

Integer Programming: Optimization and Evaluation are Equivalent

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Abstract We show that if one can find the optimal value of an integer programming problem $\min\{cx : Ax \geq b, x \in \mathbb{Z}_+^n\}$ in polynomial time, then one can find an optimal solution in polynomial time. We also present a proper generalization to general integer programs and to local search problems of the well-known result that optimization and augmentation are equivalent for 0/1-integer programs. Our results imply that, among other things, PLS-complete problems cannot have “near-exact” neighborhoods, unless $\text{PLS} = \text{P}$.

1 Introduction

The following question arises naturally in the study of optimization problems and has often been posed (see, e.g., [12, Ch. 15.2]): Is computing the value of an optimal solution as hard as actually finding an optimal solution? Crescenzi and Silvestri initiated the formal study of the relative complexity of evaluating the optimal cost of an optimization problem versus constructing an optimal solution and provided sufficient and necessary conditions for the existence of optimization problems for which obtaining an optimal solution is harder than computing the optimal value [4]. Ausiello et al. and Johnson pointed out that evaluation is actually as hard as finding an optimal solution for all optimization problems whose associated decision problems are NP-complete [2,8]. Schulz reports on the relative complexity of 15 problems related to 0/1-integer programming,³ including augmentation, optimization and evaluation, and shows that they are all oracle-polynomial-time equivalent [16]. In this paper, we prove that evaluation and optimization are polynomial-time equivalent for all integer linear programming problems. That is, given a matrix $A \in \mathbb{Z}^{m \times n}$ and a vector $b \in \mathbb{Z}^m$, a polynomial-time algorithm for finding the optimal value of $\min\{cx : Ax \geq b, x \in \mathbb{Z}_+^n\}$, for any $c \in \mathbb{Z}^n$, implies the existence of such an algorithm for finding an optimal solution, $\arg \min\{cx : Ax \geq b, x \in \mathbb{Z}_+^n\}$. In fact, our result is slightly stronger than this. As long as we are given bounds on the values that individual variables may attain, the matrix A and the vector b need not be explicitly known. An evaluation oracle, which accepts as input any objective function vector c and returns the optimal objective function value, suffices. Our proof is constructive.

³ A 0/1-integer programming problem is one where the variables can have values 0 or 1 only.

The proof itself gives rise to a new problem, related to questions typically brought up in postoptimality analysis of optimization problems, which we call the “unit increment problem:” Given an optimal solution x^* with respect to an objective function vector c , find an optimal solution for $c+e_j$, where e_j is the j -th unit vector.⁴ We show that an integer linear program can be solved in polynomial time if and only if its unit increment problem can be solved in polynomial time. For 0/1-integer programs, we prove that the unit increment problem is equivalent to the augmentation problem.⁵ Hence, we have a proper generalization of a result by Grötschel and Lovász and Schulz et al. who showed that optimization and augmentation are polynomial-time equivalent for 0/1-integer programs [6,17]. A relaxation to the augmentation problem, the ϵ -augmentation problem, can be defined as follows: Given an objective function vector c and a feasible solution x , find a feasible solution with better objective function value or assert that x is ϵ -optimal. (Here, $\epsilon > 0$, and a solution x is ϵ -optimal if $cx \leq (1 + \epsilon)cx'$ for all feasible solutions x' .) The corresponding unit increment problem, the ϵ -unit increment problem, is defined as follows: Given an index j and an ϵ -optimal solution with respect to an objective function vector c , find an ϵ -optimal solution for $c+e_j$. We show that an ϵ -optimal solution can be obtained in polynomial time if and only if the ϵ -unit increment problem can be solved in polynomial time. Moreover, we show that for 0-1 integer programs, the ϵ -augmentation problem and the ϵ -unit increment problem are polynomial-time equivalent as well.

The concepts of unit increment and augmentation extend naturally to local search, with interesting implications. For an integer programming or combinatorial optimization problem with a neighborhood function N , the local augmentation problem, given a feasible solution x and an objective function vector c , asks for a solution in the neighborhood of x , $N(x)$, of better objective function value, if one exists. The local unit increment problem is defined similarly: Given an index j and a locally optimal solution x with respect to c , find a locally optimal solution for $c + e_j$. We show that for a given neighborhood function, a locally optimal solution can be computed in polynomial time if and only if the local unit increment problem can be solved in polynomial time. However, in contrast to the cases of global optimization and ϵ -optimization, for 0/1-integer programs, the local unit increment problem and the local augmentation problem are not known to be equivalent. In fact, it follows from our results that if polynomial solvability of the local augmentation problem implies the polynomial solvability

⁴ In this part of the paper we assume, for convenience, that all objective function coefficients are nonnegative. Most of our results hold true in general, if the unit increment problem is extended to finding optimal solutions for $c \pm e_j$.

⁵ The augmentation problem is defined as follows: Given a feasible solution and an objective function vector, find a feasible solution of better objective function value, if one exists.

of the local unit increment problem, then all PLS-complete⁶ problems can be solved in polynomial time.

A neighborhood function is said to be “exact” if every locally optimal solution is guaranteed to be globally optimal. A neighborhood function is said to be “near exact” if every locally optimal solution is no worse than all but a polynomial number of feasible solutions. Near-exact neighborhoods are related to the domination number of local search heuristics [7]. We show that, for 0/1-integer programs, polynomial solvability of the local augmentation problem implies polynomial solvability of the local optimization problem whenever the corresponding neighborhood is near exact. This implies that no PLS-complete problem can possess a near-exact neighborhood, unless $\text{PLS} = \text{P}$.

The rest of the paper is organized as follows. In Section 2 we establish that optimization and evaluation are polynomial-time equivalent for integer programming problems. In Section 3, we show that the unit increment problem and the optimization problem are polynomial-time equivalent. We also give a direct proof that, for 0/1-integer programming problems, augmentation and unit increment are polynomial-time equivalent as well. In Section 4 we extend this to the cases of ϵ -optimization, ϵ -augmentation, and ϵ -unit increment. Section 5 contains our results on local search; in particular, we show that even for 0/1-integer programs, a local unit increment oracle is stronger than a local augmentation oracle, unless $\text{PLS} = \text{P}$.

2 Evaluation versus optimization

It is well-known (see, e.g., [15, Ch. 17.1]) that if an integer program has a finite optimum, then it has an optimal solution of size (i.e., encoding length) polynomially bounded by the size of the input. Hence, instead of considering $\min\{cx : Ax \geq b, x \in \mathbb{Z}_+^n\}$, we may restrict ourselves to solving $\min\{cx : Ax \geq b, x \leq u, x \in \mathbb{Z}_+^n\}$, for a vector $u \in \mathbb{Z}_+^n$ whose encoding length is polynomial in that of A , b , and c . So from now on, we consider a family \mathcal{F} of integer programming problems that is described as follows. For each instance of the family we are given a vector $u \in \mathbb{Z}_+^n$ such that the set $X \subseteq \mathbb{Z}^n$ of feasible solutions is contained in $\{0, 1, \dots, u_1\} \times \{0, 1, \dots, u_2\} \times \dots \times \{0, 1, \dots, u_n\}$. We are also given an evaluation oracle that contains the only additional information that we have on X .⁷ (In particular, we do not explicitly need to know a matrix A and a vector b such that $X = \{x \in \mathbb{Z}_+^n : Ax \geq b, x \leq u\}$.) For input vector $c \in \mathbb{Z}^n$,

⁶ The complexity classes PLS and PLS-complete were introduced by Johnson et al. to capture the difficulty of finding local optima [9]. Prominent PLS-complete problems include the max-cut problem with the flip neighborhood and the graph partitioning problem with the swap neighborhood [14], the traveling salesman problem with the k -exchange neighborhood (for sufficiently large, but constant k) [10], and the problem of finding pure-strategy Nash equilibria in congestion games [5].

⁷ We may assume, without loss of generality, that there exists a feasible solution, i.e., $X \neq \emptyset$. Otherwise both problems, evaluation and optimization, would have to detect infeasibility.

the oracle returns the optimal objective function value of $\min\{cx : x \in X\}$. The following is our main result.

Theorem 1. *Given a family \mathcal{F} of integer programming problems described by an evaluation oracle, there is an oracle-polynomial time algorithm for solving the optimization problem.*

Proof. Let $\min\{cx : x \in X\}$ be the optimization problem to be solved, given by an evaluation oracle and a vector $u \in \mathbb{Z}_+^n$ such that $X \subseteq \{0 \leq x \leq u\}$. The main idea is as follows. Among all optimal solutions, let x' be the one that is lexicographically minimal. We perturb c in such a way that x' remains optimal for the perturbed vector c' and is also optimal for the objective function vectors $c' + e_j$, for all $j = 1, 2, \dots, n$. With $n + 1$ calls of the evaluation oracle we can then recover x' via $x'_j = (c' + e_j)x' - c'x'$, for $j = 1, 2, \dots, n$. If the size of c' is sufficiently small, this yields an oracle-polynomial time algorithm.

Here are the details. Let $U := \max\{u_j : j = 1, 2, \dots, n\} + 1$. We define c' as follows:

$$c'_j := U^{2n+1}c_j + U^{2(n-j)+1}, \text{ for } j = 1, 2, \dots, n.$$

Note that the encoding length of c' is indeed polynomial in that of c and u . We first show that, (i), every solution x^* that is optimal for c' is also optimal for c . In fact, for any $x \in X$, we obtain that

$$cx^* \leq \frac{c'x^*}{U^{2n+1}} \leq \frac{c'x}{U^{2n+1}} = cx + \frac{\sum_{j=1}^n U^{2(n-j)+1}x_j}{U^{2n+1}} < cx + 1.$$

Together with the integrality of c , x^* , and x , this implies that $cx^* \leq cx$, proving (i). We now show that, (ii), if x is an optimal solution for c that is different from x' , then $c'(x' - x) \leq -U$. Let i be the first index for which $x'_i < x_i$. Then,

$$\begin{aligned} c'(x' - x) &= U^{2(n-i)+1}(x'_i - x_i) + \sum_{j=i+1}^n U^{2(n-j)+1}(x'_j - x_j) \\ &\leq -U^{2(n-i)+1} + \sum_{j=i+1}^n U^{2(n-j)+2} \\ &\leq -U. \end{aligned}$$

It remains to show that x' is optimal for $c' + e_j$, for an arbitrary, but fixed index $j \in \{1, 2, \dots, n\}$. So, let x be some feasible solution different from x' . We distinguish two cases. If x is optimal for c , then, with the help of (ii), we get

$$(c' + e_j)(x' - x) = c'(x' - x) + (x'_j - x_j) \leq -U + U = 0.$$

If x is not optimal for c , we have

$$\begin{aligned} c'(x' - x) &= U^{2n+1}c(x' - x) + \sum_{j=1}^n U^{2(n-j)+1}(x'_j - x_j) \\ &\leq -U^{2n+1} + \sum_{j=1}^n U^{2(n-j)+2} \leq -U. \end{aligned}$$

Hence, $(c' + e_j)(x' - x) \leq 0$. Thus, x' is optimal for c' and $c' + e_j$, and $x'_j = (c' + e_j)x' - c'x'$. In particular, the j -th component of x' can be computed by two calls of the evaluation oracle. \square

3 The unit increment problem and global optimization

In this section we assume that all objective function vectors are nonnegative, for convenience. All results can be extended in a straightforward way to arbitrary objective function vectors. If c is an objective function vector, let $C := \max\{c_j : j = 1, 2, \dots, n\}$ and $\alpha := 1 + \lceil \log C \rceil$. Then each c_j can be represented as a binary number using α bits. Let b^j be this representation. Moreover, let c_j^k be the number represented by the k leading bits of b^j . That is, $c_j^k := \sum_{i=1}^k 2^{k-i} b_i^j$. Thus $c_j^1 \in \{0, 1\}$, $c_j^\alpha = c_j$, and $c_j^{k+1} = b_{k+1}^j + 2c_j^k$ for all $k = 1, 2, \dots, \alpha - 1$ and $j = 1, 2, \dots, n$.

Let $\min\{cx : x \in X\}$ with $X \subseteq \mathbb{Z}_+^n$ be an instance of the optimization problem. We assume that an oracle UNIT-INC is available which with input j , $c + e_j$, and an optimal solution x^1 w.r.t. c computes an optimal solution x^2 for $\min\{(c + e_j)x : x \in X\}$. We consider the following algorithm.

Algorithm UI

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begin
  Let  $x^1$  be any feasible solution.
  set  $c_j^* := 0$  for  $j = 1$  to  $n$ 
  for  $k = 1$  to  $\alpha$  do
    for  $j = 1$  to  $n$  do  $c_j^* := 2c_j^*$ 
     $S := \{j : b_k^j = 1\}$ 
    while  $S \neq \emptyset$  do
      choose  $j \in S$ 
       $S := S \setminus \{j\}$ 
       $c_j^* := c_j^* + 1$ 
      If  $x_j^1 > 0$  then
        Call UNIT-INC( $c^*, x^1, x^2, j$ )
         $x^1 := x^2$ 
      endif
    endwhile
  endfor
  output  $x^1$ 
end
    
```

Theorem 2. *Let a family of optimization problems with linear objective functions be given by a unit-increment oracle. Then algorithm UI computes an optimal solution in oracle-polynomial time.*

Proof. Assume that at the beginning of the k -th iteration of the main loop, x^1 is an optimal solution to $\min\{c^{k-1}x : x \in X\}$. Then x^1 continues to be an optimal

solution if we change the objective function to $2c^{k-1}$. Thus at the end of the while loop, the oracle UNIT-INC guarantees that x^1 is an optimal solution to $\min\{c^k x : x \in X\}$. The correctness of the algorithm follows by induction over k . \square

For the assignment problem on a bipartite graph on $2n$ nodes and m edges, the general algorithm described above terminates in $O(nm \log C)$ time. This is because the unit increment problem for this special case can be solved in $O(m)$ time. Although there are special purpose algorithms with better worst case bounds to solve the assignment problem, it is interesting to note that the general algorithm UI achieves a good time bound.

Algorithm UI is a generalization of the bit scaling algorithm studied extensively in the network flow literature (see, e.g., [1]). The new feature here is the use of the unit increment oracle. This allows us to compare the relative difficulties of optimization and unit increment problems and also provides a framework for our study of ϵ -optimization and local optimization. Theorem 2 establishes that an optimization problem with linear objective function can be solved in polynomial time if and only if the corresponding unit increment problem can be solved in polynomial time. Alternatively, if the optimization problem is NP-hard then the corresponding unit increment problem is also NP-hard. Thus the additional information available for the unit increment problem is not of much help for NP-hard problems. This provides additional evidence that postoptimality analysis is typically hard for NP-hard problems (see, e.g., [3,18,13] for related results).

We now examine the relationship between the unit increment problem and the augmentation problem. Let x^0 be an optimal solution to $\min\{cx : x \in X\}$, and let j be a given index, $1 \leq j \leq n$.

Lemma 3. *If x^* is an optimal solution to $\min\{(c + e_j)x : x \in X\}$, then $(c + e_j)(x^0 - x^*) \leq x_j^0$.*

Proof. Since x^* is a feasible solution to $\min\{cx : x \in X\}$, $cx^0 \leq cx^*$. Thus $(c + e_j)x^0 = cx^0 + x_j^0 \leq cx^* + x_j^0 \leq (c + e_j)x^* + x_j^0$. \square

The next theorem is, in principle, a consequence of the before-mentioned equivalence between augmentation and optimization for 0/1-integer programs, and Theorem 2. However, the following proof provides a Karp reduction from the unit increment problem to the augmentation problem.

Theorem 4. *For 0/1-integer programs, the unit increment problem and the augmentation problem are polynomial-time equivalent.*

Proof. Assume that $X \subseteq \{0,1\}^n$ is given by an augmentation oracle. Consider the instance $\min\{cx : x \in X\}$ and its unit increment version $\min\{(c + e_j)x : x \in X\}$ together with respective optimal solutions x^0 (given) and x^* (unknown). By Lemma 3,

$$(c + e_j)(x^0 - x^*) \leq 1. \quad (1)$$

Since $c \in \mathbb{Z}^n$, one application of the augmentation oracle starting with x^0 either declares that x^0 is optimal for $\min\{(c + e_j)x : x \in X\}$ or finds an improving solution which must be optimal for $\min\{(c + e_j)x : x \in X\}$ in view of (1). The other direction is implied by Theorem 2. \square

Using Lemma 3 we have the following result for general integer programs. Let $\min\{cx : x \in X\}$ be an instance and x^0 be an optimal solution. Let $\min\{(c + e_j)x : x \in X\}$ be the corresponding j -th unit increment instance.

Theorem 5. *Given $x^0 \in \arg \min\{cx : x \in X\}$ and an augmentation oracle, $\min\{(c + e_j)x : x \in X\}$ can be solved by $O(x_j^0)$ calls of the augmentation oracle.*

Theorems 2 and 5 show that for an integer linear program for which $x \leq u$ for all $x \in X$ where the components of u are bounded above by a polynomial of the remaining input data, the optimization problem can be solved in polynomial time whenever the augmentation problem can be solved in polynomial time.

4 The unit increment problem and ϵ -optimization

In this section we explore the approximate solvability of optimization problems in relation to ϵ -augmentation and ϵ -unit increment oracles. We need the assumption that $c_j \geq 0$ for all $j = 1, 2, \dots, n$. We also fix $\epsilon > 0$. Let $UI(\epsilon)$ denote the variation of the algorithm UI where the oracle $UNIT-INC$ is replaced by ϵ - $UNIT-INC$ which takes as input $c + e_j$, an ϵ -optimal solution x^1 of $\min\{cx : x \in X\}$, an index j , and computes an ϵ -optimal solution x^2 of $\min\{(c + e_j)x : x \in X\}$. Using arguments similar to that in the proof of Theorem 2 one can show the following result.

Theorem 6. *Given an ϵ -unit increment oracle and an initial feasible solution, algorithm $UI(\epsilon)$ computes an ϵ -optimal solution to $\min\{cx : x \in X\}$ in oracle-polynomial time.*

Thus if a feasible solution can be computed in polynomial time and the ϵ -unit increment problem can be solved in polynomial time, then an ϵ -optimal solution can be obtained in polynomial time. Alternatively, an optimization problem is not approximable if and only if the corresponding unit increment problem is not approximable. This result is interesting in several ways. For example, even if we have an ϵ -optimal solution to the traveling salesman problem, if one of the edge weights is increased by one, then getting an ϵ -optimal solution is still NP-hard. Also, there exists a (fully) polynomial approximation scheme for an optimization problem if and only if there is a (fully) polynomial approximation scheme for the unit increment problem.

Interestingly, we can show that the ϵ -augmentation problem and the ϵ -unit increment problem are equivalent for 0/1-integer programs.

Theorem 7. *For 0/1-integer programs, the ϵ -augmentation problem and the ϵ -unit increment problem are polynomial-time equivalent.*

Proof. Consider an instance $\min\{cx : x \in X\}$ with $X \subseteq \{0,1\}^n$ and its j -th unit increment instance $\min\{(c + e_j)x : x \in X\}$. Let x^0 and x^* be optimal solutions of the former problem and the latter problem, respectively. Let x^1 be an ϵ -optimal solution to $\min\{cx : x \in X\}$. If x^1 is declared as an ϵ -optimal solution to $\min\{(c + e_j)x : x \in X\}$ by the ϵ -augmentation oracle, the proof is over. Thus suppose that starting with the solution x^1 to $\min\{(c + e_j)x : x \in X\}$ the ϵ -augmentation oracle produces an improved solution, say x^2 . We will show that x^2 is an ϵ -approximate solution to $\min\{(c + e_j)x : x \in X\}$. By Lemma 3 we have

$$(c + e_j)x^* = cx^0 \text{ or } (c + e_j)x^* = cx^0 + 1. \quad (2)$$

Since x^1 is ϵ -optimal for $\min\{cx : x \in X\}$,

$$\frac{cx^1 - cx^0}{cx^0} \leq \epsilon.$$

Case 1: $x_j^1 = 1$. In this case $(c + e_j)x^1 = cx^1 + 1$. Since x^2 is an improved solution for $\min\{(c + e_j)x : x \in X\}$ obtained from x^1 , $(c + e_j)x^2 < (c + e_j)x^1$ and hence

$$(c + e_j)x^2 \leq cx^1.$$

From (2) we have $(c + e_j)x^* = cx^0$ or $(c + e_j)x^* = cx^0 + 1$. If $(c + e_j)x^* = cx^0$, then

$$\frac{(c + e_j)x^2 - (c + e_j)x^*}{(c + e_j)x^*} \leq \frac{cx^1 - cx^0}{cx^0} \leq \epsilon.$$

If $(c + e_j)x^* = cx^0 + 1$, then

$$\frac{(c + e_j)x^2 - (c + e_j)x^*}{(c + e_j)x^*} \leq \frac{cx^1 - cx^0 - 1}{cx^0 + 1} \leq \frac{cx^1 - cx^0}{cx^0} \leq \epsilon.$$

Case 2: $x_j^1 = 0$. In this case $(c + e_j)x^1 = cx^1$. We will show that x^1 is an ϵ -optimal solution to $\min\{(c + e_j)x : x \in X\}$. If $(c + e_j)x^* = cx^0$, then

$$\frac{(c + e_j)x^1 - (c + e_j)x^*}{(c + e_j)x^*} = \frac{cx^1 - cx^0}{cx^0} \leq \epsilon.$$

If $(c + e_j)x^* = cx^0 + 1$ then,

$$\frac{(c + e_j)x^1 - (c + e_j)x^*}{(c + e_j)x^*} = \frac{cx^1 - cx^0 - 1}{cx^0 + 1} \leq \frac{cx^1 - cx^0}{cx^0} \leq \epsilon.$$

Thus if the ϵ -augmentation oracle does not declare x^1 as ϵ -optimal, the improved solution x^2 is guaranteed to be ϵ -optimal for $\min\{(c + e_j)x : x \in X\}$.

The converse of the theorem is straightforward. \square

One consequence of the above theorem is that a 0/1-integer program has a (fully) polynomial-time approximation scheme if and only if the corresponding augmentation problem has a (fully) polynomial-time approximation scheme. The same result was obtained by Orlin et al. using quite different arguments [11].

5 The unit increment problem and local optimization

In this section we consider the complexity of computing a locally optimal solution with respect to a given neighborhood function N . Recall that the *local* augmentation problem has as input a feasible solution x and an objective function vector c , and it outputs a solution $y \in N(x)$ with $cy < cx$, unless x is already a local optimum. The *local* unit increment problem accepts as input j and a locally optimal solution x^1 with respect to c , and returns a locally optimal solution x^2 with respect to $c + e_j$.

As in the case of (global) optimization and ϵ -optimization, we first observe that if a feasible solution can be obtained in polynomial time and the local unit increment problem can be solved in polynomial time, then a local optimum can be computed in polynomial time. To establish this, we simply modify algorithm UI by replacing the unit increment oracle, UNIT-INC, with a local unit increment oracle, LOCAL-UNIT-INC. We call the resulting algorithm LUI.

Theorem 8. *Given a LOCAL-UNIT-INC oracle, algorithm LUI computes a locally optimal solution in oracle-polynomial time.*

The proof of Theorem 8 is similar to that of Theorem 2. Theorem 8 establishes that the complexity of finding a local optimum is captured by that of the local unit increment problem.

Unlike the case of optimization and ϵ -optimization, we are not able to establish the equivalence of the local unit increment problem and the local augmentation problem for 0/1-integer programs. In fact, if they are equivalent, then, by Theorem 8, there is a polynomial-time algorithm for finding a local optimum for any problem in PLS, including PLS-complete problems. In other words, this would imply $\text{PLS} = \text{P}$. However, the two problems are equivalent if the neighborhood is exact. This follows from Theorem 4. Interestingly, we can show that this is also true for near-exact neighborhoods. Recall from the introduction that a neighborhood is called near exact if the objective function value of every local optimum is worse than that of at most a polynomial number of other feasible solutions.

Theorem 9. *For 0/1-integer programs with near-exact neighborhoods, a polynomial-time algorithm for local augmentation implies a polynomial-time algorithm for the local unit increment problem.*

Proof. Let x^0 be a locally optimal solution with respect to the near-exact neighborhood N and the objective function vector c . As usual, X denotes the set of feasible solutions.

Since N is near exact, there exists $X^* \subseteq X$ such that $cx^0 \leq cx$ for all $x \in X^*$ and $|X \setminus X^*| \leq f(n)$ for some polynomial f . By Lemma 3, $(c + e_j)(x^0 - x) \leq 1$ for all $x \in X^*$. Thus in one augmentation step, starting from x^0 , we get a solution which is no worse than any solution in X^* w.r.t $(c + e_j)$. This solution may or may not be a local optimum with respect to N . But outside X^* there are only $f(n)$ solutions and hence the local augmentation oracle cannot be called more than $f(n)$ more times before reaching a local optimum. \square

Corollary 10. *If there exists a near-exact neighborhood for a PLS-complete problem, then there is a polynomial-time algorithm that finds a local optimum for all problems in PLS. That is, $PLS = P$.*

However, near-exact neighborhoods are unlikely to exist, at least for the TSP [7].

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