6.084 | Problem Set 3 *Due Wed 2006-09-27*

Warmups

This section contains problems for you to check your understanding. They won't be collected or graded. If you have trouble with any of them, be sure to sort it out and ask any questions in class, by email, or at office hours.

1. In terms of the basic dimensions L, M, and T, find the dimensions of force, power, momentum, angular momentum, and energy flux.

2. Estimate the mass of the earth.

3. If you drop an object from height h and it free falls, which expression is dimensionally correct for the speed when it hits the ground: $v \sim \sqrt{g/h}$, $v \sim \sqrt{h/g}$, or $v \sim \sqrt{gh}$?

Problems

Turn in your solutions for these problems.

1. The diffusion equation in one dimension is

$$\frac{\partial n}{\partial t} = \kappa \frac{\partial^2 n}{\partial x^2}$$

where n is the concentration of a substance. Find the dimensions of the constant of proportionality κ. What other quantity has those dimensions?

2. Use dimensional analysis to find the volume of an ellipsoid with half-axes a, b, and c.

3. Estimate the energy in a 9-volt battery and the (dimensionless!) ratio

$$\frac{\text{cost of energy from a 9V battery}}{\text{cost of energy from the houeshold electricity}}.$$

4. Find the dimensions of dynamic viscosity η using the following definition. (The viscosity we used in class is the *kinematic* viscosity ν.) The force on a surface produced by viscosity is proportional to the velocity gradient near the surface (velocity gradient is the spatial derivative of velocity) and to the area of the surface, with the viscosity being the constant of proportionality:

$$\text{viscous force} = \eta \times \text{velocity gradient} \times \text{area}.$$

This viscosity is related to the kinematic viscosity $\eta = \rho\nu$. Use that relation to check your dimensions for η.

5. A raindrop's size r depends on surface tension σ, on its density ρ, and on gravity g. (Surface tension is energy per area.) Use dimensionless groups to find a symbolic estimate for r. Use $\sigma \sim 0.1\,\mathrm{J\,m^{-2}}$ for water to find a numerical estimate. How reasonable is your value?

6. Kepler's third law relates the period and radius of a planet's orbit. In this problem you use dimensional analysis to find the relation when the orbit is circular. Let the orbit's radius be r. What other quantities does the period T depend on? [Hint: The mass of the planet is not relevant. Why not?] How many dimensionless groups do you expect? Find the independent group(s) and therefore the relation between T and r.

7. Think of (but don't solve!) any approximation problem, perhaps from your research or from something you've noticed and wondered about in the world around you. It need not have anything to do with the topic du jour (dimensions). Let your imagination roam. Particularly interesting ones will be solved in class or become raw material for other homework problems.

For fun

This section contains harder problems, also not graded, for fun if you have time and interest. (If you want my comments, turn in your solutions or ask me via email or in office hours.)

1. Find a physical explanation of your result for the raindrop size.

2. In the Kepler problem, show that the relation is true even for elliptical orbits, if you replace r by the semimajor axis a. [Hint: Kepler's 2nd law is helpful.]