies are dominated by the stellar contribution to the potential; in the sample of Faber et al. (1997), $V_{\rm esc} \gtrsim 450$ km s^{-1} even without accounting for DM. This exceeds even the upper limits in Figure 1. Hence, the kicks should almost never unbind BHs from E galaxies. The tight correlations observed between the BH mass and bulge luminosity (McLure & Dunlop 2002; Erwin, Graham, & Caon 2004) and the velocity dispersion (Ferrarese & Merritt 2000; Gebhardt et al. 2000) could probably not be maintained if escape occurred with any significant frequency from luminous galaxies. The upturn in escape velocity for galaxies brighter an $M_V \sim -20$ is a consequence of the increase in the occupation number of their host halos. The dashed line in Figure 2 shows the escape velocity from the combined luminous+DM potential for the E galaxies, using the scaling relation derived above to describe the luminous component.

The existence of DM significantly affects the escape probability from dE and dSph galaxies, implying kicks of ~300 and ~100 km s^{-1}, respectively, for escape. In the absence of DM, these numbers would be ~100 and ~20 km s^{-1}, respectively. Hence, kicks of order 200 km s^{-1} would unbind BHs from dSph galaxies whether or not they contain DM, while dE galaxies could retain their BHs if they are surrounded by DM halos.

Evidence of intermediate-mass BHs at the centers of galaxies fainter than $M_V \approx -19$ is sketchy (e.g., van der Marel 2004), although there is indirect (nondynamical) evidence of BHs in faint Seyfert bulges (Filippenko & Ho 2003). We note that the dense nuclei associated with BHs in galaxies like M32 ($M_V \approx -19$) become progressively less frequent at magnitudes fainter than $M_V \approx -16$ and disappear entirely below $M_V \approx -12$ (van den Bergh 1986). If the dense nuclei are associated with nuclear BHs (e.g., Peebles 1972), their absence could signal loss of the BHs via ejection. It is intriguing that these nuclei are sometimes observed to be displaced far from the galaxy center (Binggeli, Barazza, & Jerjen 2000). Figures 1 and 2 imply that even kicks at the lower limits of Paper I would almost always unbind BHs from GCs.

3. EJECTION IN HIERARCHICAL MERGING SCENARIOS

The kicks have serious implications for models in which massive BHs grow from mergers of less massive seeds. In some of these models, the precursors are stellar- or intermediate-mass

BHs produced in the collapse of the first stars (Population III), and the merging commenced in minihalos at redshifts as large as ∼20 (Madau & Rees 2001; Volonteri, Haardt, & Madau 2003; Islam, Taylor, & Silk 2003). We evaluate the plausibility of such models in light of the estimates of $V_{\rm kick}$ derived in Paper I. Kicks from gravitational wave emission may compete with high-velocity recoils from (Newtonian) three-body interactions. While the Newtonian recoil occurs only when three BHs are present, which is contingent on the galaxy merger rate and the BH binary orbital decay rate, radiation recoil is present whenever BHs coalesce.

The confining effect of DM halos in a hierarchical universe was smaller at higher redshifts when the average halo mass was smaller. We estimate the maximum redshift at which DM halos can confine the progenitors of the present-day BHs. Ferrarese (2002) derived a relation of the present-day BH mass $M_{\rm BH}(z=0)$ to the mass of the host halo $M_{\rm vir}(z=0)$. We extrapolate the host mass back in redshift via the accretion history model (Bullock et al. 2001) calibrated by Wechsler et al. (2002) on a set of numerical simulations of DM clustering in a ΛCDM universe. The accretion trajectory $M_{\rm vir}(z) \propto e^{-\alpha z}$, where α is itself a function of the halo mass at $z=0$, can be interpreted as the mass of the most massive and thus the most easily confining parent halo at redshift z. We can then calculate the escape velocity $V_{\rm esc}(z)$ of the most massive progenitor halo as a function of redshift. Finally, we solve for $z_{\rm eject}$ such that $V_{\rm kick} = V_{\rm esc}(z_{\rm eject})$; this is the maximum redshift at which the progenitors of the present-day BHs could have started merging. We also modeled the effect on $z_{\rm eject}$ of including the potential due to a stellar component, idealized as an isothermal sphere with core radius $r_h = 2GM_{\rm BH}/\sigma^2$ and outer cutoff $10^3 r_h$.

The results for five representative choices of $V_{\rm kick}$ are shown in Figure 3. For $V_{\rm kick} \sim 150$ km s^{-1}, we find $z_{\rm eject} < 11(14)$ over the entire range of $M_{\rm BH}$; the latter value is from the models that include a stellar component. For $V_{\rm kick} \sim 300$ km s^{-1}, the assembly of a $10^8\ M_\odot$ BH must have started at $z \lesssim 8(10)$. Models that grow supermassive BHs from mergers of seeds of much lower mass at redshifts $z \gtrsim 10$ are thus disfavored because of the difficulty of retaining the kicked BHs.

4. FALLBACK TIMES

A BH that has been kicked from the center of a stellar system with a velocity less than $V_{\rm esc}$ falls back, and its orbit decays via dynamical friction against the stars and gas. We define the fallback time $T_{\rm infall}$ as the time required for a BH to return to a zero-velocity state after being ejected. The velocity with which the BH is ejected from the site of the merger is $V_{\rm eject} = (M_{\rm BH}/M_{\rm eff})V_{\rm kick} < V_{\rm kick}$; here $M_{\rm eff} = M_{\rm BH} + M_{\rm bound}$, with $M_{\rm bound}$ the mass in stars that remain bound to the BH after it is kicked. For recoil in a singular isothermal sphere nucleus $\rho \propto r^{-2}$, $M_{\rm eff}/M_{\rm BH} \approx (1.9, 1.5, 1.05, 1.00)$ when $V_{\rm kick}/\sigma = (0.5, 1, 2, 3)$, where σ is the one-dimensional stellar velocity dispersion; $M_{\rm bound}/M_{\rm BH} \propto (V_{\rm kick}/\sigma)^{-4}$ for $V_{\rm kick} \gg \sigma$.

We evaluated $T_{\rm infall}$ for BHs kicked from the centers of Dehnen (1993) law galaxies for which the central density obeys $\rho \propto r^{-\gamma}$. Bright E galaxies have $0 \lesssim \gamma \lesssim 1$ (Gebhardt et al. 1996), and cusps steeper than this are likely to be softened by the binary BH prior to coalescence (Milosavljević & Merritt 2001) and by the ejection itself (§ 5). Given values for $M_{\rm eff}$ and $V_{\rm eject}$, the fallback time in a spherical galaxy is given by the orbit-averaged dynamical friction equation. For $V_{\rm eject}/V_{\rm esc} \lesssim 0.6$, infall times were found to be well approximated by $T_{\rm infall} \approx T_{1/2}(V_{\rm eject}/V_{\rm esc})^{2.5(1+\gamma)}$ for $M_{\rm eff} = 0.001M_{\rm gal}$, where the period $T_{1/2}$ of a circular orbit at the galaxy's half-light radius is given in terms of the galaxy's visual luminosity by $T_{1/2} \approx 2 \times 10^8$ yr $(L_V/10^{11}\ L_\odot)^{1/2}$ (Valluri & Merritt 1998). Thus, return of a BH to a stationary state requires of order a few times 10^8 yr or less over a wide range of cusp slopes and galaxy luminosities for $V_{\rm eject} \lesssim V_{\rm esc}/2$. As indicated in Figure 2, this is the likely situation in the bright E galaxies. Infall times are especially short for $\gamma \geq 1$ since the BH experiences a strong impulsive frictional force as it passes repeatedly through the dense center. When $V_{\rm eject} \lesssim \sigma$, the BH never moves far from its central position, and it carries much of the nucleus with it. We carried out N-body simulations of this regime and found that return to zero velocity occurs in roughly one orbital period when $V_{\rm eject} \lesssim \sigma$. In fainter dE and dSph galaxies, ejection would more often occur near $V_{\rm esc}$, and infall times could be arbitrarily long, determined primarily by the mass distribution at large radii.

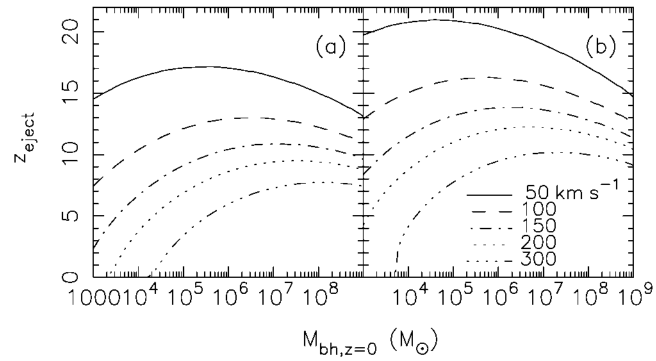

FIG. 3.—Maximum redshift $z_{\rm eject}$ at which (*a*) DM halos only, and (*b*) DM halos and the central galaxies combined, can confine BHs as a function of the $z=0$ BH mass, for five values of the kick velocity. The depth of the galactic contribution to the potential was calculated by identifying the velocity dispersion of the stellar spheroid with the circular velocity of the halo (Ferrarese 2002).

In a nonspherical galaxy, an ejected BH does not pass precisely through the dense center on each return, delaying the infall. To test the effect of nonspherical geometries on the infall time, we carried out experiments in the triaxial generalizations of the Dehnen models (Merritt & Fridman 1996). Results were found to depend only weakly on the axis ratios of the models. Decay times in the triaxial geometry exhibit a spread in values depending on the initial launch angle, bounded from below by the decay time along the short axis. We found a mean at every $V_{\rm eject}/V_{\rm esc}$ that is ∼3–5 times greater than in a spherical galaxy with the same cusp slope.

5. OBSERVABLE CONSEQUENCES OF THE DISPLACEMENT

Displacement of the BH also transfers energy to the nucleus and lowers its density within a region of size $\sim r_h$, the radius of the BH's sphere of influence (defined here as the radius of a sphere containing a mass in stars equal to twice that of the BH). The simplest case to consider is $V_{\rm eject} \gtrsim V_{\rm esc}$; the BH and its entrained mass depart the nucleus on a timescale that is of order the crossing time at r_h or less and do not return. The effect on the nucleus can be approximated by constructing a steady state model of a galaxy containing a central point mass, then removing the point mass instantaneously and allowing the remaining particles to relax to a new steady state. Figure 4*a* shows the results for three values of $M_{\rm eff}/M_{\rm gal}$. Initial conditions consisted of 10^6 particles representing stars in a $\gamma = 1$ Dehnen model. We find that a core of roughly constant density forms within a radius of $\sim 2r_h$. Setting $\gamma = 2$ (not shown) results in

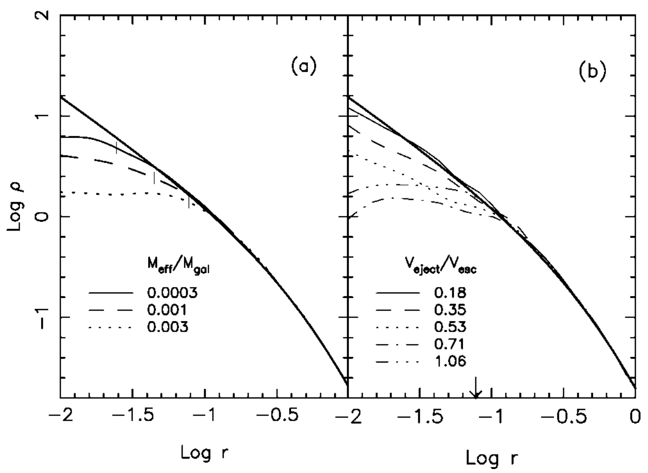

FIG. 4.—Effect on the nuclear density profile of BH ejection. The initial galaxy model (*thick solid line*) has a $\rho \sim r^{-1}$ density cusp. (*a*) Impulsive removal of the BH. Tick marks show the radius of the BH's sphere of influence r_h before ejection. A core forms with radius $\sim 2r_h$. (*b*) Ejection at velocities less than V_{esc}. The BH has mass $0.003M_{gal}$; the galaxy is initially spherical, and the BH's orbit remains nearly radial as it decays via dynamical friction. The arrow marks r_h.

a core of size $\sim r_h$. Figure 4*b* shows the change in the nuclear density profile for simulations with $V_{eject} < V_{esc}$. Significant changes in the central density require $V_{eject} \gtrsim 0.25V_{esc}$. We conclude that the recoil could affect the observable structure of nuclei since radii of $\sim 2r_h$ are resolved in many nearby galaxies (Merritt & Ferrarese 2001).

The "mass deficits" seen at the centers of bright galaxies (Milosavljević et al. 2002) may be due to the combined effects of slingshot ejection and BH displacement, although we note that the large cores observed in some bright galaxies could probably not be produced by either mechanism (Milosavljević et al. 2002).

The X-shaped radio sources associated with giant E galaxies are plausible sites of recent BH coalescence (Merritt & Ekers 2002). Displacement of the merged BHs from the galaxy center prior to ignition of the "active" lobes would imply a distortion of the X-morphology, in the sense that the "wings" (the inactive lobes) would be noncollinear near the center of the X. Such distortions are in fact a common feature of the X-sources (Gopal-Krishna, Biermann, & Wiita 2003), although the linear scale of the distortions in some of the X-sources (e.g., ~10 kpc in NGC 236; Murgia et al. 2001) suggests that orbital motion of the merging galaxies may be a more likely explanation.

We thank J. Bullock, A. Cooray, A. Klypin, M. Santos, G. Tormen, and F. van den Bosch for helpful comments. D. M. was supported by grants NSF AST 02-0631 and NASA NAG5-9046. Funding information for M. F., S. A. H., and D. E. H. is given in Paper I.

REFERENCES

Binggeli, B., Barazza, F., & Jerjen, H. 2000, A&A, 359, 447
Binggeli, B., & Jerjen, H. 1998, A&A, 333, 17
Bullock, J. S., Kolatt, T. S., Sigad, U., Somerville, R. S., Kravtsov, A. V., Klypin, A. A., Primack, J. R., & Dekel, A. 2001, MNRAS, 321, 559
Damour, T. 2001, Phys. Rev. D, 64, 124013
Dehnen, W. 1993, MNRAS, 265, 250
Djorgovski, S. G., Gal, R. R., McCarthy, J. K., Cohen, J. G., de Carvalho, R. R., Meylan, G., Bendinelli, O., & Parmeggiani, G. 1997, ApJ, 474, L19
Erwin, P., Graham, A. W., & Caon, N. 2004, in Carnegie Obs. Astrophys. Ser. 1, Coevolution of Black Holes and Galaxies, ed. L. C. Ho (Cambridge: Cambridge Univ. Press), 12
Faber, S. M., et al. 1997, AJ, 114, 1771
Faber, S. M., & Jackson, R. E. 1976, ApJ, 204, 668
Favata, M., Hughes, S. A., & Holz, D. E. 2004, ApJ, 607, L5 (Paper I)
Ferrarese, L. 2002, ApJ, 578, 90
Ferrarese, L., & Merritt, D. 2000, ApJ, 539, L9
Filippenko, A. V., & Ho, L. C. 2003, ApJ, 588, L13
Fitchett, M. J. 1983, MNRAS, 203, 1049
Gebhardt, K., et al. 1996, AJ, 112, 105
———. 2000, ApJ, 539, L13
Gopal-Krishna, Biermann, P. L., & Wiita, P. J. 2003, ApJ, 594, L103
Islam, R. R., Taylor, J. E., & Silk, J. 2003, MNRAS, 340, 647
Kidder, L. E. 1995, Phys. Rev. D, 52, 821
Łokas, E. L., & Mamon, G. A. 2001, MNRAS, 321, 155
Madau, P., & Rees, M. J. 2001, ApJ, 551, L27
Mateo, M. 1998, ARA&A, 36, 435
McLure, R. J., & Dunlop, J. S. 2002, MNRAS, 331, 795
Merritt, D., & Ekers, R.D. 2002, Science, 297, 1310
Merritt, D., & Ferrarese, L. 2001, in ASP Conf. Ser. Vol. 249, The Central Kiloparsec of Starbursts and AGN: The La Palma Connection, ed. J. H. Knapen, J. E. Beckman, I. Schlosman, & T. J. Mahoney (San Francisco: ASP), 335
Merritt, D., & Fridman, T. 1996, ApJ, 460, 136
Milosavljević, M., & Merritt, D. 2001, ApJ, 563, 34
Milosavljević, M., Merritt, D., Rest, A., & van den Bosch, F. C. 2002, MNRAS, 331, L51
Murgia, M., Parma, P., de Ruiter, H. R., Bondi, M., Ekers, R. D., Fanti, R., & Fomalont, E. B. 2001, A&A, 380, 102
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1996, ApJ, 462, 563
Peebles, P. J. E. 1972, ApJ, 178, 371
Valluri, M., & Merritt, D. 1998, ApJ, 506, 686
van den Bergh, S. 1986, AJ, 91, 271
van der Marel, R. P. 2004, in Carnegie Obs. Astrophys. Ser. 1, Coevolution of Black Holes and Galaxies, ed. L. C. Ho (Cambridge: Cambridge Univ. Press), 37
Volonteri, M., Haardt, F., & Madau, P. 2003, ApJ, 582, 559
Webbink, R. F. 1996, VizieR On-line Data Catalog, 7151, 0
Wechsler, R. H., Bullock, J. S., Primack, J. R., Kravtsov, A. V., & Dekel, A. 2002, ApJ, 568, 52
Yang, X., Mo, H. J., & van den Bosch, F. C. 2003, MNRAS, 339, 1057