A linear dynamic model for microgrid voltages in presence of distributed generation

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ANALYSIS OF COMPLEX SYSTEMS
Large-scale complex systems

**Complex systems**

Systems comprising a large number of agents, spatially distributed, interacting one with the other.

Putting together classical models for each subsystem, usually leads to huge and useless global models, difficult to analyze and sometimes also to simulate.
Agents and interaction

Simple model are generally adopted for each agent.

The complexity of the entire system is given by the interconnection of a large number of simple systems.

Emerging behavior

Typical result in the analysis of such systems consist in understanding how the complex behavior emerges, possibly gaining some insight about the dependence of such behavior on some characteristic parameters.

The goal is not simulating the behavior of the single agent.
MICROGRID CONTROL
A microgrid is a portion of the power distribution network which is populated by a large number of microgenerators, interfaced via power inverters.

Preliminary works are exploring the possibility of controlling these devices in a synergistic way to provide ancillary services to the microgrid: voltage support, reactive power compensation,...

The control algorithms for ancillary services receive as

- **input** voltage phasor measurements from PMUs

and produce, as

- **output** complex power references for the inverters.

### Algorithms for ancillary services

#### Tertiary control
Market operation, generation-demand matching, load scheduling

- aggregated demands and power flows
- active power dispatching (generation and demand)

#### Secondary control
Ancillary services, voltage support, losses minimization, reactive power compensation

- voltage and current phasors $i(t), v(t)$
- complex power references $s(t) = p(t) + jq(t)$

#### Primary control
Frequency stability, safety procedures, inverter control

- instantaneous voltages $v(t)$
- instantaneous currents $i(t)$

**Power distribution network**
These algorithms are typically distributed (each inverter decide according to local measurement and data from neighbors) and iterative (alternated execution of measurement and actuation).

It is necessary to estimate the propagation time of the control action across the microgrid: a dynamical model for the measured signals (voltages) as a function of the complex power references sent to the microgenerators.
MICROGRID DYNAMICAL MODEL
Simplified models capable of catching the relevant behaviors.

PCC – node 0

\[ u_0 = U_0 \]

Loads and microgenerators

Constant power devices with first order dynamics

\[ \tau_v \frac{di_v}{dt} = -i_v + \frac{\bar{s}_v}{u_v} \]

The steady state corresponds to the constant power relation

\[ u_v \bar{i}_v = s_v. \]
Grid model

The **interconnection** between these subsystems is given by the **electrical topology** of the microgrid.

Let $X$ be a matrix defined by its elements

$$X_{hk} = \sum_{e \in \mathcal{P}_h \cap \mathcal{P}_k} z_e$$

where $\mathcal{P}_h$ is the path from the PCC to node $h$.

**Network equation**

$$
\begin{bmatrix}
\vdots \\
u_v \\
\vdots 
\end{bmatrix}
= X 
\begin{bmatrix}
\vdots \\
i_v \\
\vdots 
\end{bmatrix}
+ 
\begin{bmatrix}
\vdots \\
U_0 \\
\vdots 
\end{bmatrix}
$$
Pluggin this expression for the node voltages into the model for each nodes, one gets the **nonlinear dynamical system**

\[
\tau_v \frac{d i_v}{dt} = -i_v + \frac{\bar{s}_v}{1_v^T \bar{X} i + \bar{U}_0}
\]

in which the coupling between all nodes appears in $1_v^T \bar{X} i$.

**Second-order Taylor expansion for large $U_0$**

\[
\tau_v \frac{d i_v}{dt} = -i_v + \frac{1}{U_0} \bar{s}_v - \frac{1}{U_0^2} \bar{s}_v 1_v^T \bar{X} i.
\]
We obtained an approximate linear system of the input-output relation between complex power references $s_v$ (input) and voltages $u_v$.

This result allows us to employ the tools of linear system analysis (eigenvalues, Bode plots, step response, ...) to study the dynamic behavior of such system, as a function of relevant parameters.

**An example**

System eigenvalues are

$$
\Lambda = \left\{ -\frac{1}{\tau_v} \pm \frac{|s_v| |X_{vv}|}{U_0 \tau_v} \right\}.
$$
SIMULATIONS
We consider the IEEE 37 testbed.

Let node 30 be a microgenerator, to which we command a step of 60 kVAR in reactive power supply.

Microgenerators are equipped with fast inverters, however slow loads are present in the microgrid.

We are interested in the settling time of voltages to their steady state in two points of the network: at the same inverter (30) and at another inverter (22).
Response to step in reactive power
CONCLUSION
We focused on a specific problem: obtaining a dynamic model for the input-output relation between the complex power references commanded to the microgenerators dispersed in the microgrid, and their phasorial voltage measurements.

The obtained model has remarkable features:

- It is a linear system.
- Network topology and parameters are recognizable in the model.
- It catches the complex phenomena emerging from the interconnection of many simple systems.
- It gives important insight for the design of iterative control algorithms.

Thanks!

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