Bosonic broadcast channel capacity and a new minimum output entropy conjecture

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The lossy bosonic channel provides a quantum model for optical communication systems that rely on fiber or free-space propagation. For the pure-loss case, the classical-information capacity of this channel has been shown to be achievable with single-use coherent-state encoding with a Gaussian prior [1]. For the more general thermal-noise channel, the Holevo information of single-use coherent-state encoding with a Gaussian prior is a lower bound on the channel capacity [2], and would equal that capacity if a recent conjecture about that channel's minimum output entropy were true [3]. Less is known about the classical-information capacity of multi-user bosonic channels. For multiple-access bosonic communications—in which two or more senders communicate to a common receiver over a shared propagation medium—single-use coherent-state encoding with a Gaussian prior and optimum measurement achieves the sum-rate capacity but does not realize the single-user capacity [4]. Moreover, the capacity region that is lost when coherent detection is employed instead of the optimum measurement has been quantified for this multiple-access channel. In this paper, we consider the classical-information capacity of the degraded bosonic broadcast channel, in which a single sender communicates to two or more receivers through a lossless 1:M coupler whose auxiliary inputs are in their vacuum states.

When coherent-state encoding is employed in conjunction with coherent detection, our bosonic broadcast channel is equivalent to a classical degraded Gaussian broadcast channel whose capacity region is known, and known to be dual to that of the classical Gaussian multiple-access channel [5]. Thus, under these coding and detection assumptions, the capacity region for the bosonic broadcast channel is dual to that for the multiple-access bosonic channel with coherent-state encoding and coherent detection. To treat more general transmitter and receiver conditions, we use a limiting argument to apply the degraded quantum broadcast-channel coding theorem for finite-dimensional state spaces [6] to the infinite-dimensional bosonic channel with an average photon-number constraint. We consider the two-receiver single-use case in which Alice (A) simultaneously transmits to Bob (B), via the transmissivity $\eta > 1/2$ port of a lossless beam splitter, and to Charlie (C), via that beam splitter's reflectivity $1-\eta < 1/2$ port, using coherent-state encoding with a Gaussian prior at average photon number $\bar{n}$. The capacity region with optimum measurement is then

$$R_B \leq g(\eta\beta\bar{n}), R_C \leq g((1-\eta)\bar{n}) - g((1-\eta)\beta\bar{n}), \quad (1)$$

where $g(x) \equiv (x+1)\log(x+1) - x\log(x)$ and $\beta \in [0,1]$. Interestingly, this capacity region is *not* dual to that of the bosonic multiple-access channel with coherent-state encoding and optimum measurement that was found in [4]. Equation (1) is an inner bound on the bosonic broadcast channel's capacity region when arbitrary encoding and optimum measurement are employed. We have shown that Eq. (1) will be that capacity region if the following new minimum output entropy conjecture is correct.

Conjecture: Let a lossless beam splitter have input $\hat{a}$ in its vacuum state, input $\hat{b}$ in a zero-mean state with von Neumann entropy $g(K)$, and output $\hat{c}$ from its transmissivity-η port. Then the von Neumann entropy of output $\hat{c}$ is minimized when input $\hat{b}$ is in a thermal state with average photon number K.

We will present evidence supporting this conjecture.

References

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