Commentary for an issue of Theoretical Linguistics on:
“Stratified Reference - the Common Core of Distributivity, Aspect, and Measurement’ by Lucas Champollion.

Partitives and Duratives
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1. Introduction
Champollion detects similarities in the interpretation of three seemingly unrelated forms: partitives, measure adverbials and distributivity operators. Stratified reference is a high level description of the unique meaning component that lies at the core of these similarities. I found it helpful to think of ‘stratified reference’ as having the same type of status as ‘maximality’ which is implicated in the interpretation of definite descriptions, degree constructions, interrogatives and elsewhere. Converging on a single statement with which to describe the meanings of diverse forms enables us, as Champollion puts it, to link problems.

Mereological parts make up the domains of quantification for stratified reference statements. I inquire here about the nature of the quantification: what kind of quantificational force do we want?, how might the parthood relation be restricted? and are there constraints on the size of the domain of quantification? I’ve come to appreciate Champollion’s mechanism by taking it apart and trying to put it back together with a few pieces missing. I hope the reader is able to learn something from this exercise. My comments are exclusively directed toward section 2 Aspect and section 3 Measurement (sometimes referred to below with the symbols ‘C.§2’, ‘C.§3’ respectively).

2. Universal force of stratified reference claims
To say that a predicate has stratified reference is to make a certain kind of claim about the extension of that predicate. (1) below is an example in which the claim is made about the plural predicate books. (1) is associated with the expression 5 pounds of books (compare C.§3,(49)). Following Champollion’s practice, the claim is expressed in a. and paraphrased in b.

(1) a. \(\forall x \text{books}(x) \rightarrow x \in \#\lambda y \text{books}(y) \land \varepsilon(\lambda d \text{pounds}(d) = 5)(\text{weight}(y))\).
   
   b. Every sum of one or more books can be divided into one or more parts, each of which is a sum of one or more books whose combined weight is very small compared with 5 pounds.
(1) says something about every entity in the extension of the predicate ‘books’. Before getting into the content of the claim, I want to question the wisdom of making it universal. To make the discussion plainer, I first motivate a simpler, but equivalent version of (1).

Suppose that all books weigh a lot less than 5 pounds. In that case, any sum of books would be divisible into parts that are books, each of which weighs a lot less than 5 pounds, as (1) says. In other words, if (2) below is true, then (1) is true as well.

(2) All books weigh a lot less than 5 pounds.

Going the other way now, note that (1)b begins with “every sum of one or more books”. Suppose we restrict attention to sums consisting of one book. According to (1), such a sum can be divided into parts that are books each of which weighs a lot less than 5 pounds. But since such sums have just one part that is a book\(^1\), it follows that the sum, the book, weighs a lot less than 5 pounds. In other words, if (1) is true, then (2) must be true as well. Since (1) entails and is entailed by (2), they are equivalent. (2) is also equivalent to the claim in (3) below about the singular predicate book, a claim associated with the expression *5 pounds of book* (see C.§3, (41)).

(3) a. \(\forall x[\text{book}(x) \rightarrow x \in *\lambda y (\text{book}(y) \wedge \varepsilon(\lambda d[\text{pounds}(d) = 5])(\text{weight}(y)))].\)

b. Every book can be divided into one or more parts, each of which is a book whose weight is very small compared with 5 pounds.

Contrary to what is said in C.§3, I can felicitously report that I’ve bought 5 pounds of books without committing to something as strong as (2). It is sufficient that each of the books I’ve bought weighs less than 5 pounds. At various points in Champollion’s discussion, quantification is limited to typical exemplars. This helps in some cases and hurts in others. Suppose I find and purchase one of those atypical, 5 pound books. We want to rule out *5 pounds of book* to describe my purchase, but we can’t do it if all we have to work with is a version of (2) that is limited to typical books. Again, what goes wrong here is not about books in general, but about the particular book I’m describing. That book is not composed of books weighing a lot less than 5 pounds.

I’ve made my case using expressions from the Measurement section of the paper, but similar considerations apply to data appropriate to the Aspect section. We can say ‘Jack walked for 2 minutes’. On the universal view, that would be because all events of walking are divisible into one or more walkings that are short compared with 2 minutes. And yet there are bi-pedal robots that walk. Suppose that one of them, Robby, takes his first steps very very slowly – say 2 minutes a step. It follows now that not all events of walking are short compared with 2 minutes, but this clearly doesn’t affect the felicity of ‘Jack walked for 2 minutes’. Here too, limiting the universal to typical events won’t help, since we do want to rule out ‘Robby walked for 2 minutes’ as a description of the first two minutes of Robby’s walk.

\(^1\) If you are worried about the existence of a book which is formed by summing together two or more other books, then everywhere read ‘book’ as ‘book that isn’t formed from other books’.
The problem I’m raising is there in the beginning of the Aspect section. One scenario is discussed in which it is odd to say *John pushed carts all the way to the store for fifty meters*. On the proposed account, this is explained in terms of the nature of events described with ‘John pushed carts all the way to the store’. But if that explanation relies on a claim about all events in the extension of the predicate, then a change in scenario shouldn’t matter. But it does. In the scenario imagined by H. M. Gärtner, the predicate modified with for fifty meters is acceptable. Discussion of Gärtner’s scenario is introduced as an example in which the predicate “is restricted to events that can be subdivided along the relevant spatial dimension” later referred to as a “contextual restriction”. If the idea is to leave the universal quantifier but limit its domain to include the event in question and perhaps nothing more – then we may as well dispense with the universal altogether.

A non-universal version of the proposed meaning for for would look like this:

(4) Stratified Reference (Definition)

\[ \text{SR}_{f,e(K)}(P)(x) \equiv x \in *\lambda y (P(y) \land e(K(f(y)))) \]

(5) \[ \llbracket \text{for} \rrbracket = \lambda x \lambda M \lambda P \lambda e: \text{SR}_{\tau,\epsilon(M)}(P)(e). \ P(e) \land M(\tau(e)) \]

In (4)-(5), I’ve removed the universal from the definition of SR and I’ve adjusted the arguments supplied to it in \[ \llbracket \text{for} \rrbracket \]. I’ve preserved the basic assumption that for two hours has a modifier type, \( \langle v,t \rangle, \langle v,t \rangle \). It should be mentioned that that type assignment is hard to square with its ability to take scope above negation, as in ‘John didn’t laugh for two hours’ discussed in Champollion (2015:§3), where a temporal subinterval analysis of for two hours is employed.

3 The content of a stratified reference claim

3.1 Introduction

About an entity in the extension of a mass noun it is often said that every part of that entity is also in the extension of the noun. Using ‘x’ for the entity, ‘P’ for the noun and ‘\( \leq \)’ for ‘is a part of’, we can write that as:

(6) \( \forall y (y \leq x \rightarrow P(y)) \)

Similar kinds of things have been said about eventualities in the extensions of activity and stative predicates. Stratified reference is a sophisticated and innovative application of this way of thinking.

(6) is a quantificational statement. It is headed by an unrestricted universal quantifier over parts and it has ‘P(y)’ as its nuclear scope. In the coming sections, I’ll consider alternative quantifiers, then possible restrictions on the quantification and finally alternative nuclear scopes. The goal will be to gain a better understanding of stratified reference by locating it within the space of alternatives considered.

\(^{2}\) The earliest reference I’ve seen to this idea is Bello (1847).
3.2 Quantificational force

The quantifier in (7) below mixes existential quantification over sets containing parts of \( x \) with universal quantification over the elements of the set.

\[
\exists S \ (S \text{ is a set of parts of } x \text{ that sum up to } x \land \forall y \ (y \in S \rightarrow P(y)))
\]

This is the kind of quantification that is used in stratified reference. It’s encapsulated in the formulation ‘\( x \in \lambda y... \)’. The existential in (7) is the source for ‘can be divided’ in the paraphrase in (1)b. (7) is far from stratified reference, but my interest at the moment is just the quantificational force, and (7) suffices for that.

Our earlier (6) is universal and unrestricted and so it entails that \( P \) applies to absolutely any parts of \( x \), no matter how small. That is the minimal parts problem discussed in C.§2. The problem does not arise in (7) nor does it arise in stratified reference, in part because of the choice of quantifier. We’ll have a bit more to say about minimal parts at the end.

The existential formulation in (7) circumvents the ‘undergeneration’ problem associated with Champollion’s (33), repeated below:

(8) Five feet of snow fell on Berlin.

At issue is a proposal according to which the partitive subject brings with it a requirement that the portion of snow said to have fallen on Berlin, let’s call it \( a \), satisfies the following:

\[
\forall y \ (y \leq a \rightarrow \text{HEIGHT}(y) < \text{HEIGHT}(a))
\]

The snow that fell on West Berlin presumably has the same height as the snow that fell on Berlin – so (9) is falsified. The problem goes away when we replace the universal with the quantifier used in (7):

\[
\exists S \ (S \text{ is a set of parts of } a \text{ that sum up to } a \land \forall y \ (y \in S \rightarrow \text{HEIGHT}(y) < \text{HEIGHT}(a)))
\]

3.3 Restrictor

Neither (6) nor (7) places any restriction on the parts of \( x \) in the domain of quantification. One obvious restriction is to proper parts. The analysis of for adverbials in Kratzer (2007), cited in C.§2, incorporates (7) restricted to proper parts:

\[
\exists S \ (S \text{ is a set of proper parts of } x \text{ that sum up to } x \land \forall y \ (y \in S \rightarrow P(y)))
\]

Another possible restrictor limits the quantification to natural parts – those that correspond in some way to how we interact with the entity being divided or to how we perceive its formation. Schwarzschild (2006) provides empirical evidence that context and convention influence the selection of a measure function in a partitive. A context-dependent restrictor on the parts quantifier is a way to capture these effects.

The restriction to proper parts might be described as a formal restriction. The restriction to natural parts could be described as pragmatic. Now, I’d like to turn to a
possible metaphysical restriction having to do with the nature of events. To fix terminology, if \( e' \) is a subevent of \( e \) and the runtime of \( e' \) is less than that of \( e \), I'll call \( e' \) a temporal part of \( e \). Similarly, if \( e' \) is a subevent of \( e \) and the spatial trace of \( e' \) is less than that of \( e \), I'll call \( e' \) a spatial part of \( e \). I should add that an event \( e' \) could be both a temporal and a spatial part of \( e \). The possibility I want to raise is that adverbial stratified reference is limited to temporal parts of events, perhaps because proper event parts are necessarily temporal parts. The examples of non-temporal atelicity offered in the paper don’t allow us to detect this limitation because in all those cases, the temporal and the non-temporal are correlated. Recall the Gärtner scenario in which every few meters along the width of the store, there are carts at some distance from the wall and John’s task is to walk along the wall and push the carts in. In that case, each pushing is a part of John’s task that occupies both less space and less time than the task overall. Likewise, when John and Mary waltz for a couple of meters (13b), temporal parts of that event are spatial parts. In (39), stratified reference is applied to the verb phrase cool for five degrees and it makes reference to event parts characterized by small changes in temperature. Here there is a correlation between time and temperature change. Subevents in which there is a small drop in temperature are temporal parts of the event of cooling for five degrees. What we need are examples in which time is divorced from another dimension. Consider the following case. Jill lines up 36 equally sized bamboo poles, parallel and adjacent to each other, forming a square whose sides are 5 feet long. She has a block of concrete whose surface is 5’ x 5’ tied to the ceiling above the poles. At noon, she cuts the rope, the concrete falls and the poles are crushed at once. That event has spatial parts corresponding to pairs of poles, at least 18 of them, and, being spatial parts, they each take up less space than the entire event. Assuming we count two rods as considerably smaller than 5’ x 5’ we now should be able to say:

(12) \( * \) Jill crushed bamboo poles for five square feet.

Or consider Jack who places pretzel dough along a 2-foot peel at intervals of a few inches. He puts the peel in the oven and bakes for eight minutes. The result is an event of pretzel baking divisible into subevents corresponding to one or two pretzels, so we ought to be able to say:

(13) \( * \) Jack made pretzels for two feet.

Turning to a different dimension, suppose that in the laboratory, the water’s temperature decreased in a flash by 5 degrees. The event took almost no time, but if events truly have parts that aren’t temporal parts, then this event presumably has parts corresponding to small drops in temperature and so we ought to be able to say:

(14) \( * \) The water cooled for 5 degrees instantaneously.

I find the examples odd. A possible explanation is that for adverbials require stratified reference and the only parts that an event has, at least for this purpose, are temporal parts:

(15) Event partition is temporal (for adverbial stratified reference)
\[ e' < e \rightarrow \text{DURATION}(e') \prec \text{DURATION}(e). \]

(15) is a claim about the nature of events or a claim about how we measure amount in the event domain. In either case, if it is correct, we need to look to statives for examples of non-temporal atelicity. Champollion discusses the examples in (16) and (17). (18) is from Larson (2003)’s argument “that durative adverbs do not measure time, but rather event-stuff”.

(16) The road meanders for a mile. (Gawron 2005)
(17) The crack widens for 5 meters.
(18) {This amplifier is accurate up to 10,000Hz}. It is unstable for (the next) 5,000 Hz. {It fails entirely at 16,000Hz}.

For what it’s worth, I feel here “the distinctly metaphoric character” that Stump (1981:230) perceives in “the nontemporal sense of adverbially-used frequency adjectives” (‘an occasional sailor is over six feet tall’) and which he analyzes in temporal terms. Perhaps in (16)-(18), on the level of the metaphor, there are events with temporal parts that satisfy stratified reference.

The claim in (15) is that event parts are temporal parts. A more radical conclusion might have been that for adverbials are always temporal adverbials and that any putative non-temporal uses involve nonce temporal measure phrases. Just as we can use 10 minutes in a spatial way in ‘it’s 10 minutes from here’, likewise, under the right circumstances, we can use spatial and other non-temporal measure phrases to describe time intervals. On this view, when we say ‘we read for the first 10 miles of the trip’, we use 10 miles to describe the length of time it took to drive 10 miles. To test this idea, we’d like to see examples that don’t involve for adverbials (and examples without ‘the first’). I’m uncertain about the status of such cases:

(19) It took me 10 miles to solve that crossword puzzle.
(20) He finished the ice-cream in 10 miles.

3.4 Nuclear Scope
The chief locus of Champollion’s innovation is the nuclear scope of the stratified reference claim. In addition to requiring the parts that are quantified over to be in the extension of the predicate in question, there is also a requirement that the parts be smallish.

Before discussing the proposal itself, it will help to introduce some measurement jargon. Consider the following three statements:

(21) The temperature of the room was 50ºF.
(22) The temperature of the room was 10ºC
(23) The temperature of the room changed by 50ºF.

(21) reports the result of applying a measure function to the room. The measure function assigns a measurement to the room and we describe that measurement with a measure phrase, 50ºF. (21) and (22) report on the same measure function assigning the same measurement. They differ in the choice of the measure phrase used to describe that
measurement (10°C is equivalent to 50°F). (21) and (23) report on different measurements of the room and therefore different measure functions. We learn how cold the room was from (21) but not from (23). The measure function in play in (23) assigns a measure of temperature-change. Despite the fact that the measurements are different in (21) and (23), the same measure phrase is used to describe them. Champollion compares *five degrees (Celsius) of water with two degrees Celsius of global warming and concludes that “we cannot simply categorize measure functions as acceptable or unacceptable per se”. But these examples are like (21) and (23). They involve distinct measure functions but the same measure phrases. So the correct conclusion is that we can’t categorize measure phrases as acceptable or unacceptable per se. This conclusion is pertinent to the discussion in Krifka (1989:83) and Krifka (1998:202) where the extensive/intensive distinction is captured in the meanings of measure phrases.

Having clarified terms, the question before us now is this: what motivates reference to measure functions and measurements in stratified reference claims?

A good place to start on this question is (24), attributed above to Kratzer (2007) where it’s formalized as in (25)³

(24) \( \exists S \) (S is a set of proper parts of \( x \) that sum up to \( x \) \( \land \forall y \) (\( y \in S \rightarrow P(y) \))

(25) \( x \in \lambda y (P(y) \land y < x) \)

One can get pretty far with this formulation and even a bit further if we add the restriction in (15) to temporal parts⁴. But the lack of reference to measure renders this formulation insufficient to rule out *90° of water or *John drove for thirty miles an hour. And it won’t capture the squish in Zweig’s *twelve pounds of six-pound weights (C.§3, (48)). The intuition there was that although we can divide the six-pounders into two parts that fall under the extension of six-pound weights, the parts are too big for the overall measurement, 12 lbs. twelve pounds of four-pound weights is better because the parts are four-pounders which are sufficiently small for the overall measurement. Given these shortcomings in (24)/(25), let’s consider (26) which includes ‘f’ a variable over measure functions:

(26) \( \exists S \) (S is a set of parts of \( x \) that sum up to \( x \) \( \land \forall y \) (\( y \in S \rightarrow P(y) \land f(y) < f(x) \))

The restriction to proper parts is unnecessary with this formulation because if \( y \) is a part of \( x \) that measures less than \( x \), it must be a proper part. Except for the squish, I think every data point in C.§2 and C.§3 is covered with this formulation and would even be accommodated by the simpler version in (27):

³ There’s a similar idea in Zucchi and White (2001:252). The formulation in (24) brings out the similarity with the verification procedure in Carlson (1981:46) “Think for instance how one would go about assessing the truth of ‘He ran for four minutes.’ say, in an athletic event. What happens is that some sufficiently fine PARTITION (or perhaps just COVER) of the period of four minutes in terms of its proper subperiods is chosen, and the truth of ‘He runs.’ is ascertained in each member of it.”

⁴ (15) would be needed to rule out *Jack made pretzels for eight minutes’ in the scenario above where he baked a single peel of pretzels in eight minutes.
Consider the example of spatial and temporal atelicity. When John spends 50 minutes continually traversing the 50 meters from parking lot to store, each time taking a few carts with him, he executes an event that divides nicely into subevents of pushing carts all the way to the store each of which is not 50 minutes long. Stratified reference is satisfied in ‘John pushed carts all the way to the store for fifty minutes’. But since each of these subevents is, in some sense, 50 meters long, stratified reference is violated when minutes is changed to meters. As for granularity (examples (14) and (15) in C.§2), divide up a thousand years of transmission of cultural arts into component transmissions and the resulting parts are bound to be longer than the subloops making up a 36 second script loop. Big wholes may have big parts, but small wholes must have small parts.  

(27) \( \exists S \ (S \text{ is a set of parts of } x \text{ that sum up to } x \land \forall y \ (y \in S \rightarrow P(y) \land f(y) \neq f(x))) \)

Do partitives and for-adverbials require many parts or very small parts? These often come to the same thing – the more you cut the pie, the smaller the pieces get. But that is not always the case. It is possible to divide the pie in half and then continue to work exclusively on one of the halves. Suppose that Buffy throws a 6-pound weight in his bag, followed by a 2-pounder and then four 1 pounders. His bag now weighs 12 pounds. If 12 pounds of weights is acceptable as a description of Buffy’s bag contents, then the problem with Zweig’s twelve pounds of six-pound weights was not that the weights were both too big, but that there were too few of them. For an aspectual example, we return to Robby the robot. His first few steps are painfully slow but eventually he gets going and walks at a human rate. If he starts with a loooong 2 minute step and finishes up with 2 minutes of normal walking, I think we can say that he walked for 4 minutes. One or two of the big steps will not do the trick, but their supersize is not a problem as long as the overall step-count is high.

In (26)-(28), I made use of measure functions but not measurements. By contrast, the definition of stratified reference in (4) includes the term ‘\( \varepsilon(K)(f(y)) \)’ in which ‘\( K \)’

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5 There is another granularity effect that is not addressed here or in the paper. It has to do with the interstices between the events making up the mereological wholes whose dimensions are measured. A series of yawns, separated by a few minutes, might together form a yawning event that measures 10 minutes. But two yawns an hour apart do not sum to a one-hour yawning event. Change from yawning to the transmission of cultural arts and the spaces get bigger.

6 “de Hoop and de Swart showed that clauses must delimit multiple cases in order to restrict an adverbial quantifier” Fernald (2000:20) [emphasis mine].
stands for a measurement. I was able to use only measure functions because I began by rejecting the universal quantifier over the predicate’s extension. If we put it back in (26) we get:

\[ \forall x \ (P(x) \rightarrow \exists S \ (S \text{ is a set of parts of } x \text{ that sum up to } x \land \forall y \ (y \in S \rightarrow P(y) \land f(y) < f(x))) \]

(29) has the unwanted entailment that \( P \) is infinitely divisive – anything that is \( P \) has a proper part that is \( P \). This is the aforementioned minimal parts problem. Placing the numerosity restriction from (28) under the universal would lead to the same negative consequence:

\[ \forall x \ (P(x) \rightarrow \exists S \ (S \text{ is a set of parts of } x \text{ that sum up to } x \land |S| > 2 \land \forall y \ (y \in S \rightarrow P(y)))) \]

4. Summary

At the end of his paper, Champollion cites the potential of stratified reference in a solution to the problem of non-quantized predicates that “empirically pattern with telic predicates”. In this final section, I discuss an instance of that problem. The discussion makes reference to key points made earlier and should serve as a summary my comments.

Jack has been tasked with painting colored lines on the back wall of every room in the hotel. It takes him nearly an hour to paint the 16 lines each wall requires. He creates a device consisting of a long rod on which 16 paint brushes are mounted. He drags the device across the wall from top to bottom and the result is 16 lines painted on the wall. The whole affair now takes only 30 minutes. After completing the first wall of the day, Jack declares “I painted 16 lines on the wall in 30 minutes”. He would not say:

\[ ^{31} * \text{I painted 16 lines on the wall for 30 minutes.} \]

and even (32) sounds a bit odd in this context:

\[ ^{32} * \text{I painted lines on the wall for 30 minutes.} \]

The event of painting the wall divides into a sufficient number of subevents in each of which two lines are painted but these subevents fail to satisfy stratified reference. In (31), they’re ruled out because, among other things, they are not events of painting 16 lines. This problem is removed in (32) but others remain. These two-line subevents are not temporal parts of the big event and even if they were, their duration is not shorter than the duration of the entire wall painting, as stratified reference requires. But suppose now that we divide the wall painting into three distinct 10-minute units: top, middle and bottom. If we grant that part of a line is a line, then these subevents are events of painting lines and, given Jack’s apparatus, they are events of painting 16 lines. Since these events are small temporal parts of the main event, stratified reference should be

\[ ^{7} \text{ A value for ‘K’ can be a set of temporal intervals, spatial extents, degrees, or numbers (see text between (16) and (17)). If this doesn’t correspond exactly to the notion of measurement introduced above, it’s close enough for current discussion.} \]
satisfied by them in both (31) and (32). It should be satisfied, but it apparently isn’t. This is the problem of non-quantized predicates that “empirically pattern with telic predicates”. In (32), the problem arises with a bare plural object. This is unexpected on the accounts in Carlson (1981:61) and Zucchi and White (2001) but probably not on the account in Rothstein (2004:Chs 6-7) (see also Filip 2008).

References