Schedule of Talks

Talks highlighted in **Red** are survey/tutorial style talks.

Time Slot	*Monday, July 10, 2006*	*Tuesday, July 11, 2006*	*Wednesday, July 12, 2006*
9:00 – 10:00	*BREAKFAST*	*BREAKFAST*	*BREAKFAST*
10:00 – 10:30	*Opening Remarks*	Steven Smith	James Sethna
10:30 – 11:00	**Arthur Baggeroer**	Debashis Paul	Marc Timme
11:00 – 11:30	Dennis McLaughlin	Robb Muirhead	Plamen Koev
11:30 – 12:00	**Iain Johnstone**	Yasuko Chikuse	Matthew Harding
12:00 – 1:00	*LUNCH*	*LUNCH*	*LUNCH*
1:00 – 1:30	Noureddine ElKaroui	**Arno Kuijlaars**	**Duck Tours Excursion**
1:30 – 2:00	Momar Dieng	**Arno Kuijlaars**	**Duck Tours Excursion**
2:00 – 2:30	**Roland Speicher**	Petr Seba	**Duck Tours Excursion**
2:30 – 3:00	**Roland Speicher**	Jinho Baik	**Dessert at Finale**
3:00 – 3:30	*BREAK*	*BREAK*	**Dessert at Finale**
3:30 – 4:00	Christ Richmond	Joseph Guerci	**Dessert at Finale**
4:00 – 4:30	Olivier Besson	Alexei Onataski	
4:30 – 5:00	Daniel Bliss	Sourav Chatterjee	
5:00 – 5:30	Jack Silverstein	**Percy Deift**	
5:30 – 6:00	Reserved	**Percy Deift**	

Time Slot	*Thursday, July 13, 2006*	*Friday, July 14, 2006*
9:00 – 10:00	*BREAKFAST*	*BREAKFAST*
10:00 – 10:30	Brian Sutton	Roland Speicher
10:30 – 11:00	Brian Rider	Michael Anshelevich
11:00 – 11:30	Güler Ergün	Vladislav Kargin
11:30 – 12:00	Zdzislaw Burda	*Closing Remarks*
12:00 – 1:00	*LUNCH*	
1:00 – 1:30	Eitan Bachmat	
1:30 – 2:00	Tomohiro Sasamoto	
2:00 – 2:30	**Michael Anshelevich**	
2:30 – 3:00	**Michael Anshelevich**	
3:00 – 3:30	*BREAK*	
3:30 – 4:00	James Mingo	
4:00 – 4:30	Ioana Dumitriu	
4:30 – 5:00	Reza Rashidi Far	
5:00 – 5:30	Nizar Demni	

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Opportunities at the interface of signal processing and stochastic eigen-analysis

Arthur Baggeroer

Massachusetts Institute of Technology

The history of signal processing, (we think of Wiener), is filled with examples of mathematicians and engineers working together on compelling real-world applications. These collaborations clarified, elaborated and elevated the theory and practice of signal processing to its present day stature.

The recent developments in stochastic eigen-analysis appear to be on the verge of similarly triggering a new wave of applications. This talk will provide some examples from the past and offer additional real-world scenarios (with an emphasis on array signal processing) where collaborations between mathematicians and engineers can yield immediate dividends. The hope is that, as before, they will eventually pave the way for compelling applications in other areas of science and engineering.

Random Matrix Applications in Environmental Data Assimilation

Dennis McLaughlin, Adel Ahanin, and Dara Entekhabi

Massachusetts Institute of Technology

The dramatic increase in data from remote and in situ sensing presents important opportunities and challenges for environmental science. In order to properly interpret all this data, large numbers of diverse observations must be processed to give descriptions of environmental variables that are dynamically consistent over time and space. This is typically accomplished by using dynamic models (usually based on nonlinear partial differential equations) to merge observations (a process commonly called "data assimilation" or "data fusion"). Since uncertainties are significant data assimilation problems are often posed in a Bayesian probablistic framework.

An important aspect of these problems is the need to propagate probabilistic information (e.g. probability densities or some of their moments) over time. The most commonly used options are i) Monte Carlo (ensemble) methods, ii) methods based on linearization and implicit Gaussian assumptions, and iii) hybrid combinations of these two approaches.

Ensemble and hybrid methods provide sample estimates of probabilistic quantities that are based on random matrices with a relatively small number of columns, each corresponding to a particular replicate of a very large (order 10^6 or more) state vector. Results from random matrix theory may be used to assess the accuracy of these sample estimates and to decide how many replicates are needed for a given application.

In this paper we consider how hybrid propagation methods may improve the validity of implicit Gaussian assumptions used both in approximate Bayesian conditioning calculations and in accuracy assessments that rely on random matrix theory. This investigation suggests some possible topics for future collaboration between mathematicians interested in sampling and random matrix theory and environmental scientists working on large nonlinear data assimilation problems.

Monday 11:30am - 12:00pm

Random Matrices and Multivariate Statistical Analysis

Iain Johnstone

Stanford University

The talk aims at an introductory account of some of the topics of classical multivariate statistical analysis to which the approaches, tools and results of random matrix theory have contributed. A number of canonical multivariate methods are founded on eigen-decompositions of one or two independent Wishart matrices. Using hypothesis tests as an organizing theme, we review how the null hypothesis distributions are linked to the Laguerre and Jacobi ensembles of RMT. Results for linear statistics and extremes of the eigenvalues yield information about likelihood ratio and union-intersection tests respectively.

*survey

Recent results on the behavior of the largest eigenvalue of large dimensional sample covariance matrices and statistical applications

Noureddine El Karoui

University of California Berkeley

In modern statistical practice, one often encounters $n \times p$ data matrices X with n and p both large. Classical results from multivariate statistical analysis (Anderson 1963) fail to give good approximations in this setting.

Using random matrix theory, Johnstone (2001) recently shed light on some theoretical aspects of Principal Component Analysis in this setting. He specifically showed that under some conditions on the dimensions of the matrix X, if it had i.i.d Normal(0,1) entries, the largest eigenvalue of the sample covariance matrix X'X, properly rescaled and recentered converges to a Tracy-Widom law.

We will discuss several related results. First, we will explain that this convergence result holds as long as n and p go to infinity, removing some of the dimensionality conditions. In the case of complex normal entries, which were investigated by Forrester (93), Johansson (2000) and Johnstone (2001), we will discuss rates of convergence issues which practically improve the quality of the asymptotic approximation.

We will also discuss the case where the entries of the matrix are correlated with a fairly general covariance structure, including for instance well-behaved Toeplitz matrices. We will see that Tracy-Widom limits also appear in this fairly general alternative setting, and give numerically explicit formulas for the centering and scaling of the largest eigenvalue.

Finally, time permitting, we will illustrate how these and related theoretical insights might be used in practice.

Edgeworth –Type Expansions for the Distribution Functions in the Gaussian Ensembles

Momar Dieng

The University of Arizona.

Recently L. Choup derived expansions for Hermite and Laguerre kernels at the edge of the spectrum of the finite n Gaussian Unitary Ensemble (GUE_n) and the finite n Laguerre Unitary Ensemble (LUE_n), respectively. This yields Edgeworth-type expansions for the largest eigenvalue distribution function of GUE_n and LUE_n. These expansions have the F_2 Tracy–Widom distribution as their leading term and provide corrections to it. We discuss these results and the analogues we obtained for the OE and SE cases. This is joint work with L. Choup.

Free Probability Theory and Random Matrices

Roland Speicher

Queen's University

Assume you are given two symmetric random matrices A and B and you know the eigenvalue distribution of A and the eigenvalue distribution of B. What can you say about the eigenvalue distribution of $A + B$ or of AB? In general, not much since the relation between the eigenspaces of A and the eigenspaces of B is relevant. However, in typical cases and when the size of the matrices tends to infinity the eigenspaces of A and B are in generic position and this problem has a deterministic solution; there are precise formulas for calculating the eigenvalue distribution of $A + B$ or of AB from the eigenvalue distribution of A and the eigenvalue distribution of B. These formulas (and much more) are provided by Voiculescu's theory of free probability.

In this survey talk I will give an idea what free probability is and how it relates to random matrices. In particular, I want to show how tools from free probability (like R-transform or S-transform) provide the answers to the above mentioned problems.

*survey

Adaptive Array Detection, Estimation and Beamforming

Christ D. Richmond

MIT Lincoln Laboratory

Adaptive radar/sonar systems typically consist of an arrays of receivers deployed in environments dominated by limiting interference sources. The operational objectives of such systems often include detection and parameter estimation of signals of interest. The design and analysis of these systems requires theoretical quantification of detection performance (receiver operation characteristics) and estimation performance (accuracy of parameter estimates). The use of random matrix theory to provide insights into the performance of such systems is reviewed for a popular class of adaptive algorithms, and some persistent open problems of practical significance are discussed.

On some detection problems in radar array processing involving eigenvalues of Wishart and related random matrices

Olivier Besson*, Louis L. Scharf†, and Shawn Kraut‡

*ENSICA

†Colorado State University

‡MIT Lincoln Laboratory.

We consider a radar application in which an array of sensors is used to detect the presence of a signal of interest (SOI), in the presence of colored noise with possibly unknown covariance matrix. We consider a situation where the SOI's signature is not known perfectly, but lies on an unknown line in a known linear subspace. Additionally, we consider a partially homogeneous environment, for which the covariance matrix of the primary data -which may contain the SOI- and the covariance matrix of the secondary data -which contain noise only- have the same structure, but possibly different levels. The generalized likelihood ratio test (GLRT) is formulated in the case where the noise covariance matrix is known, and in the case where it must be estimated from secondary data. In either case, the GLRT involves the largest eigenvalue of a Wishart or F-distributed random matrix, normalized to its trace. In the Wishart case, we derive the probability density function (p.d.f.) of such a statistic under both hypotheses. Some numerical results are presented to validate the theoretical results, and to assess the performances of the detectors.

Asymptotic Random Matrix Applications in Communication: MIMO and More

Daniel Bliss Jr.

MIT Lincoln Laboratory

The tools of asymptotic random matrices have been employed to address a number of wireless communication problems. The analysis of code-division multiple-access (CDMA) systems, networks with multiple-antenna receivers, and multiple-input multiple-output (MIMO) links have all benefited from these tools. Here, the focus is on MIMO communication, but a short discussion of some of the other addressed problems is provided.

Wireless communication using MIMO enables increased spectral efficiency for a given total transmit power. MIMO capacity is a strong function of the singular value distribution of the channel matrix. The channel matrix contains the complex attenuation from each transmit to receive antenna. Bounds on the maximum spectral efficiency of MIMO systems in which both the transmitter and receiver know the channel (channel-state-informed transmitter) and in which only the receiver knows the channel are introduced. MIMO performance for environments with interference are also considered.

On the Signal-to-Interference-Ratio of CDMA Systems in Wireless Communications

Jack W. Silverstein

North Carolina State University

Let $\{s_{ij} : i, j = 1, 2, \ldots\}$ consist of i.i.d. random variables in $\mathbb{C}$ with $E(s_{11}) = 0$, $E|s_{11}|^2 = 1$. For each positive integer N let $s_k = s_k(N) = (s_{1k}, s_{2k}, \ldots, s_{Nk})^T$, $1 \le k \le K$, with $K = K(N)$ and $K/N \to c > 0$ as $N \to \infty$. Assume for fixed positive integer L, for each N and $k \le K$ $\alpha_k = (\alpha_k(1), \ldots, \alpha_k(L))^T$ is random, independent of the s_{ij}, and the empirical distribution of $(\alpha_1, \ldots, \alpha_K)$, with probability one converges weakly to a probability distribution H on $\mathbb{C}^L$. Let $\beta_k = \beta_k(N) = (\alpha_k(1)s_k^T, \ldots, \alpha_k(L)s_k^T)^T$ and set $C = C(N) = (1/N)\sum_{k=2}^K \beta_k \beta_k^*$. Let $\sigma^2 > 0$ be arbitrary. Then with probability one $\mathrm{SIR}_1 \equiv \lim_{N\to\infty}(1/N)\beta_1^*(C + \sigma^2 I)^{-1}\beta_1 = \sum_{\ell,\ell'=1}^L \bar{\alpha}_1(\ell)\alpha_1(\ell')a_{\ell,\ell'}$ where $A = (a_{\ell,\ell'})$ is nonrandom, Hermitian positive definite, and is the unique matrix of such type satisfying $A = \left(c \, \mathrm{exp} \, \frac{\alpha\alpha^*}{1+\alpha^* A\alpha} + \sigma^2 I_L\right)^{-1}$ where $\alpha \in \mathbb{C}^L$ has distribution H. The quantity SIR_1 is used as a model for the best signal-to-interference ratio for one user with respect to other $K-1$ users, for K large in a direct-sequence code-division multiple-access system in wireless communications. The result generalizes those previously derived but under more restricted assumptions, allowing for the analysis of user location with respect to the antennas in arbitrary settings (joint work with Zhidong Bai).

Intrinsic Estimation Bounds With Signal Processing Applications

Steven Thomas Smith

MIT Lincoln Laboratory

Signal processing algorithms and systems analysis for signal detection, location, and classification all rely on covariance, subspace, and matrix-based methods. The applicability of standard, linear estimation theory is limited in many such settings, where the estimation problem necessarily involves nonlinear parameter spaces, such as spheres, orthogonal and unitary matrices, Grassmann manifolds, Stiefel manifolds, and positive-definite matrices. Furthermore, unlike the standard estimation problem, there is no prescribed or natural coordinate system on these spaces with which to express ones answers. Therefore, an intrinsic approach to estimation theory is necessitated in these (and other) applications.

This talk addresses the problem of intrinsic or nonlinear estimation at its deepest level, and provides powerful new tools and insights, as well as well as some startling surprises. The covariance matrix problem is framed as an intrinsic estimation problem on the space of positive definite (covariance) matrices, which has the structure of a homogeneous or quotient space, not a vector space—the necessary setting for classical Cramér-Rao bounds. Covariance matrix estimation accuracy bounds are derived from an intrinsic derivation of the Cramér-Rao bound on arbitrary Riemannian manifolds (another new development), and compared to the accuracy achieved by standard methods involving the sample covariance matrix (SCM). Estimator efficiency is discussed from different, novel, viewpoints.

Remarkably, it is shown that that from an intrinsic perspective, the SCM is a biased and inefficient estimator; the bias corresponds to the SCM's poor estimation quality at low sample support—this contradicts the well-known fact that $E[\mathrm{SCM}] = \mathbf{R}$ because the linear expectation operator implicitly treats the covariance matrices as points in a real vector space, compared to the intrinsic treatment of positive-definite Hermitian matrices used in this talk. The analysis approach developed is directly applicable to many other estimation problems on manifolds encountered in signal processing and elsewhere, such as estimating rotation matrices in computer vision and estimating subspace basis vectors in blind source separation. Finally, the intrinsic Cramér-Rao bounds will be compared to other, related intrinsic bounds reported in the literature.

Spiked covariance model and some extensions

Debashis Paul

University of California, Davis

A spiked population model for large dimensional vector-valued measurements is considered. The covariance matrix of the measurements is thought of as having a few large eigenvalues well separated from the rest of the eigenvalues. The question of how the leading sample eigenvalues and eigenvectors depend on their population counterparts as well as the ratio of the dimension to sample size is considered. Some extensions of the model that include a time dependence of the observations are also studied.

On Powers of a Random (Haar) Orthogonal Matrix

Robb Muirhead[a], and Morris Eaton[b]

[a]Pfizer Global R&D.

[b]University of Minnesota.

In a talk at SEA'05, Tom Marzetta of Bell Labs presented simulations indicating that when the north pole of a unit sphere in 3-dimensions is rotated twice by the same random (Haar) orthogonal matrix, the resulting point is more likely to lie in the northern hemisphere than in the southern hemisphere. An algebraic proof of this result, which actually holds for any dimension greater than or equal to three, is outlined, through a description of the distribution of the "north-pointing" coordinate. (The techniques of the proof involve some basics of groups acting on spaces, and the notion of an invariant distribution – such as the characterization of the uniform distribution on a sphere being the "unique orthogonally invariant probability distribution on the sphere".) We also describe the distribution of this "north-pointing" coordinate in any dimension following three rotations with the same random (Haar) orthogonal matrix.

Asymptotic Statistical Analysis on Special Manifolds

Yasuko Chikuse

Kagawa University,

The special manifolds of interest are the Stiefel manifold and the Grassmann manifold. Formally, the Stiefel manifold $V_{k,m}$ is the space of k-frames in the m-dimensional real Euclidean space R^m, represented by the set of $m \times k$ matrices X such that $X'X = I_k$, where I_k is the $k \times k$ identity matrix. For $m = k$, $V_{k,m}$ is the orthogonal group $O(m)$ of $m \times m$ orthogonal matrices. The Grassmann manifold $G_{k,m-k}$ is the space of k-planes (k-dimensional hyperplanes) in R^m. We see that the manifold $P_{k,m-k}$ of $m \times m$ orthogonal projection matrices idempotent of rank k corresponds uniquely to $G_{k,m-k}$. This paper is concerned with statistical analysis on the manifolds $V_{k,m}$ and $P_{k,m-k}$ as statistical sample spaces consisting of matrices.

For the special case $k = 1$, the observations from $V_{1,m}$ and $G_{1,m-1}$ are regarded as directed vectors on a unit hypersphere and as axes or undirected lines, respectively. There exists a large literature of applications of directional statistics and its statistical analysis, mostly occurring for $m = 2$ or 3 in practice, in the Earth Sciences, Astrophysics, Medicine, Biology, Meteorology, Animal Behavior and many other fields. Examples of observations on $G_{k,m-k}$ arise in the signal processing of radar with m elements observing k targets. The Grassmann manifold is a rather new subject treated as a statistical sample space.

A random matrix X on $V_{k,m}$ is said to have the matrix Langevin (or von Mises-Fisher) distribution, denoted by $L(m, k; F)$, if its density function is given by

$$\exp(\mathrm{tr} F'X)/{}_0F_1(\frac{1}{2}m; \frac{1}{4}F'F), \text{ with } F \text{ an } m \times k \text{ matrix,}$$

where the ${}_0F_1$ is a hypergeometric function with matrix argument. Writing the singular value decomposition of F as

$$F = \Gamma \wedge \Theta', \text{ with } \Gamma \in V_{k,m}, \ \Theta \in O(k), \text{ and } \wedge = \mathrm{diag}(\lambda_1, \ldots, \lambda_k), \ \lambda_1 \geq \ldots \lambda_k \geq 0,$$

the distribution has the modal orientation $\Gamma\Theta' = V_{k,m}$ and the λ_i's control the concentrations about the mode. The $L(m, k; F)$ distribution is a most useful distribution for statistical analysis on $V_{k,m}$.

The solutions of statistical inference, estimation and testing hypotheses, on the parameters $F = \Gamma \wedge \Theta'$ of the $L(m, k; F)$ distribution are given in terms of the hypergeometric functions with matrix argument, which make the problems intractable. We will develop asymptotic theorems for large sample, large and small concentrations, and high dimension m.

Similar analyses can be developed on the Grassmann manifold. See Chikuse [Statistics on Special Manifolds, Lecture Notes in Statistics, Vol. 174, Springer, 2003].

Universality in random matrix theory

Arno Kuijlaars

Katholieke Universiteit Leuven

Dyson's universality conjecture says that the local eigenvalue correlations of random matrices, in the limit as the size of the matrices tends to infinity, depend on the symmetry class, but not on the precise form of the random matrix ensemble. During the last decade this conjecture has been proved for various types of unitary, orthogonal and symplectic ensembles. At special points in the spectrum (such as edge points) different eigenvalue behavior occurs which however also turns out to be universal. I will give an overview of some of these results.

*survey

Traffic systems and random matrix theory

Petr Seba[a,b]

[a] Academy of Sciences of the Czech Republic

[b] University Hradec Kralove,

We show that the clearance distribution observed on high way traffic is well described by eigenvalue distributions of random matrices. The same observation applies also for the bumper-to-bumper distribution of parking cars. Using the collected data we will demonstrate that the parking habit is capable to change the universality class of the related random matrix ensemble.

Two random matrix central limit theorems

Jinho Baik[a] and Toufic M. Suidan[b]

[a]University of Michigan

[b]University of California, Santa Cruz

We will discuss two types of probability models: directed last passage site percolation models and non-intersecting random walks. For a few special cases (for example, longest increasing subsequences and random hexagon tiling), the limiting distributions are known to be described in terms of random matrices such as the Tracy-Widom distribution. However, such random matrix central limit theorems are believed to be universal in these models. We will discuss the universality for particular regimes.

A generalized projection on to matched subspaces approach for adaptive waveform design

Joseph Guerci

DARPA

The advent of digital arbitrary waveform generators (DAWGs) and continued advances in high speed embedded computing have opened the door to the possibility of multidimensional channel-dependent adaptive waveform design for sensing and communication. Given an estimate of a channel (multidimensional transfer function and/or colored noise covariance), it has been previously shown that the optimal waveform from an SINR perspective is the eigenfunction associated with the largest eigenvalue of the channel kernel function [1]-[3]. In practice, however, this optimal waveform often does not meet other critical constraints such has constant modulus, bandwidth/resolution, sidelobe rejection, etc.

In this talk, a new method for constrained adaptive waveform estimation is introduced which exploits the relatively high SINR properties of the dominate subspace of the channel kernel, i.e., the K dominant eigenfunctions where K is chosen based on a minimally acceptable level of channel gain relative to a quiescent (non-matched) waveform. By projecting constraints onto this matched subspace, an acceptably constrained yet “matched” waveform can be produced. The method is illustrated with applications from a multichannel radar operating in a colored noise environment.

[1] J. R. Guerci and P. Grieve, “Optimum Matched Illumination-Reception Radar”, U.S. Patent 5 121 125, June 1992, and U.S. Patent 5 175 552, December 1992.

[2] S. U. Pillai, H. S. Oh, J. R. Guerci, and D. C. Youla, “Optimum Transmit-Receiver Design in the Presence of Signal-Dependent Interference and Channel Noise” IEEE Trans. on Information Theory, Vol. 46, No. 2, March 2000 pp. 577-584.

[3] S. U. Pillai and J. R. Guerci, “Multichannel Matched Transmit-Receiver Design in Presence of Signal-Dependent Interference and Noise”, Proceedings of the IEEE Sensor Array and Multichannel (SAM) Processing, March 2000, Boston, MA.

Asymptotic distribution of principal components estimator of large spherical factor models

Alexei Onatski

Columbia University

This paper studies asymptotic distribution of k largest eigenvalues and corresponding eigenvectors of the sample covariance matrix of data having k-factor structure when the dimensionality of the data, n , and the number of observations, T, go to infinity proportionally. We show that, in contrast to the classical case when only T goes to infinity, the principal eigenvectors of the sample covariance matrix are inconsistent estimates of the population eigenvectors . However, the components of the sample eigenvectors centered by their probability limits and scaled by $\sqrt{T}$ are asymptotically normal, and we give an explicit formula for the limits and the variance of the asymptotic distribution. The asymptotic distribution of the principal sample eigenvalues is also obtained. Both sample eigenvalues and the components of the sample eigenvectors exhibit a phase transition phenomenon when, if the true cumulative effect of factors on the cross-sectional units is below certain threshold, then sample eigenvectors and eigenvalues do not carry any information about their population analogs, but if the cumulative effect passes the threshold and becomes very large, all information is eventually recovered. As a Monte Carlo analysis shows, the obtained asymptotic distribution of the components of the sample eigenvectors approximate the finite sample distribution much better than the classical asymptotic distribution does, even for n and T as small as about 15.

Concentration of measure, free probability and Markov chains

Sourav Chatterjee

University of California, Berkeley.

We shall present a new measure concentration technique that uses couplings and rates of convergence of Markov chains to obtain concentration inequalities. A concentration result for the empirical distribution of the eigenvalues of sums of random hermitian matrices (arising in free probability theory) will be derived as an application of the method. This also gives an example of concentration for discontinuous functions, which can be an important area of application for the new technique.

Riemann-Hilbert Problems: Applications

Percy Deift

Courant Institute.

The speaker will describe the application of Riemann-Hilbert techniques to a variety of problems in mathematics and mathematical physics. The nonlinear steepest descent method plays a key role.

The sloppy model universality class and the Vandermonde matrix

James P. Sethna[a], Joshua J. Waterfall[a], Fergal P. Casey[b], Ryan N. Gutenkunst[a], Kevin S. Brown[c], Christopher R. Myers[d], Piet W. Brouwer[a], and Veit Elser[a]

[a]Laboratory of Atomic and Solid State Physics, Cornell University.

[b]Center for Applied Mathematics, Cornell University

[c]Harvard University

[d]Cornell University

We explain why multiparameter nonlinear systems so often are *sloppy*; the system behavior depends only on a few 'stiff' combinations of the parameters and is unchanged as other 'sloppy' parameter combinations vary by orders of magnitude. We contrast examples of sloppy models (from systems biology, variational quantum Monte Carlo, and data fitting) with systems which are not sloppy (multidimensional linear regression, random matrix ensembles). We observe that the eigenvalue spectra for the sensitivity of sloppy models have a striking, characteristic form, with a density of logarithms of eigenvalues which is roughly constant over a large range. We suggest that the common features of sloppy models indicate that they may belong to a common universality class. In particular, we motivate focusing on a *Vandermonde ensemble* of multiparameter nonlinear models and show in one limit that they exhibit the universal features of sloppy models.

Speed Limits in Biological Neural Networks

Marc Timme [a,b,c]

[a]Cornell University

[b]Max Planck Institute for Dynamics and Self-Organization

[c]Bernstein Center for Computational Neuroscience

Precisely coordinated spatio-temporal spiking dynamics have been observed experimentally in different neuronal systems and are discussed to be an essential part of computation in the brain. Their dynamical origin, however, remains unknown.

Here we study the dynamics of spiking neural network models and reveal basic mechanisms underlying the neurons' precise temporal coordination. We focus on the synchronization dynamics of neural networks exhibiting a complicated connection topology. In such networks, an irregular, balanced state coexists with a synchronous state of regular activity. Using a random matrix approach we predict the speed of synchronization in such networks in dependence of properties of individual neurons and their interaction network. We find that the speed of synchronization is limited by the network connectivity and remains finite, even if the coupling strengths between neurons become infinitely large. We offer an intuitive explanation of this phenomenon.

Computing Eigenvalue Distributions of Random Matrices with Applications

Plamen Koev

Massachusetts Instiute of Technology

I will present our new algorithm for efficiently computing distributions of the eigenvalues of various random covariance matrices based on the hypergeometric function of a matrix argument.

The latter distributions are critical in many multivariate statistical tests (Canonical Correlation Analysis, Principal Component Analysis, MANOVA, etc.) and thus in many practical applications in genomics, wireless communications, 3D target classification, etc.

I will explain the connections between Schur functions, Zonal polynomials, representation theory, and random matrix theory that make our new algorithm possible and in turn enable the above applications, which I will also discuss.

Applications of Random Matrix Theory to Economics, Finance and Political Science

Matthew C. Harding

MIT and Institute for Quantitative Social Science, Harvard University

Random Matrix Theory introduces social scientists to new methods of analyzing large N, T panel data sets which commonly occur in economics, finance and political science. In this paper we discuss a number of stochastic eigen-analysis techniques which enhance traditional econometric and statistical approaches to social science research using large data sets. We explore applications of free probability, moments of eigenvalue distributions and eigen-inference, largest increasing subsequences and random incidence matrices to real world problems in portfolio selection, global factors in stock and bond markets, regional risk sharing, precursors of extreme events, financial contagion in emerging markets and strategic voting in the US Congress.

From Random Matrices to Stochastic Operators

Brian D. Sutton

Randolph-Macon College

Random matrices are usually treated as arrays of numbers, not operators. Utilizing the structured matrix models of Dumitriu and Edelman, we show that classical random matrix models are, in fact, finite difference schemes for stochastic boundary value problems. The stochastic Airy, Bessel, and sine operators play a privileged role in this theory.

The BVP's provide a unifying effect. For example, the stochastic Airy operator is discretized by both the Gaussian/Hermite matrix model and the Wishart/Laguerre matrix model when scaled at the right edge of the spectrum. Therefore, the largest eigenvalues of the two ensembles have the same distribution as $n \to \infty$, a fact observed by Johnstone and others.

β-ensembles, Stochastic Airy spectrum, and a diffusion

Brian Rider[a], José Ramírez[b]

[a]University of Colorado, Boulder

[b]Universidad de Costa Rica

Building on earlier work of A. Edelman, I. Dumitriu, and B. Sutton we prove that the largest eigenvalues of the general beta-ensemble of Random Matrix Theory, properly centered and scaled, converge in distribution to the law of the low lying eigenvalues of a random operator of Schrödinger type. The latter is $-\frac{d^2}{dx^2} + x + \frac{2}{\sqrt{\beta}} b'(x)$ acting on $L^2(R_+)$ with Dirichlet boundary condition at the origin. Here $b'(x)$ denotes a standard White Noise, and the $\beta > 0$ is that of the original ensemble. Based on this convergence, we provide a new characterization of the Tracy-Widom type laws (for all β) in terms of the explosion probability of a one-dimensional diffusion.

Applications of Structured Random Matrices to Complex Systems

G. Ergün

University of Bath

Many complex systems whether technological, social or biological have network structures. Topological features of such structures may be described by the associated adjacency matrices. The eigenvalues of adjacency matrices are related to many basic topological invariants of networks. I will present a generic random matrix model to study coupled systems or a system with modules, where introduction of a full coupling between two systems/modules of contrasted intra-interactions changes the semi circular shape of the density of states (DOS) to an onion dome shape. However, if the intra-interactions are the same but a few strong inter-couplings are introduced then the density of the spectra has a triangular shape, which has been observed in scale free networks of many kinds.

Random Lévy Matrices

Z. Burda[a], J. Jurkiewicz[a], M.A. Nowak[a], G. Papp[b] and I. Zahed[c]

[a] Jagellonian University

[b]Eötvös University, Budapest

[c]SUNY Stony Brook

We compare two ensembles of large symmetric random matrices exhibiting a power law spectrum with the stability index $0 < \mu < 2$: Free Random Lévy (FRL) matrices with a rotationally symmetric measure, and Bouchaud, Cizeau (BC) random matrices based on a measure whereby each independent matrix entry is chosen from a symmetric stable Lévy distribution. Both the ensembles are spectrally stable with respect to matrix addition. We illustrate the relation between the two types of stability and show that the addition of rotated BC matrices leads by a matrix central limit theorem to FRL spectra.

Causal sets and their applications

Eitan Bachmat

Ben-Gurion University

Causal sets are random partially ordered sets of a geometric origin. They can be considered as discrete analogs of space-time manifolds. We show how causal sets appear naturally in the consideration of scheduling, card game, crystal growth, airplane boarding and pattern recognition problems. We then explain how in some cases their finer asymptotic properties are related to random matrix theory. Finally we will provide some applications.

Fluctuations of the asymmetric exclusion process and random matrix theory

Tomohiro Sasamoto

Chiba University

The one-dimensional asymmetric simple exclusion process (ASEP) is a stochastic process of many particles on a one-dimensional lattice and is defined as follows (Fig. 1). Due to the volume exclusion interaction among particles, each lattice site can be either occupied by a particle or is empty. During infinitesimal time duration dt, each particle performs the asymmetric diffusion; it tries to hop to the right neighboring site with probability dt if the target site is empty. We could also allow particles to hop to the left but restrict our attention to the case of total asymmetry.

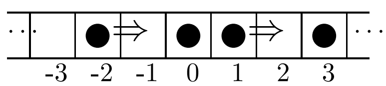

Fig. 1 : asymmetric simple exclusion process(ASEP)

Though it may look too simple, the ASEP shows many interesting phenomena and has a lot of applications. The ASEP can be regarded as a simple model of traffic flow or molecular motor system. The ASEP also plays a role of testing ground in the theory of nonequilibrium statistical physics.

Another important aspect of the ASEP is its mathematical tractability. In particular, in recent years, many properties of physical interest have been revealed by utilizing the methods of random matrix theory. The major breakthrough was the Laguerre ensemble representation for the current fluctuation by Johansson (2000) for the special initial configuration of particles $\ldots 111000 \ldots$ (0 and 1 stand for an empty and occupied site respectively). The result was obtained by mapping the problem of the ASEP to some combinatorial problem and a lot of progress has been made since then by pursuing this connection.

There is a closely related but somewhat different approach using the determinantal expression for the transition probability. This was first applied to recover (and slightly generalize) the Laguerre unitary ensemble representation of the current fluctuation (Nagao and Sasamoto 2004). It was then utilized to compute the multi-point fluctuation for a case where other methods seem to be difficult to apply (Sasamoto 2005). The details of the methods will be explained with simple examples. More recent developments and some open problems are also discussed.

Free Meixner distributions and random matrices

Michael Anshelevich

Texas A&M University

The Meixner family of distributions consists of the Gaussian, Poisson, gamma, negative binomial, hyperbolic secant, and (perhaps) the binomial distribution. There is a number of different characterizations that distinguish these among all probability distributions. In this talk, I will describe the "free" version of the Meixner family. I will define it, explain what "free version of" means, and describe a number of properties it has. Some of the free Meixner distributions are familiar as the limiting mean eigenvalue distributions for the GUE, Wishart, and Jacobi ensembles. This talk is intended as a tutorial, so I will try to minimize the amount of background necessary.

Fluctuations of Random Matrices and Second Order Freeness: Part I

James A. Mingo and Roland Speicher

Queen's University

Whereas free probability theory allows to deal with the average eigenvalue distribution of random matrices, the theory of second order freeness is intended to address in a similar fashion the global fluctuations of the eigenvalues (i.e., the covariances of traces of powers of the matrices). We will motivate the notion of second order freeness by asking how the fluctuations of $A+B$ are related to the fluctuations of A and the fluctuations of B, in a generic situation. We will describe some properties of second order freeness and, in particular, present our second order analogue of R-transform formulas, which allows to calculate fluctuations of $A+B$ from fluctuations of A and B effectively.
(This is, in part, joint work with B. Collins and P. Śniady.)

Global fluctuations for the β-Hermite and β-Laguerre ensembles

Ioana Dumitriu[a] and Ofer Zeitouni[b]

[a]University of Washington

[b]University of Minnesota, Minneapolis

The β-Hermite and β-Laguerre ensembles are some of the most studied general-parameter distributions in classical random matrix theory, and include the Gaussian orthogonal, unitary, and symplectic ensembles, together with the Wishart real, complex, and quaternion ones.

In a much celebrated paper from 1998, Johansson computed the global fluctuations of a linear statistic for a large class of ensembles, which includes the β-Hermite ones; the proof used methods from potential theory and was based exclusively on the eigenvalue distributions.

With the discovery of matrix models by D. and Edelman in 2001, we have embarked on a journey to (re)prove and extend Johansson's results using a matrix-based approach. The first step was achieved by D. and Edelman (2006), for a polynomial statistic and for both kinds of ensembles.

This talk will present the second (and last) leg of this journey. We have used concentration results obtained by Guionnet and Z., together with perturbation theory, to extend our computation of global fluctuations to a larger class of functions than the one considered by Johansson.

Spectra of large block matrices

R. Rashidi Far*[a], R. Speicher[a] W. Bryc[b], and T. Oraby[b]

[a]Queen's University, [b]University of Cincinnati.

Motivated by some engineering problem, we investigate the limit eigenvalue distribution of random block matrices. We present a method, coming from operator-valued free probability theory, to get the limit distribution of the eigenvalues for such matrices.

For a matrix composed of d square-block matrices in each row and $z \in \mathbb{C}^+$, we show that $z\mathcal{G}(z) = I_d + \eta(\mathcal{G}(z)) \cdot \mathcal{G}(z)$ where I_d is the identity matrix, $\eta(\cdot)$ is an operator on the entries of $\mathcal{G}(z)$ and $\mathcal{G}(z) \in M_d(\mathbb{C})$ with the following properties: $\lim_{|z|\to\infty} z\mathcal{G}(z) = I_d$ and the normalized trace of $\mathcal{G}(z)$ is the Cauchy transform of the desired distribution.

Then applying the Stieltjes inversion formula, we numerically calculate the distribution from the above mentioned formula for some cases and show how they match with the simulation results.

Matrix-valued Processes, Free Probability and Root Systems

Nizar Demni

LPMA Universite de Paris VI

Our motivation began by extending well known properties to matrix-valued stochastic processes (Wishart and Laguerre). The latters are the dynamic version of real and complex Wishart variables already studied by James, Muirhead , Chikuse. in multivariate statistics. While studying this kind of processes, we are dealing with multivariate hypergeometric functions and orthogonal polynomials. In the next step, we focus on the limit as the size of the matrix goes to infinity. By this way, we define free processes in a von Neumann algebras. An other aspect is provided by the eigenvalues processes which evolve like a radial Dunkl processes associated to a root system for which we derive some properties such as existence, uniqueness and the first collision time of the Weyl chamber. Besides, it is worthnoting that at time $t = 1$ and starting at 0, these ones coincide with the Beta ensembles defined and studied by A. Edelman and I. Dumitriu.

Friday 10:00am - 10:30am

Fluctuations of Random Matrices and Second Order Freeness: Part II

James A. Mingo and Roland Speicher

Queen's University

Whereas free probability theory allows to deal with the average eigenvalue distribution of random matrices, the theory of second order freeness is intended to address in a similar fashion the global fluctuations of the eigenvalues (i.e., the covariances of traces of powers of the matrices). We will motivate the notion of second order freeness by asking how the fluctuations of $A+B$ are related to the fluctuations of A and the fluctuations of B, in a generic situation. We will describe some properties of second order freeness and, in particular, present our second order analogue of R-transform formulas, which allows to calculate fluctuations of $A+B$ from fluctuations of A and B effectively.
(This is, in part, joint work with B. Collins and P. Śniady.)

Multivariate orthogonal polynomials in non-commuting variables

Michael Anshelevich

Texas A&M University

In this talk, I will describe some preliminary results on orthogonal polynomials in several non-commuting variables. After deriving a general recursion relation, I will concentrate on a sub-class of polynomials with a resolvent-like generating function, that can be called "multivariate free Meixner polynomials." Under further restrictions, all such polynomials are simply products of one-variable polynomials, after a change of variable. This parallels known results for the multivariate Meixner polynomials in commuting variables. On the other hand, an example will also be provided of polynomials that are not products, and that have no analog in the commutative case.

Norm of sums and products of free random variables

Vladislav Kargin

New York University

This talk is about large deviation results in free probability theory. Consider $S_n = n^{-1/2}(X_1 + ... + X_n)$ where X_i are identically distributed, bounded, and free random variables. In classical probability theory the tails of the distribution of S_n (i.e. sets of x, where $|\mathcal{F}_{S_n}(x)| > c$) would be asymptotically exponentially small but not zero. In contrast, in free probability theory, there is a threshold, after which the tail of the spectral distribution of S_n is exactly zero In other words the probability of large deviations is smaller for sums of free random variables. This effect was named superconvergence by Bercovici and Voiculescu. I will talk how this result can be generalized to non-identically distributed variables, and about the explicit rate of superconvergence.

For products, I will discuss how the norm of products $\Pi_n = X_1 X_2 ... X_n$ behaves, where X_i are free identically-distributed positive random variables. In addition, I will talk about the behavior of the norm of $Y_n = X_1 \circ X_2 \circ ... \circ X_n$, where $\circ$ denotes the symmetric product of two positive operators: $A \circ B =: A^{1/2} B A^{1/2}$.

It turns out that the condition $\|X_i\| \leq L$ implies that $\|\Pi_n\| \leq cL\sqrt{n}s^{n-1}$, where s is the standard deviation of the random variables X_i. For Y_n our result is that $\|Y_n\| \leq cLn$. These results are significantly different from the analogous results for commuting random variables.