

## 6.084 | Problem Set 5

*Due Wed 2006-11-01*

1. Estimate the Reynolds number for a fly going about its business (flying around).
2. Some practice with log–log plots and extreme cases:
  - a. [Warmup] Sketch  $2x$ ,  $8/x$ , and  $x^2$  on log–log axes.
  - b. Sketch  $\ln \cosh x$  on regular and then on log–log axes. [Note:  $\cosh x \equiv (e^x + e^{-x})/2$ .]
3. An inchworm climbs up a 100–cm wood pole by day and night. He begins one daytime, and every daytime he climbs 10 cm. Every night he sleeps and loses his grip, slipping 9 cm. When does he reach the platform (stability at last!) at the top?
4. Estimate the terminal velocity  $v$  of a raindrop with radius  $r$  and density  $\rho_{\text{water}}$ , as follows.
  - a. First, neglect dimensionless constants such as  $\pi$ , 4,  $c_d$ ,  $1/2$ , etc. and get a symbolic expression for the terminal velocity. What assumption about the drag force are you making? Then put in reasonable numbers for  $r$  and whatever else you need in order to get a numerical estimate for  $v$ .
  - b. Check whether your assumption about how drag depends on  $v$  is consistent with your result. [Hint: There’s a useful dimensionless number. . .]
  - c. Then include the dimensionless constants to get a more accurate numerical estimate for  $v$ . Does the updated  $v$  make your assumption more consistent or less consistent? How reasonable is your estimate for  $v$ ?
5. The Fibonacci numbers  $f_n$  begin (starting with  $n = 0$ )

$$0, 1, 1, 2, 3, 5, \dots,$$

where each new term is the sum of the previous two:

$$f_n = f_{n-1} + f_{n-2}.$$

Check whether the following function (in Python) computes  $f_n$ . If it does, explain why. If it doesn’t, explain how to fix it.

```
def f(n):
    if n == 1:
        return 1
    else:
        return f(n-1) + f(n-2)
```

6. The motion of a pendulum depends on the mass  $m$  of the bob, on the length  $l$  of the (light) rod connecting the bob to the pivot, and on gravity  $g$ . Find the period  $T$  in terms of these variables by using universal functions:
- What are the dimensions of these variables ( $m$ ,  $g$ ,  $l$ , and  $T$ )? Explain why only one independent dimensionless group can be made from them.
  - Find a decent group and then an expression for the period  $T$ . Do you know, perhaps from other courses, the value of the constant?
  - You should have found that the mass  $m$  dropped out. Explain, physically, why  $m$  dropped out.
7. Continuing the previous problem, now add the initial amplitude  $\theta_0$  to the mix of variables.
- How many independent dimensionless groups are there? Find a reasonable group or set of groups.
  - Use your group or groups to write an expression for the period  $T$ .
  - As  $\theta_0$  increases, should the period  $T$  increase, decrease, or stay the same? *Sketch* your dimensionless group containing  $T$ , as a function of  $\theta_0$ . Label as much as you can on the graph, for example, axis scales, limiting values, asymptotes, slopes, . . . . You won't be able to figure out everything using the techniques taught so far (for example, perhaps not the shape when  $\theta_0 \approx 0$ ). Do as much as you can, and in one or two more units we'll revisit this problem.
8. In this problem you apply approximation techniques to integration.
- Use extreme cases to judge this formula:

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\pi/\alpha}.$$

- Use dimensions to judge these formulas [hint: Imagine that  $x$  has dimensions of length.]:

$$\int_0^{\infty} e^{-bx} dx = \frac{1}{b}.$$

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \frac{\sqrt{\pi}}{\alpha}.$$

9. [Harder, **optional**] An object rolls without slipping down an inclined plane. What is its linear acceleration down the plane? Here are the variables: moment of inertia  $I$ , radius  $r$ , mass  $m$ , strength of gravity  $g$ , inclined-plane angle  $\theta$ , and acceleration  $a$ .
- Analyze the problem using universal functions and extreme cases, guessing and conjecturing whatever you need to.