Opportunities at the interface of signal processing and stochastic eigen-analysis

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Outline

- Covariance based methods
  - “Array” signal processing
- Beamforming
  - The role of random matrices
  - “Classical” Capon beamforming algorithm
- Opportunities for collaboration
- The art of adapting theory to practice
  - Importance of “wiggle room”
- Some mathematical questions

Covariance based methods

- Correlation techniques = foundation of signal processing
  - *From theory (Wiener, et.al) to practice*
  - *"… the potential significance of **correlation techniques** had fired the imagination …. the general enthusiasm was due to experimental evidence ….. that **weak signals could be recovered in the presence of noise by using correlation techniques**. From that point the field evolved very rapidly." -- Professor Henry J. Zimmermann, MIT RLE Director, 1958.*

- *Covariance based methods extended to array processing*

Task	Example
Detection	Is a plane present?
Estimation	Location of plane
Classification	What kind of plane?

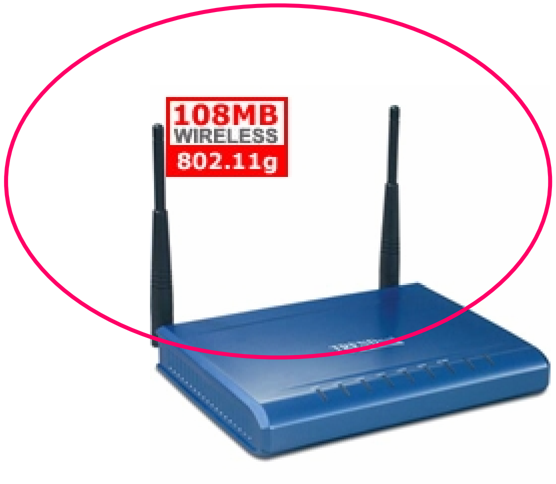
Taxonomy of Array Processing Problems Which Use Random Matrices

- Beamforming
  - Objective: Output Time Series
  - Examples
    - Digital Communications => satellites, land links
    - Post processing for radar, sonar, seismics, *e.g.* synthetic aperture and other doppler based imaging methods
- Directional spectrum estimation
  - Objective: Estimate distibution of power *versus* k-ω or angle(s) - (waveform or signal function not needed)
  - Examples
    - Ambient noise for sonar, radio astronomy
    - Sea surface wave spectra, seismicity

Taxonomy of Array Processing Problems Which Use Random Matrices (cont)

- Directional of Arrival Estimation
  - Objective: Estimate angles for a set of signal emitters
  - Examples
    - source localization (power irrelevant)
    - tracking (localization *versus* time)

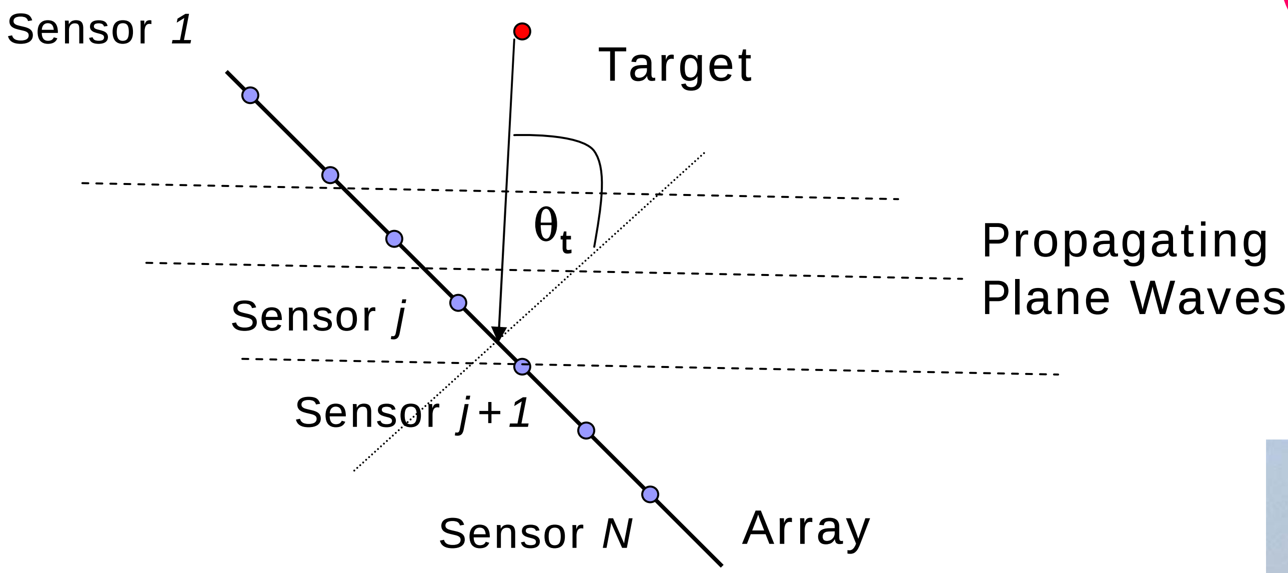
Array processing applications

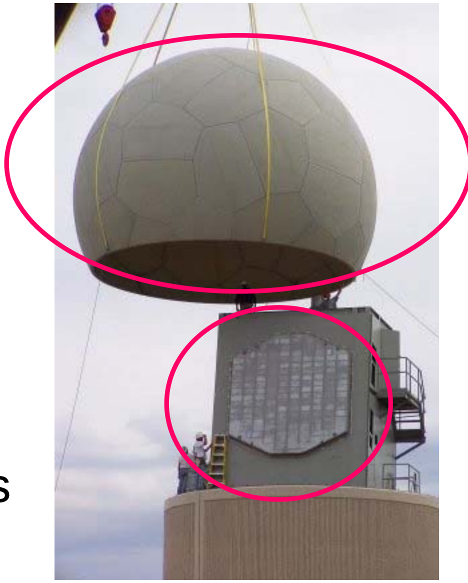


- Wireless communications
  - e.g. Multiple antennas (“MIMO”) in routers
  - e.g. Cellphone radio towers

- Biomedical imaging
  - MEG
- Emerging applications
  - Targeted therapy for lesions

“Array” Signal Processing





- “Array” = Sensors placed together
- Main idea: Adding sensors (generally) enhances capabilities
    - Example: Human ear (at high frequencies)
- Many geometries (often concurrently)
- Built on the mathematics of correlation techniques!

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- Beamforming
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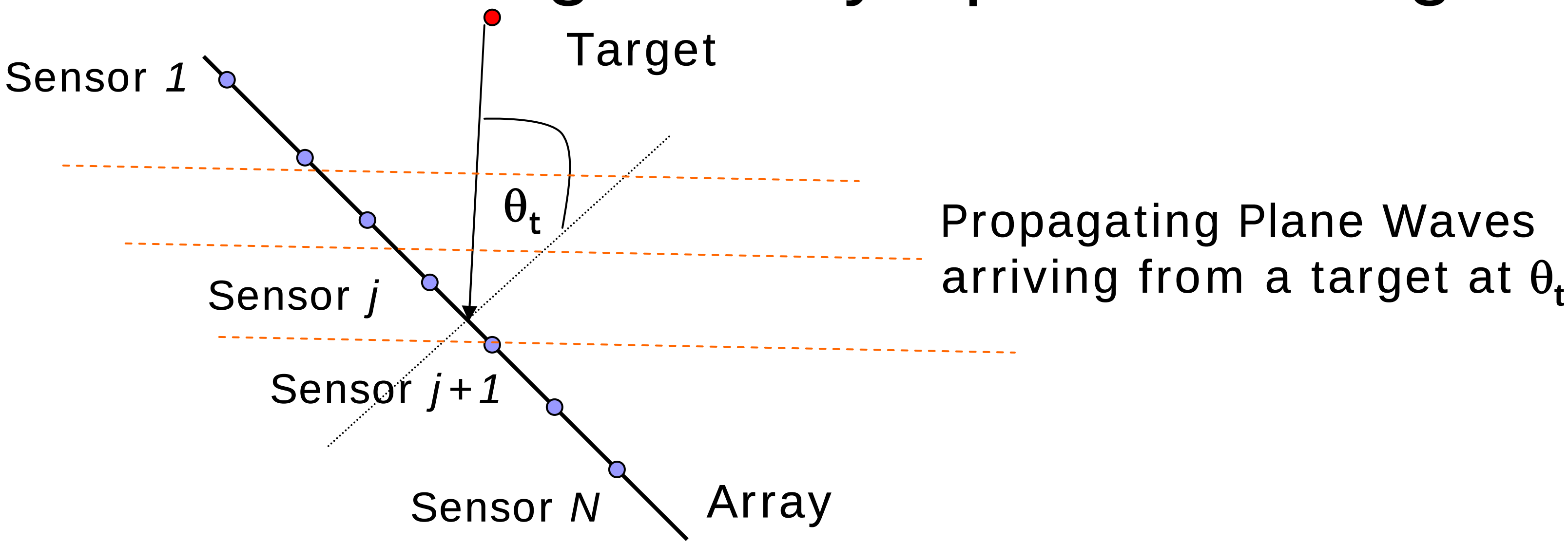
  - "Classical" Capon, or Minimum Variance, Distortionless Response (MVDR) processing
  - Many variants, but most involve

$$w = \alpha \hat{R}^{-1} \hat{s}$$

- w/ $\hat{R}$ the sample covariance matrix and $\hat{s}$ the steering "replica" vector
- to practice
  - Importance of "wiggle room"
- Some mathematical questions

Beamforming = Array Spatial filtering



- **Idea**: Measurement @ time t at sensor j is **correlated** with Measurement @ time $t+\Delta t$ at sensor $j+1$. Hence exploit this relationship!
- **Physics**: Δt *and the degree of correlation* depends on
  - *Array geometry (placement of sensors)*
  - *Propagation physics (air vs. water or sediment, free space vs. dispersive, linear vs. nonlinear)*
  - *Multiple targets and randomness add/create the challenge.*

Capon beamformer for Seismic Signal Processing

Capon 1969

Array signals, replicas S(θ,ϕ), power estimates

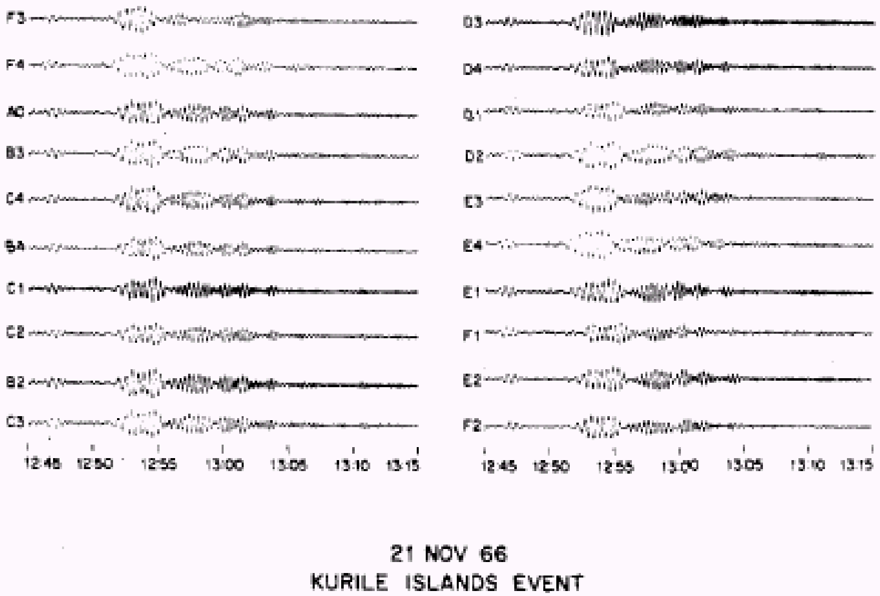
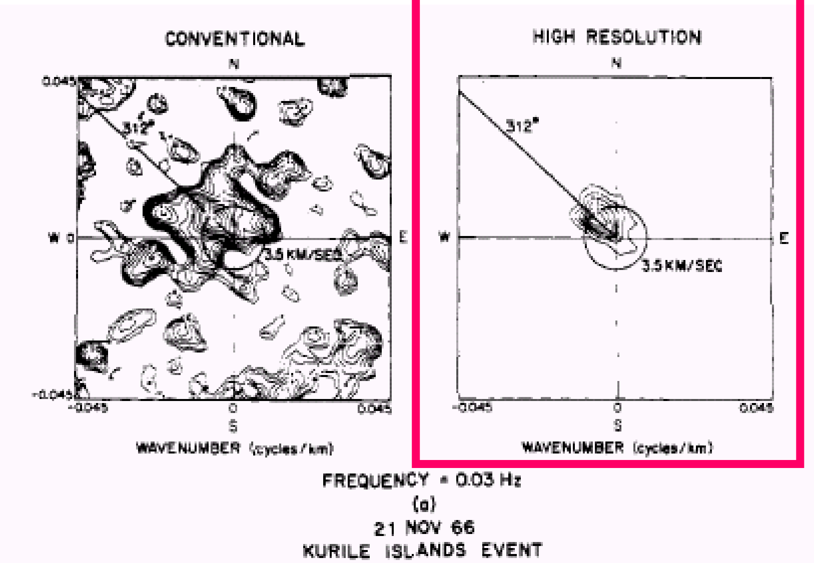
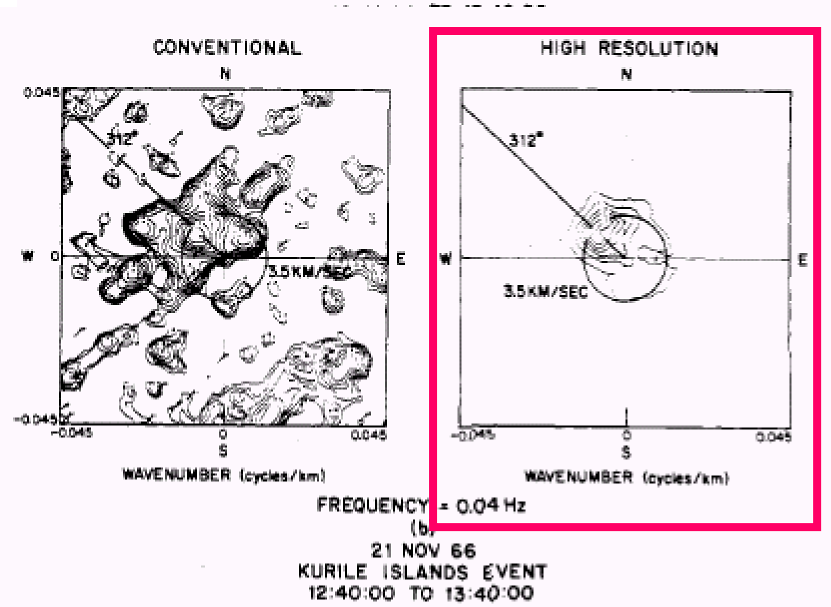


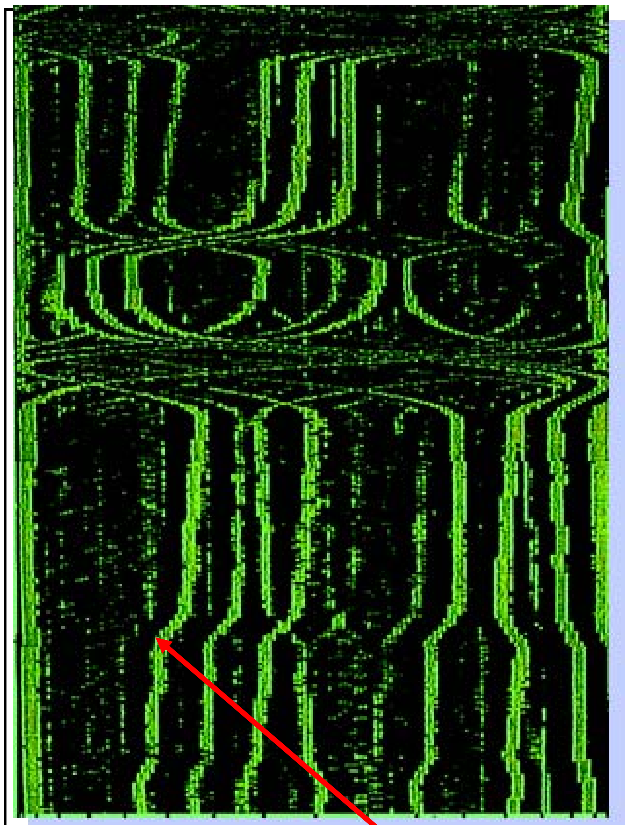
Fig. 6. The long-period waveforms for 21 November 1966 Kurile Islands event.



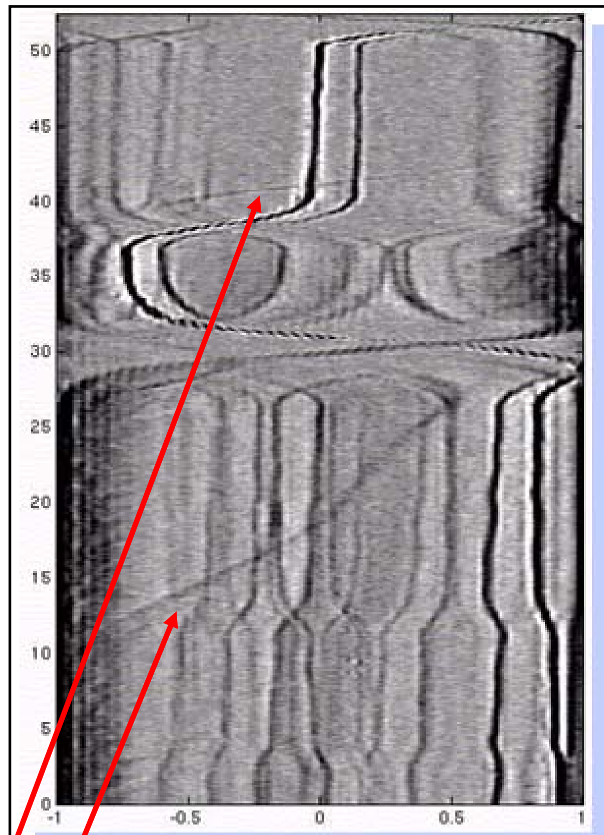


Capon: High-Resolution Frequency-Wavenumber Spectrum Analysis, 1969. (Two figures indicate time changing paths and refraction of continental margins)

APB-98/ARCI Phase 2: Signal Processing and OMI Gains



Legacy Display



A-RCI - 2

Contact
Vs.
No Contact

- ARCI-2 Leveraged Advanced BB Algorithms:
  - VLF Emphasis
  - Sub Band Peak Energy Detect & SCOT (Adaptive Equalize)
  - Cross-Correlation for Improved Bearing Resolution & Detection
- BQQ-5 + AFTAS Uses Legacy LR & CC with Fixed Spectral Shaping

Array Snapshot Processing

$$\mathbf{x} = \mathbf{z} + \mathbf{w}$$

Snapshot Signal Noise

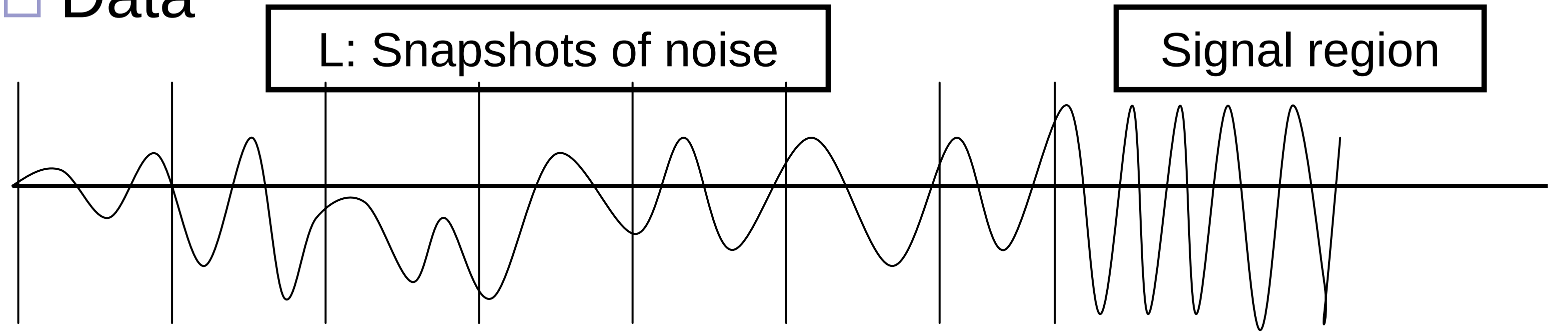
- **x** is the *N x 1* “snapshot” vector collected at a single time instant
- **x** is modeled as multivariate (complex) Gaussian
- E[$\mathbf{x}\,\mathbf{x}^H$] = **R** = True Covariance Matrix
- True Covariance Matrix encodes:
  - Spatial correlation of measurements due to ALL the targets
  - Effect of random noise field
- In practice use Sample Covariance Matrix (**R^**) instead

Array Snapshot Training Problem

- Sample Covariance

$$\hat{R} = \frac{1}{L}\sum_{l=1}^{L} x^l x^{l^H}$$

- Data



Array Snapshot Training (cont)

- L (# of snapshots) often << N (# of sensors)
  - Do not want signal in noise sample covariance estimate => self nulling of the desired signal
  - OK for spectral estimation problems, but Capon estimator is for power, not power desnity
  - Sources of limit on L
    - Non stationarity (source motion through resolution cell)
    - Array transit time and coherent relative phase addition over bandwidth

Direction of Arrival Estimation

- K targets located at $\theta_1, \theta_2, \ldots, \theta_K$: **Canonical radar problem**
- If **R** is known a priori

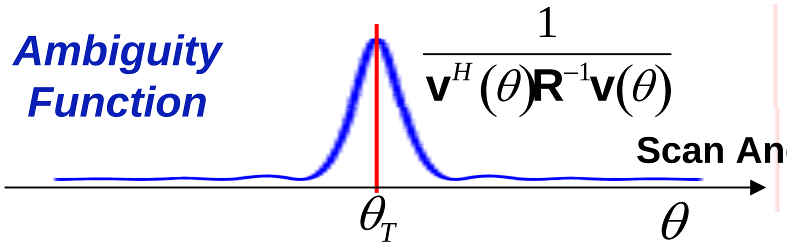
$$P_{\text{Capon}}(\theta) = \frac{1}{\mathbf{v}^H(\theta)\mathbf{R}^{-1}\mathbf{v}(\theta)}$$

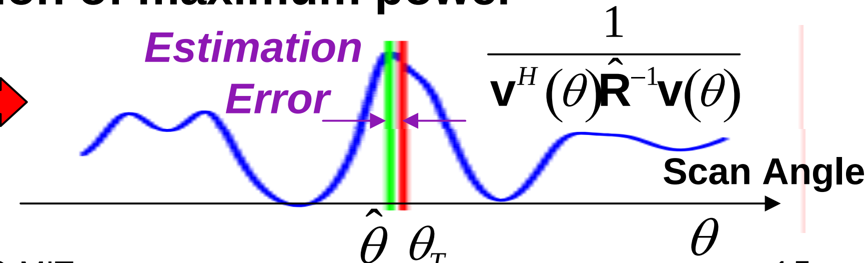
If **R** is estimated

$$\hat{P}_{\text{Capon}}(\theta) = \frac{1}{\mathbf{v}^H(\theta)\hat{\mathbf{R}}^{-1}\mathbf{v}(\theta)}$$

- $v(\theta)$ encodes how the plane wave impinges for each θ
- Scan for different values of θ and Pick K largest peaks
- Insufficient data causes higher sidelobes in the scan angle (θ) space

- **Parameter estimate $\hat{\theta}$ given by location of maximum power**





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State-of-the-art array processing

- Since 1950's much progress in exploiting structure:
  - Unitary invariance
  - Centro-symmetry
  - Dynamics of the target being tracked
  - etc.
- Techniques refined and tested over years
- Central role of sample covariance matrices
  - New mathematics yet to be fully taken advantage of!

Opportunities for collaboration

- Large # sensors, small # samples common for decades!
  - 16-100 common in radar
  - 1000's in sonar
  - Systems being planned: 10,000's

- Modes of collaboration:
  - Probabilistic *analysis* of algorithms (lots of low-hanging fruit!)
  - *Statistical optimization* of existing algorithms
  - *Inspiring* new algorithms

- Extensive real-world data sets available for testing
  - e.g. Global warming studies in Arctic with large arrays (1980's)

- Array processing problems as test-bed for refining methodologies
  - Other applications (almost surely) will follow

Fundamental challenge/question

- Sample covariance matrices appear in many applications

- Hope that the “new” mathematics of random matrices inspire fundamentally new signal processing paradigms

- A word of caution:
  - Role of mismatch between assumed model and reality

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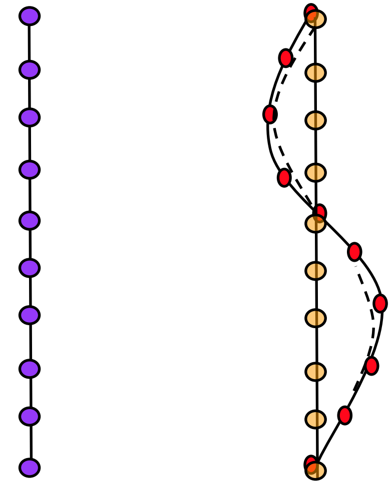
Role of model mismatch : Example



- Theory: Array is a straight line ("linear array")
- In practice: Array elements wiggle a bit
  - (e.g. when being towed behind a ship)
- Over-zealous mathematical solutions:
  - Model the wiggling as well?
  - Fancy optimization over worst case wiggling?
- Real-world solution: "simple" modifications of simple algorithms work best!
  - e.g. $\hat{P}_{\text{Capon}}(\theta) = \dfrac{1}{\mathbf{v}^H(\theta)\hat{\mathbf{R}}^{-1}\mathbf{v}(\theta)}$

> - Testing with data sets only way to know for sure
> - Moral: Remain confident in mathematical solution but resist temptation to over-optimize!

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Mathematical Questions (abridged)

- Given true **R**, subspace spanned by a set of eigenvectors is (often) sufficient to extract signals
- Large sensors, Small data situation:
  - Blurring of eigenvalues + Blurring of eigenvectors
- Much more known about eigenvalues then eigenvectors
  - Eigenvectors more important in practice (eigenvalues can always be tweaked!)

- Analysis: Blurring of eigenvectors
  - Models of interest:
    - Rectangular random matrices
    - Matrix valued processes
- Design:
  - Parametric deblurring of sample eigenvalues: Possible
  - Parametric deblurring of sample eigenvectors: Possible?
- Non-parametric approaches to combat model sensitivity: Possible?

Summary

- Lots of existing well-defined problems to get feet wet!
- New ideas from random matrix theory appear particularly relevant
- Mathematics fundamental enough to eventually be useful
- Collaborations accelerate the process