Chapter 2

Precursors

2.1 Introduction

Thus far I have presented an approach to the semantics of plurals in the form of two rather similar grammars for a fragment of English. And I have given a few examples of the kinds of things one can say within this approach. Although the grammars we have are not borrowed wholesale from any previous work, they are meant to capture two prominent positions taken in other papers. Of course, these other papers were concerned with various issues, not only the ontological question. The purpose of this chapter is to profile a few of those earlier papers to give the reader a taste of the context in which the ontological question has been raised. Link’s (1983) paper was chosen in part because it has become a standard reference for the union approach. Hoeksema’s paper was chosen in part because it covers the question of how to embed the semantic theories of chapter 1 in a generalized quantifier framework. The last paper reviewed here is Landman (1989a). One of the decisions I made in an effort to create a coherent and focussed story was to introduce two opposing views and to pin the difference between them on the meaning of the conjunction and. Landman’s paper is a useful ‘antidote’ in this regard because he sketches an analysis that is in some sense intermediate between the sets and the union approaches and because some of the important properties of the sets approach are achieved there without a sets meaning for and. After reviewing that paper we will consider whether we’re missing anything by telling the story begun in chapter 1.

2.2 Link (1983)

For now, we turn to Link (1983) the goal of which is to produce a logic with a model theoretic interpretation that can handle both plural and mass terms. Link is interested in capturing with a single formalism
properties that are shared by both kinds of terms. An example of such a property is what Link calls cumulative reference and what we have called, at least when the property is manifested in the plural domain, cumulativity. Both plural and mass nouns form cumulative predicates, as this example from Link’s paper is meant to show:

(46) a. If a is water and b is water then the sum of a and b is water.
    b. If the animals in this camp are horses, and the animals in that camp are horses, then the animals in the two camps are horses.

A mass term (noun phrase) is not interpreted as denoting a set of entities because "inherent in the notion of a set is atomicity which is not present in the linguistic behavior of mass terms" (Link 1983:305). Therefore, in order to preserve the "structural analogy between the two cases," Link rejects the use of sets to map plural terms. Instead he employs lattice theoretical notions in his semantics to establish the related mass, singular and plural domains. Nonetheless, the point has been made that at least the plurals portion can be redone in terms of sets (see Lasersohn 1988:131, Landman 1989a:568-571) without doing violence to the mass-plural connection. Our union theory is essentially a set theoretic version of Link’s (1983) interpretations of plural noun phrases.

2.3 Hoeksema (1983)

The paper by Hoeksema, entitled Plurality and Conjunction, covers a host of topics, two of which, a semantic theory of number concord and a semantics for conjunction, are reviewed here. We dwell at some length on Hoeksema’s work because it addresses the issue of how to embed the theories we outlined above in a framework in which all or some noun phrases are interpreted as generalized quantifiers. Since our theories will remain in their current form in subsequent discussion, it is important to show that they are a part of and not an alternative to a more complete account of the semantics of noun phrases (for more recent discussion of plurals in a generalized quantifier setting, see van der Does 1992, 1993).

Hoeksema’s paper begins with a specification of the domain of discourse, which is that of our sets theory. Working in a generalized quantifier framework in which all noun phrases denote functions from subsets of the domain of discourse into the set of truth values, Hoeksema analyzes the infelicities in (47) and (48) as follows:

(47) #John are walking.
(48) #The boys is walking.
John denotes a function from subsets of D (the domain of singularities) into truth values. In any model in which the predicate are walking denotes a non-empty set, it fails to denote a subset of D. The function \( \| \text{John} \| \) is then undefined for this argument, hence the infelicity in (47). Similarly for (48), \( \| \text{the boys} \| \) is undefined for its argument, the set of singularities denoted by \( \text{is walking} \). The idea of having a semantic theory of number concord is certainly attractive, though difficult problems arise once a broader range of data is considered. Lasersohn (1988:Ch.4) considers a number of these problems and proposes a more elaborate theory taking Hoeksema’s theory as a point of departure. We will return later in this section to this part of Hoeksema’s work after presenting his analysis of NP conjunction.

The two theories proposed in chapter 1 differ solely by the interpretation they give for NP conjunction. Because of the importance we have placed on NP conjunction, Hoeksema’s analysis will be covered in some detail here. The discussion here, based in part on revisions found in Hoeksema (1987a), will begin with a description of the semantic rules, followed by some examples to show how they work and then a discussion of the type of data Hoeksema is trying to account for.

Hoeksema’s semantics is done in a PTQ type framework in which expressions of English are translated into a lambda calculus. I will presuppose familiarity with that framework. Relevant and important aspects of this system will be revealed as we go along. I will be using the symbol \( A \rightarrow B \) to relate (subscripted) expressions of English to formulas of the translation language. \( A \rightarrow B \) is shorthand for "B is the translation of A or is logically equivalent to the translation of A."

Hoeksema proposes two translations for the NP conjunction and. I present them in (49) and (50) below with subscripts i or c on the English word and indicating which translation rule applies to it. This subscripting is not part of the formal system but it has expository value. The term "intersective conjunction" refers to the conjunction and under its intersective reading; similarly for "collective conjunction."

\[ (49) \text{Intersective conjunction.} \]
\[ \text{and}_i \Rightarrow \lambda \pi \lambda \Phi \lambda P [ \Phi(P) \land \pi(P)] \]

\[ (50) \text{Collective conjunction. (cf. Hoeksema 1987a:35)} \]
\[ \text{a. Grp is a two place function.} \]
\[ \| \text{Grp} [a,b] \| = \{ \| a \|, \| b \| \}. \]
\[ \text{b. and}_c \Rightarrow \lambda \pi \lambda \Phi \lambda P [ \Phi(\lambda x.\pi(\lambda y. P(\text{Grp}[x,y])))] \]

The interpretation for intersective conjunction given in (49) is the standard
PTQ-style NP conjunction. Applying a predicate to an intersective conjunction of NPs is equivalent to applying the predicate to each of the NP-conjuncts and conjoining the result, for example:

(51) Every man and every woman solved the crossword puzzle.
⇔ Every man solved the crossword puzzle and every woman solved the crossword puzzle.

The interpretation for collective conjunction given in (50) is the counterpart of our sets-theory interpretation for and in a generalized quantifier system. Hoeksema arrived at (50) by taking the sets theory and which is, in effect, of syntactic type $<e, <e, e>>$ and producing an expression of type $<T <<T, T>>$, where $T = <<e, t>, t>$, the type of generalized quantifiers. He used the rules of type change for the version of the Lambek calculus found in van Benthem (1986).

To see the connection between the two interpretations consider what we get for the teacher and the students. Even though Hoeksema doesn’t actually give a meaning for definite plurals, I think it is fair to assume the following type $<<e, t>, t>$ versions of the interpretations presented at the beginning of this chapter:

the students $\implies \lambda R. R(S)$ (S is the set of students)
the teacher $\implies \lambda Q. Q(t)$ (t denotes the teacher).

Now plugging the translation of the students in for $\pi$ in the subformula $\lambda x. \pi(\lambda y. P(\text{Grp}[x,y])$ from (50), we get:

$$\lambda x. \lambda R. R(S) (\lambda y. P(\text{Grp}[x,y])) = \lambda x. P(\text{Grp}[S,x])$$

So and, the students translates as:

$$\lambda \Phi \lambda P [ \Phi(\lambda x. P(\text{Grp}[S,x]))]$$

and

(52) the teacher and, the students $\implies$
$$\lambda P [ \lambda Q . Q(t) (\lambda x. P(\text{Grp}[S,x]))] = \lambda P [ P(\text{Grp}[S,t])]$$

Now by (50a), the interpretation of $\text{Grp}[S,t]$ is just the set that contains the teacher and the set of all the students. So $\text{Grp}[S,t]$ is exactly the interpretation of the teacher and the students on our original sets theory.

Assume a pig is a member of the general interpretations only of the form $\text{Grp}[x,t]$. The pig and a plural here from works with the plural formula $\forall x.P(\text{Grp}[x,t])$, every pig
What (52) says is that the collective conjunction of the teacher and the students is interpreted as the generalized quantifier whose generator is just the interpretation of the conjunction of these two NPs on our sets theory. This correspondence holds for all collective conjunctions of definite NPs. One might describe Hoeksema’s system as a type-shifted version of the sets theory.

It is important to notice that it is not the generalized quantifier framework itself that forces Hoeksema into a sets theory. Assuming Quine’s Innovation, the definition in (50) could be modified slightly to arrive at a generalized quantifier version of the union theory as follows:

(53) Collective conjunction for a union theorist.
    a. $\text{Grp}$ is a two place function.
       $\| \text{Grp} [a,b] \| = \| a \| \cup \| b \|$
    b. and = $\Rightarrow \lambda x \lambda \Phi \lambda P (\Phi(x, \pi. P(\text{Grp}[x, y])))$
       ($\pi, \Phi$ are generalized quantifier variables)

Hoeksema’s theory is of course not limited to definite noun phrases, the way ours are. To give a more accurate picture of what he is up to, I will give translations for some other kinds of noun phrases, beginning with the conjunction of indefinites translated in (54):

(54) $a$ cow and, a pig $\Rightarrow$
    $\lambda P \exists x \exists y (\text{cow}'(x) \land \text{pig}'(y) \land P(\text{Grp}[x,y]))$

(a cow and, a pig) and, a horse $\Rightarrow$
    $\lambda P \exists x \exists y \exists z (\text{cow}'(x) \land \text{pig}'(y) \land \text{horse}'(z) \land P(\text{Grp}[z, \text{Grp}[x,y]]))$

Assuming Hoeksema’s interpretation for $\text{Grp}$, (54) tells us that $(a$ cow and, a pig) and, a horse will denote the set of properties that hold of some two-membered set containing a horse and a set of a pig and a cow. This is the generalized quantifier that would correspond to the sets theory interpretation for (the cow and the pig) and the horse assuming there was only one of each of these in D. If we departed from Hoeksema in taking $\text{Grp}[x,y]$ to denote the union of x and y, the denotation of $(a$ cow and, a pig) and, a horse would be the set of all properties P such that P holds of a plurality consisting of a cow, a pig and a horse.

An advantage of the formulation of collective conjunction given here from Hoeksema (1987a) over the one in Hoeksema (1983) is that it works for universal noun phrases as well. According to the present formulation every soldier and, every officer met will be true if and only if for every pair of an officer and a soldier, the soldier met the officer (and vice-
versa).

Unfortunately, there are cases in which, despite Hoeksema's (1987a:35, 38fn3) claims to the contrary, the rule in (50) makes incorrect predictions. In particular, it seems to give incorrect results for downward entailing quantifiers such as no, few and not every.

(55) No soldier and no officer met \[\rightarrow\exists x [\text{soldier'(x)} \wedge \neg\exists y [\text{officer'(y)} \wedge \text{met'}(\text{Grp}(x,y))]]
= \forall x [\neg\exists y [\text{officer'(y)} \wedge \text{met'}(\text{Grp}(x,y))]]
= \forall x [\neg\text{soldier'(x)} \vee \exists y [\text{officer'(y)} \wedge \text{met'}(\text{Grp}(x,y))]]
= \forall x [\text{soldier'(x)} \rightarrow \exists y [\text{officer'(y)} \wedge \text{met'}(\text{Grp}(x,y))]]

If every soldier met some officer then the translation in (55) is true even though in such a situation it is false that no soldier and no officer met. The meaning of the English sentence in (55), on the other hand, seems to be the negation of the meaning that would be assigned to (56) using (50):

(56) Some soldier and some officer met.

A stop-gap solution to this problem may be available if we allow different translations for the collective conjunction depending on the entailingsness of the conjuncts. In fact there is precedent for this in von Stechow (1980) where different rules are used to interpret a (collective) NP conjunction depending on the semantic character of the conjuncts. Furthermore, it cannot be a coincidence that Barwise (1979) and others give different branching quantifier interpretations for conjoined NPs depending on the entailingsness of the conjuncts. This problem will be left unsolved as we move on to the data that Hoeksema sought to explain by introducing two interpretations for conjunction.

Hoeksema observed that, depending on the type of NPs that are conjoined, sometimes a conjunction of singular NPs must combine with a plural verb phrase and other times a conjunction of singular NPs may combine with a singular verb phrase. This is illustrated in (57) and (58):

(57) a. A man and a woman {were/*was} arrested.
    b. The man and the woman {were/*was} arrested.
    c. Ray and Tess {were/*was} arrested.

(58) a. Every day and every night was spent in bed.
    b. No peasant and no pauper was ever President.

Names and definite and existential NPs are in the first class requiring a plural predicate while apparently all other NPs are in the second class.

Precursory

Hoeksema observes in the cases considered above with examples in (59) that the correct meaning is given

(59)

(60)

These illustrate the fact that a conjunction of two

NPs can have a different truth value than the two

NPs combined as a whole term, however, for the other two sentences involving a conjunction of two

NPs it seems clear that the conjunction acts as

narrowly as a singular term. The following

example illustrates this.

(61)

The verb is singular while the NPs are plural.
Furthermore, the example is obviously incorrect.

The correct meaning is given by this act of

stipulation and follows in (60), as

way in singular.
Hoeksema further noted that you can usually replace a conjunction of NPs with one of the conjuncts without changing the truth-value, if the NPs are in the second class. He contrasts (59) in which a. entails b. with (60) in which a. does not entail b.

(59) a. Every man and every woman has/have solved the crossword puzzle.
    b. Every man has solved the crossword puzzle. Every woman has solved the crossword puzzle.

(60) a. Tim and Grace has/has solved the crossword puzzle.
    b. Tim has solved the crossword puzzle. Grace has solved the crossword puzzle.

These observations are correlated with the two interpretations for conjunction as follows. Recall, according to Hoeksema singular NPs correspond to functions from subsets of D, the domain of singularities, into truth values. Given the definition in (49), an intersective conjunction of two such NPs will again be a function from subsets of D. So two singular NPs conjoined intersectively will combine with a singular verb phrase. On the other hand, a collective conjunction of two singular NPs denotes a function from sets containing pluralities into truth values and hence must combine with a plural verb phrase. With this account in mind and narrowing our gaze for a moment to just the data in (57-60), we accept the following stipulation:

(61) Singular names, definite NPs and indefinite (existential) NPs conjoin collectively, all other NPs conjoin intersectively.

The verb phrases in (57) are plural because the conjunction is collective, while those in (58) are singular because the conjunction is intersective. Furthermore the presence of an intersective conjunction in (59a) explains the entailment to (59b), while a collective conjunction in (60a) would correctly fail to licence the entailment to (60b).

I find this account of the data in (57-60) appealing. At the very least, one can view it as an interesting argument for an ambiguity in the meaning of and. Nonetheless, difficult questions remain. Before accepting this account one would want to know two things, first, where does the stipulation in (61) come from? Secondly, to what extent is the data in (57)-(60) representative?

Before turning to an explanation of (61), I want to mention one way in which the data is unrepresentative. If a conjunction of NPs takes singular agreement, it usually does so optionally:
(62) Every boy and every girl was/were happy.

Regarding such examples Hoeksema (1987a:38fn2)'s "position is that most facts about number agreement can only be explained (as opposed to described) semantically, but that there remains some arbitrariness which must be ascribed to syntactic encoding." How broadly this caveat can be construed surely depends on having a concrete theory of this "syntactic encoding" and on having a full account of the exceptions to the semantic theory. More exceptions will come to light once I have given an account of (61).

While there is a semantic characterization of the class of collectively conjoining NPs in the 1983 paper, there is no explanation there for why these NPs should conjoin collectively. In Hoeksema (1987a) an attempt is made at explaining the stipulation via an appeal to the non-quantificational view of indefinites. Hoeksema drops the assumption that all NPs are of type <e,<e,t>> and assumes instead that names and definite NPs are e-type and that indefinites introduce an e-type variable. Let us call these e-NPs. All other NPs remain of type T = <e,<e,t>> and we'll call them T-NPs. Collective conjunction is then defined as in our sets theory and is therefore of type <e <e,e>>. Intersective conjunction remains of type <T <T,T>>. The stipulation in (61) is now reformulated in (63):

(63) e-NPs conjoin with a collective <e,<e,e>> conjunction.
    T-NPs conjoin with an intersective <T,<T,T>>
    conjunction.

At the very best (63) is a default rule since exceptions arise in all directions. To begin with e-NPs can conjoin with T-NPs, (e.g. John and every other student) yet there is no <e,<T,T>> or <e,<T,e>> conjunction. Furthermore, we have seen cases of T-NPs conjoined collectively:

(64) Every soldier and,e every officer met.

Finally, languages have special mechanisms to signify the intersective conjunction of NPs, including e-NPs. An English example is the both...and... conjunction:

(65) Both Bill and Sue left (*together).

As Ross (1967:92) points out, the French equivalent involves the repetition of the conjunction (et Bill et Sue). Hoeksema (1983:82fn8) mentions
Precursors

examples of this kind in Dutch as well.

Hoeksema handles all these cases with a type-shifting mechanism to raise the types of NPs and of the collective conjunction (to achieve the rule given in (50)) and then (63) is reduced to a rule requiring the use of minimal types. Thus while Ray and Tess could be raised to type T and conjoined intersectively, this is not done in (57c). This kind of account might also help us understand why conjoined numerical NPs like two men do not seem to conjoin as T-type expressions with the collective interpretation. Thus, as Angelika Kratzer has observed to me, there doesn’t seem to be a reading of (66),

(66) Two men and two women met.

in which one of the NPs has scope over the other, as contrasted with the closely related (67):

(67) Two men met two women.

An explanation for this discrepancy⁷ might go as follows. Numerical NP expressions have e-type (or <e,T> type) group interpretations in which the number is adjectival and they have T-type interpretations in which the number is a quantifier (cf. Partee 1987). The missing reading would require opting for the higher type for the NP as well as the type-shifted form of the collective interpretation for conjunction. This reading can be blocked if we disallow ‘gratuitous’ type shifting (cf. Partee & Rooth 1983). This explanation requires extending the demand for minimal types to the interpretation of conjunction as well as to the interpretation of NPs.

Summarizing, Hoeksema presents us with an ambiguity account of NP conjunction. One meaning is the familiar PTQ-style intersective NP conjunction. The other, collective, meaning is essentially the one on which we have based our sets theory. A <T,<T,T>> version of this conjunction allowed for a more general account than the one we proposed, however, even this account was not fully general as it gave incorrect results for the conjunction of NPs denoting downward entailing quantifiers.

Positing an ambiguity in the meaning of NP conjunction enabled Hoeksema to propose a partial semantic explanation of number concord by

⁷In Kratzer’s opinion the difficulty that arises here is of a kind with that of example (55), No soldier and no officer met. In both cases, Hoeksema’s theory involves a scope relation between noun phrase conjuncts when there shouldn’t be one.
assigning different conjunction interpretations to different NP types. It explains why only some conjunctions licence the entailment pattern in (68):

(68)  \[ \text{A and B VP.} \]

\[ \begin{align*}
\text{A VP and B VP.} \\
\end{align*} \]

Finally, the presence of these two interpretations is useful in explaining the difference between the a. and b. examples below ((70) is Modern Hebrew):

(69)  \[ \begin{align*}
a. & \text{ Ray and Tess solved the puzzle.} \\
b. & \text{ Both Ray and Tess solved the puzzle.} \\
\end{align*} \]

(70)  \[ \begin{align*}
a. & \text{ Ray vl-Tess patru } \text{ 'et haxida} \\
& \text{ Ray and Tess solved ACC the puzzle.} \\
b. & \text{ Gam Ray vl-gam Tess patru } \text{ 'et haxida} \\
& \text{ also Ray and also Tess solved ACC the puzzle.} \\
\end{align*} \]

In the context of this discussion, the rules presented in chapter 1 can be thought of as the syntax and semantics of collective conjunctions of definite NPs.

2.4 Landman (1989)

Up to now I have mentioned the analysis of plurals found in Link (1983) on which our union theory is based and Hoeksema (1983,7a), a precursor of our sets theory. I want now to introduce the work of Landman (1989a) who takes what appears to be an intermediate position, not quite captured by either the union or the sets theory.

A leading idea in Landman's article is that cumulativity and distributivity are two sides of the same coin and the formal theory should reflect this. In addition, Landman seeks to elaborate the idea introduced in Link (1984) that distributivity and cumulativity are concepts relating not only singularities to pluralities, but also pluralities to higher order pluralities. Thus if a predicate is true of each member of a set of singularities, it is true, by cumulativity, of the plurality corresponding to the set. Likewise, if a predicate is true of each member of a set of pluralities, it is true, by cumulativity, to the plurality corresponding to that set. Instances of cumulativity of this second, "higher," kind defy analysis in our union theory because that theory never assigns higher than first order pluralities as the meaning of definite NPs. More will be said about this in the next chapter.

Landman's work is based in large measure on Link's. So we first
In Link's work in Chapter 2, we reconstructed a piece of Link's account of distributivity in terms of the union theory we set up in Chapter 1. The analysis will be only briefly introduced here, but it will come under more careful scrutiny in Chapter 4. According to this account, inherently distributive predicates such as \( \text{be a pop star} \) denote subsets of \( D \); they are true, in the singular, only of singularities. Plural versions of these predicates are translated with the "star-operator." \( ^*P \) denotes the closure under union of \( \| P \| \). So from \( ^*P(J+M) \) where \( \| J+M \| = \{j, b\} \) we can conclude \( P(J) \) and \( P(B) \) (where \( \| J \| = j \), \( \| B \| = b \)) and from \( P(J) \) and \( P(B) \) we can conclude \( ^*P(J+M) \). This reasoning corresponds to the deduction from \( \text{John and Bill are pop stars} \) to \( \text{John is a pop star and Bill is a pop star} \) and vice versa. In his 1984 paper, Link extends this picture to account for 'higher order' distributivity (and reciprocity). He achieves this by allowing plural noun phrases to be ambiguous, between a set-denoting interpretation and a (singular) entity denoting interpretation. The set denoted by a plural is called a sum and the non-set entity is called a group. This immediately raises the prospect of iteration: a set of groups is again a sum and the corresponding group is itself a group of groups.

Landman argued that such iteration was called for and he designed a system with higher orders of distributivity and cumulativity. His modus operandi is to model both singular individuals and groups of individuals as singleton sets. The one element in the singleton corresponding to a singular individual is that individual. To prevent confusion, I will call this singleton an "individual singleton," departing slightly from Landman's terminology. The one element in the singleton corresponding to a group is a set containing the members of that group. In order to handle distributivity, Landman's theory includes sums, which are distinct from groups and which are not singletons. A set of individuals is a sum. It is a sum (= union) of the individual singletons. The singleton containing that sum is the corresponding group. If a set \( \| P \| \) contains only singletons, groups or individual singletons, then its closure under union, \( \| ^*P \| \), will contain sums of the individual singletons or groups in \( \| P \| \). By taking the extension of every "basic" predicate (I assume "basic" means without a "*"), to include only singletons, an inference of the following sort is rendered valid:

\[(71)\] Let \( \alpha \) denote a non-singleton set. Let \( S_1, S_2, \ldots, S_n \) be a series of terms such that:

a. For each \( i, 1 \leq i \leq n \), \( \| S_i \| \) is a singleton.

b. \( \| S_1 \| \cup \| S_2 \| \cup \ldots \cup \| S_n \| = \| \alpha \| \)

then:

\( ^*P(\alpha) \leftrightarrow P(S_1) \land P(S_2) \land \ldots \land P(S_n) \)
This inference can be used to map both distributivity and cumulativity of P, regardless of whether an arbitrary S₁ denotes an individual singleton or a group (singleton containing a set of elements of D). For example, the distributive reading of John and Mary died, is captured by letting John and Mary denote a sum, letting John and Mary substitute for α in the above scheme with John substituting for S₁ and Mary substituting S₂.

At this point, I give a sketch of Landman's grammar as an extension of our union theory. In particular, and is interpreted as union on this theory. Since for Landman noun phrases are ambiguous, we first translate the part of English we are interested in into a disambiguated language which we will call Π-English. The relation between the two languages is as follows:

[A] A noun phrase of English is a noun phrase of Π-English.
[B] A verb phrase of English is a basic verb phrase of Π-English.
[C] If α is a noun phrase of Π-English, then Π(α) is a noun phrase of Π-English. 
[D] If β is a verb phrase of English then *(β) is a verb phrase of Π-English.

The semantics of Π-English is just the semantics of our union theory along with the following rules:

[9] \| Π(α) \| = \{ \| α \| \}. 

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8 Π' stands for group. Π(α)' corresponds to Landman's '↑(α)' and to Link's '<α>'.

9 Our adoption of Quine’s innovation would, I believe, improve on Landman’s theory here. For in this theory, John denotes not only {j} but also, given the rules in [C] and [9], {{j}}, {{{j}}} etc. This will have consequences if we attempt to explain the following pair:

i. #John met.
ii. The men met.

in terms of the type or order of the subject (cf. Landman 1989:593, "collective predicates like meet take groups [singleton sets of sets] but not singular individuals in their extension"). Given Quine’s innovation, j={j}={j}={{{j}}}, and so i.-ii. can be explained by saying that meet has no elements of D in its extension.
Precursors

\[10\]
\[\|*(\beta)\| = \text{The closure under union of } \|\beta\|.
\]

\[11\]
\[\forall x [x \in \|\beta\| \implies x \text{ is a singleton set}]
\]

A computation of some of the interpretations of the noun phrase \textit{the cows and the pigs} should give a sense of how this theory works. We use the symbol "\(\implies\)" to mean, "translates into \(\Gamma\)-English as." We will not distinguish singular individuals from their singleton sets here, adopting Quine’s Innovation (see Appendix).

The \(\Gamma\)-English expression \textit{the cows} denotes the set of all the cows. Let’s assume that there are only three cows and three pigs, then we can represent the cows as \{a,b,c\}. \(\|\text{the pigs}\|\) will be represented as \{m,n,p\}.

\[\begin{align*}
(72) & \quad \text{the cows and the pigs} \implies \text{the cows and the pigs} \\
& \quad \|\text{the cows and the pigs}\| = \{a,b,c\} \cup \{m,n,p\} \\
& \quad = \{a,b,c,m,n,p\}.
\end{align*}\]

\[\begin{align*}
(73) & \quad \text{the cows and the pigs} \implies \Gamma(\text{the cows}) \text{ and the pigs} \\
& \quad \|\Gamma(\text{the cows}) \text{ and the pigs}\| = \{\{a,b,c\}\} \cup \{m,n,p\} \\
& \quad = \{\{a,b,c\},m,n,p\}.
\end{align*}\]

The interpretation in (72) is just the interpretation that the union theory gives for this noun phrase. The one in (74) is the interpretation the sets theory would give. The one in (73) cannot be gotten on either theory, hence if it can be shown that we need that interpretation then we must choose this theory over the other two. Furthermore, a theory which assigns the meanings of (72) and (74) by simply positing an ambiguous \textit{and} with a sets and a union interpretation (cf. Lønning 1987:109) will still not assign the interpretation in (73).

Here is an example in which all of these interpretations come into play:

\[\begin{align*}
(75) & \quad \text{The cows and the pigs carried a piano upstairs.}
\end{align*}\]

Landman (1989a:594) would claim that this sentence has a number of readings and he would use different interpretations for the subject to capture these readings. (72) would be used for the reading in which each animal carried a piano. Although Landman doesn’t actually discuss such an example, (73) would presumably be used for a reading according to
which each pig carried a piano and the group of the cows carried a piano. (74) would be used for the reading in which there were two carryings, one by the cows and one by the pigs. In each of these cases the verb phrase would be translated: *(carried a piano upstairs)* since in each case there is distributivity of one sort or another. Let us look a little closer at the case in which the subject of (75) is translated as in (74) and the star is employed in the translation of the verb phrase. In this case we have an instance of the schema given in (71); Π*(the cows) and Π(the pigs)* substitutes for α. This expression indeed denotes a non-singleton or what Landman would call a sum. Given [11], the basic, unstarred, predicate *carried a piano upstairs* could not include this sum in its extension. But the starred, *(carried a piano upstairs)* could, given [10]. In fact, it could if and only if the singletons {{a,b,c}} and {{m,n,p}} were in || carried a piano upstairs ||. Hence on this reading, (75) entails that the cows carried a piano upstairs and the pigs carried a piano upstairs.

There is yet another collective reading in which there is one carrying by all the animals at once and for that reading the noun phrase would be translated: Π*(the cows and (the pigs))* and the verb phrase would have a basic translation, without the ***. Notice that the denotation of Π*(the cows and (the pigs))* is a singleton and rightly so since basic, starless, predicates have only singletons in their extensions.

As pointed out above, the number of readings that Landman's theory allows for seems not to be captured on either of the theories set out in chapter 1. One might therefore wonder if we shouldn't instead compare his theory to the union theory in our discussion. There are two reasons why I did not do this. The first has to do with the nature of the arguments to be presented. Thinking just in terms of the domains on the two theories, the union theory is the poorest, a subset of the other two, while Landman's is the richest. The arguments to be presented here will be arguments against the richness of the sets theory ontology. These arguments remain intact for the even richer ontology of the mixed theory. In that case, we may as well stick with the simpler comparison, as set out in chapter 1. The second reason for doing this has to do with an aspect of Landman's theory not yet mentioned. Recall, in chapter 1, it was observed that a distributively read verb phrase could be conjoined with a non-distributively read verb phrase. Since in the theory just sketched, the presence of a distributive reading is determined in part by the interpretation of the noun phrase (which 'decides' what is distributed over: animals, groups of animals etc.), these examples require an amendment to the theory. To this end, in Landman (1989a:2.4), a family of type-shifting operations are introduced. For example, there is an operation, ↓2 which 'converts' an expression with the meaning in (74) to one with the meaning in (72).
Using this operation, we can lift a predicate \( P \) which applies to a set of individuals into \( \lambda xP(\downarrow 2(x)) \) yielding the equivalence below:

\[
(76) \quad \text{let } \alpha = \{a,b,c\}, \{m,n,p\} \text{ and let } \beta = \{a,b,c,m,n,p\}, \text{ then:} \\
[\lambda xP(\downarrow 2(x))](\alpha) = P(\beta).
\]

If the meaning in (72), here expressed by \( \beta \), were not available as a noun phrase meaning, nothing would be lost, since the same readings would crop up through type-shifting using the meaning in (74), here called \( \alpha \). Similarly, there is a type-shift, which can convert \( \alpha \) into \( \Gamma((\text{the cows}) \text{ and } (\text{the pigs})) \), so the latter appears dispensable as well. It appears then, that once these type-shifts are taken into consideration, Landman's theory would not make different predictions were it built upon the sets theory. Now, the one case that is not so clear is (73). In principle, one can introduce a new function which like Landman's type-shifting operations would apply to arguments to give values of a different type and which in particular would apply to \( \{\{m,n,p\}, \{a,b,c\}\} \), the meaning in (74), to give you \( \{a,b,c\}, m,n,p \} \), the meaning in (73). One might guess that this is not a proper type-shift, like the ones Landman proposes. However, it seems that such a function would be needed to account for a case of conjunction where one VP has a reading requiring the (74) meaning and one requiring the (73) meaning. Once the full range of type-shifts are spelled out, it seems to me that the overall theory would not be changed by removing those parts of the semantics of noun phrases that make this theory different from the sets theory. On this perspective, Landman (1989a) is the sets theory along with a specific proposal on how to handle distributivity. So we remain at this point with our two basic theories and in chapter 5 the question of how they each handle distributivity will be raised.

This completes our review of the kind of work that forms the background to the two theories outlined in chapter 1. The purpose of these two theories is to capture in as simple a format as possible an important consideration that runs through much recent work on plurals despite differences in terminology, formal framework and particular linguistic concerns. In the forthcoming pages, a choice will be made between the union and the sets theory. It is hoped that the choice and the justification for it will have relevance for the work for which our two theories go proxy.