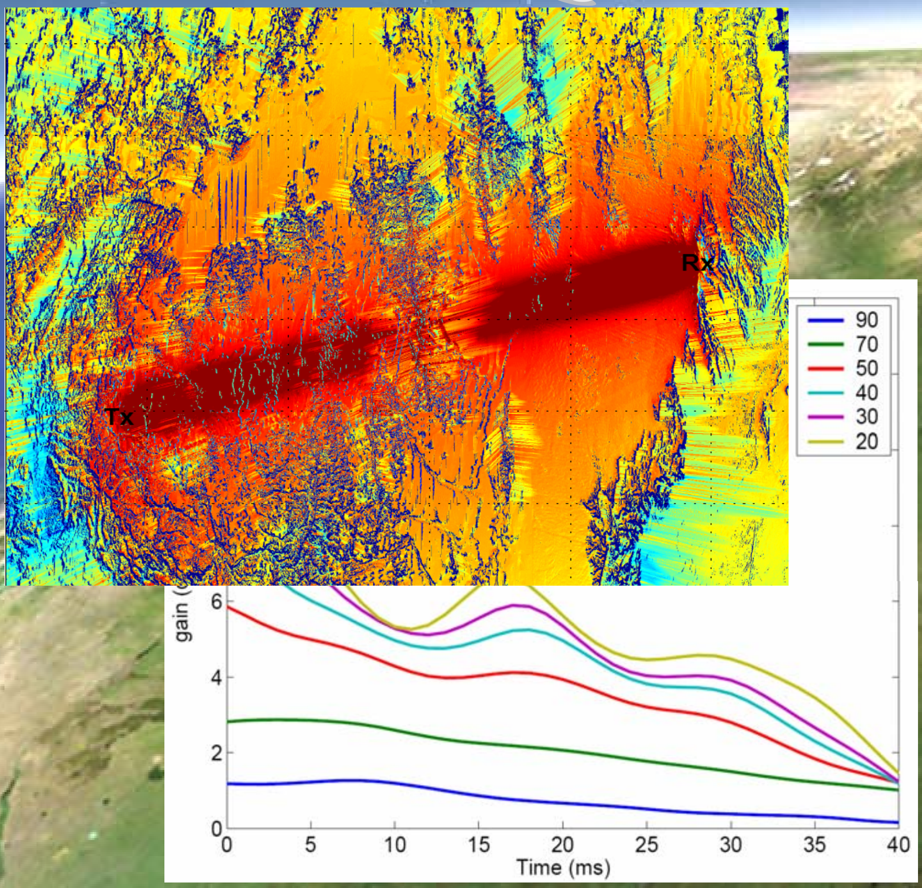
Constrained Waveform Design

Using Matched Subspaces



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10-14 July 2006

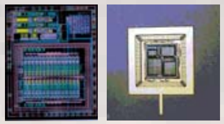
Outline

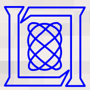
- Emerging Technologies & Applications
- Adaptive Waveform SAM Processing
  - Agility vs Adaptivity
  - Functional vs Channel Adaptivity
- Channel-Adaptive Waveform Theory
- Applications
- Implementation Issues
- Future

A Confluence of Events

Advanced HPEC

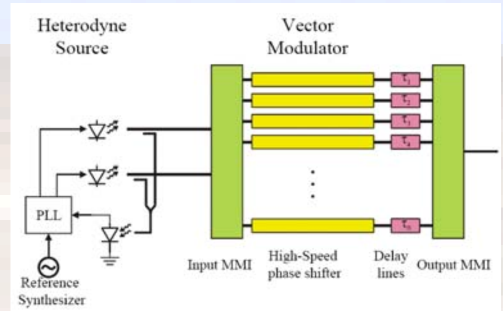
MP-510



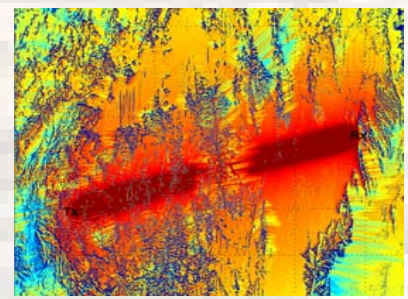


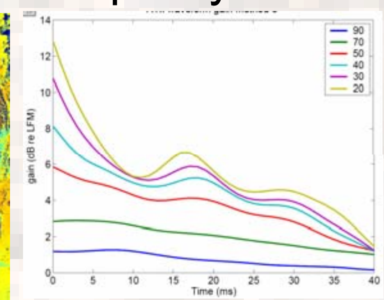
ASIC high-speed cache memory devices

Digital AWGs (DAWGs) DARPA AOSP

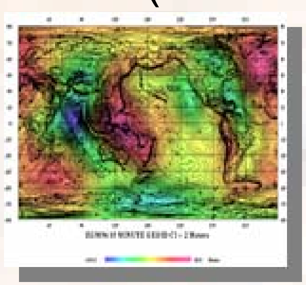


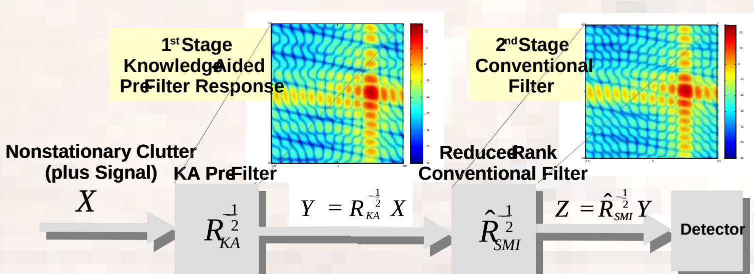
Waveform Adaptivity



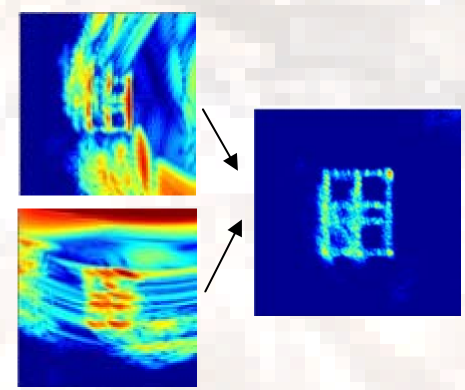


Knowledge-Aided Processing (DARPA KASSPER)





Spatio-Temporal Waveform (“Input”) Adaptivity



Functional versus Channel Adaptivity

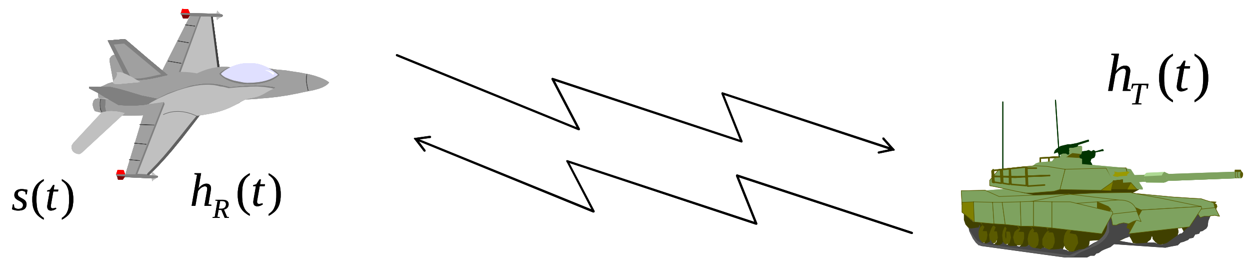
Functional Adaptivity:

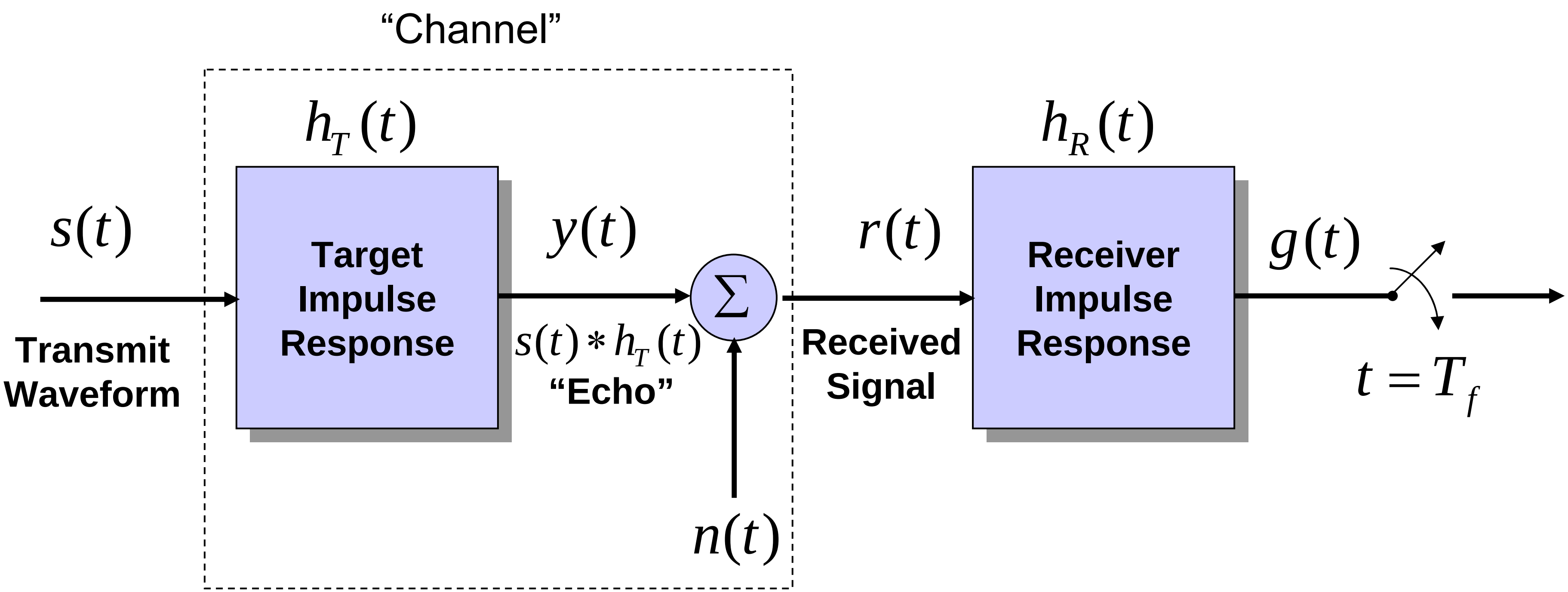
- Sensor can perform more than one function due to ability to transmit a diverse set of waveforms
  - Example: Search, Track, ID & Countermeasure
    - Nature's embodiment: The Bat!
  - Example: Radar & Comms
- Rule-Base Adaptivity
  - Rules used to select waveforms that are pre-computed

Channel Adaptivity

- Sensor can create "waveforms-on-the-fly" to adapt to varying "channel" (target, colored noise, etc.)
  - Much greater real-time computation burden

Basic Waveform Design

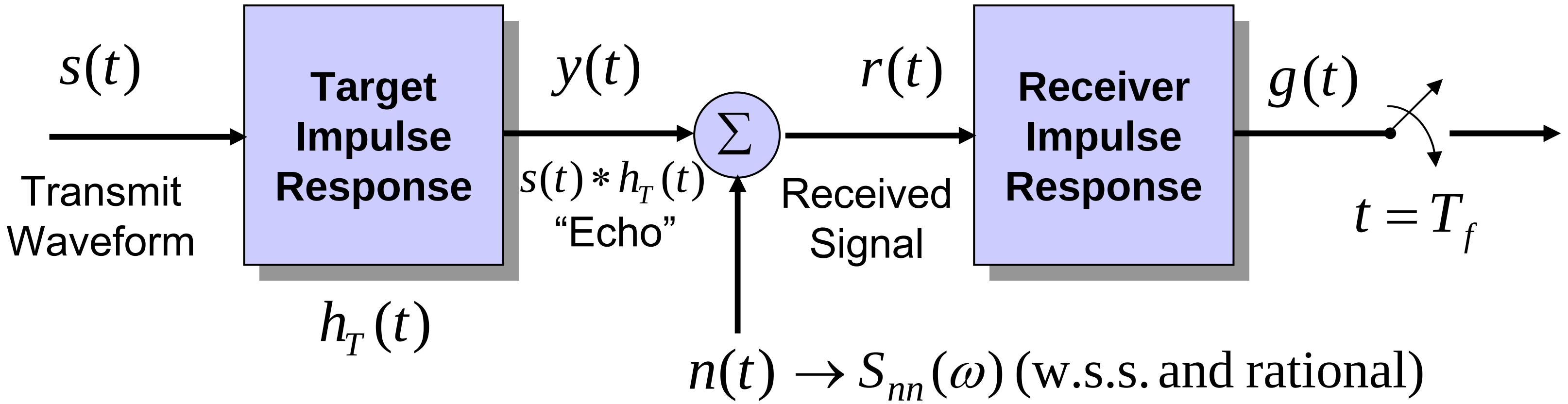




- Choose both transmit waveform ***s*(*t*)**, and receiver **$h_R(t)$** to maximize SINR (for example)

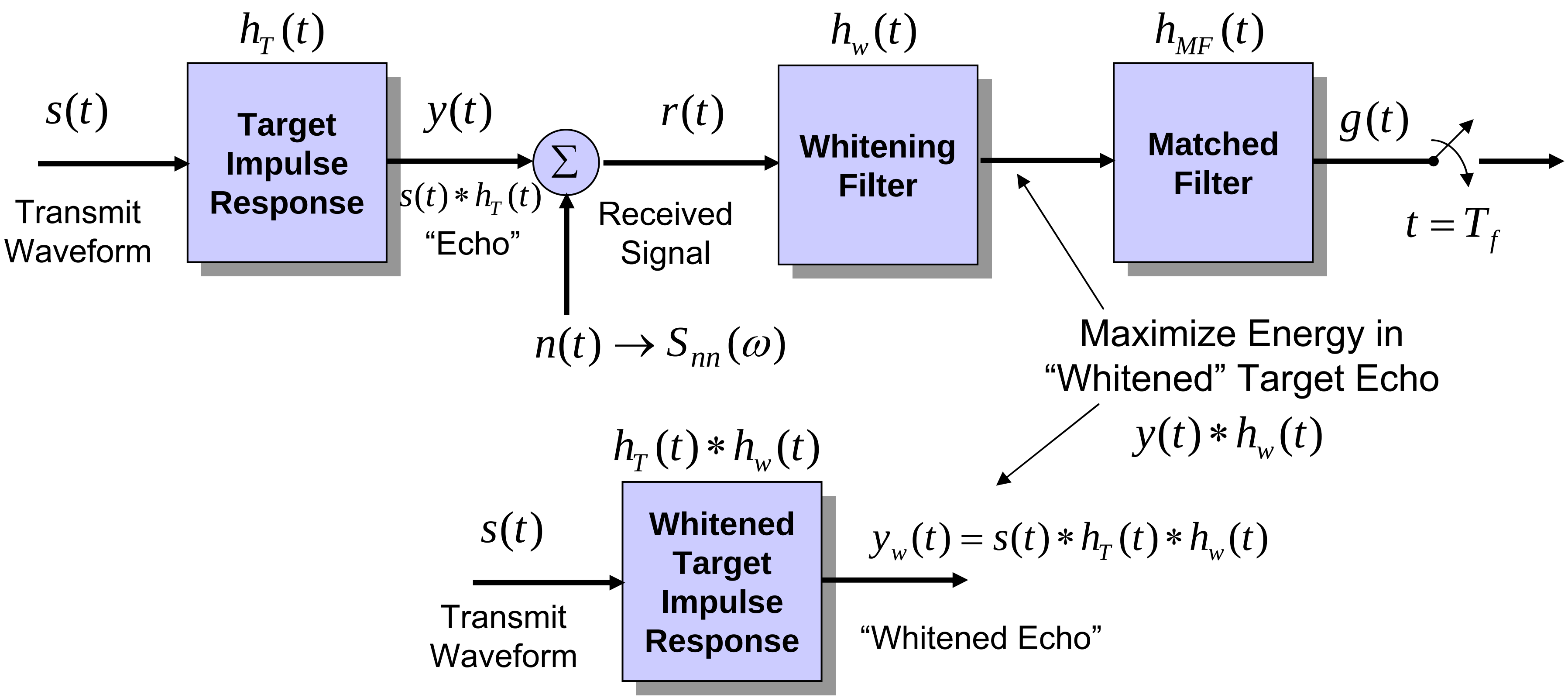
Additive Colored Noise Case

- Additive colored noise (ACN) case



- Step 1: Optimal receiver (in a max SINR sense), consists of a "whitening" filter followed by a matched filter (matched to the "whitened" target echo response)
  - Existence of causal and stable whitening filter guaranteed due to w.s.s. and rationality assumption
  - Van Trees, *Det., Est. and Mod. Theory, I,* 1968

ACN Case (cont.)



- Corresponding output SINR:

$$\text{SINR}_o = \frac{1}{\sigma_w^2} \int_{T_i}^{T_f} \left| y_w(t) \right|^2 dt = f(s(t))$$

Choose $s(t)$ to maximize SINR_o (variational calculus)

ACN Case (cont.)

- Step 2: Choose $s(t)$ to maximize SINR_o

$$\max_{s(t)} = \frac{1}{\sigma_w^2} \int_{T_i}^{T_f} \left| y_w(t) \right|^2 dt$$

$$\text{subject to} \ : \ s(t) \in [0, T], \text{ and } \int_0^T \left| s(t) \right|^2 dt < \infty$$

$$\int_{T_i}^{T_f} \left| y_w(t) \right|^2 dt = \int_{T_i}^{T_f} \left| s(t) * h(t) \right|^2 dt = \int_{T_i}^{T_f} \left(s(t) * h(t) \right) \left(s^*(t) * h^*(t) \right) dt$$

$$= \int_{T_i}^{T_f} \left(\int_0^T s(\tau_1) h(t-\tau_1) d\tau_1 \right) \left(\int_0^T s^*(\tau_2) h^*(t-\tau_2) d\tau_2 \right) dt$$

$$= \int_0^T s(\tau_1) \int_0^T s^*(\tau_2) \int_{T_i}^{T_f} h(t-\tau_1) h^*(t-\tau_2) dt \, d\tau_2 \, d\tau_1$$

$$= \int_0^T s(\tau_1) \int_0^T s^*(\tau_2) K^*(\tau_1, \tau_2) d\tau_2 \, d\tau_1$$

$$\text{where } h(t) \stackrel{\Delta}{=} h_T(t) * h_w(t) \text{, and } K(\tau_1, \tau_2) \stackrel{\Delta}{=} \int_{T_i}^{T_f} h^*(t-\tau_1) h(t-\tau_2) dt$$

ACN Case (cont.)

- Step 2: Cont.

$$\max_{s(t)} \int_0^T s(\tau_1) \int_0^T s^*(\tau_2) K^*(\tau_1, \tau_2) d\tau_2 \, d\tau_1$$

Applying Schwarz's inequality yields:

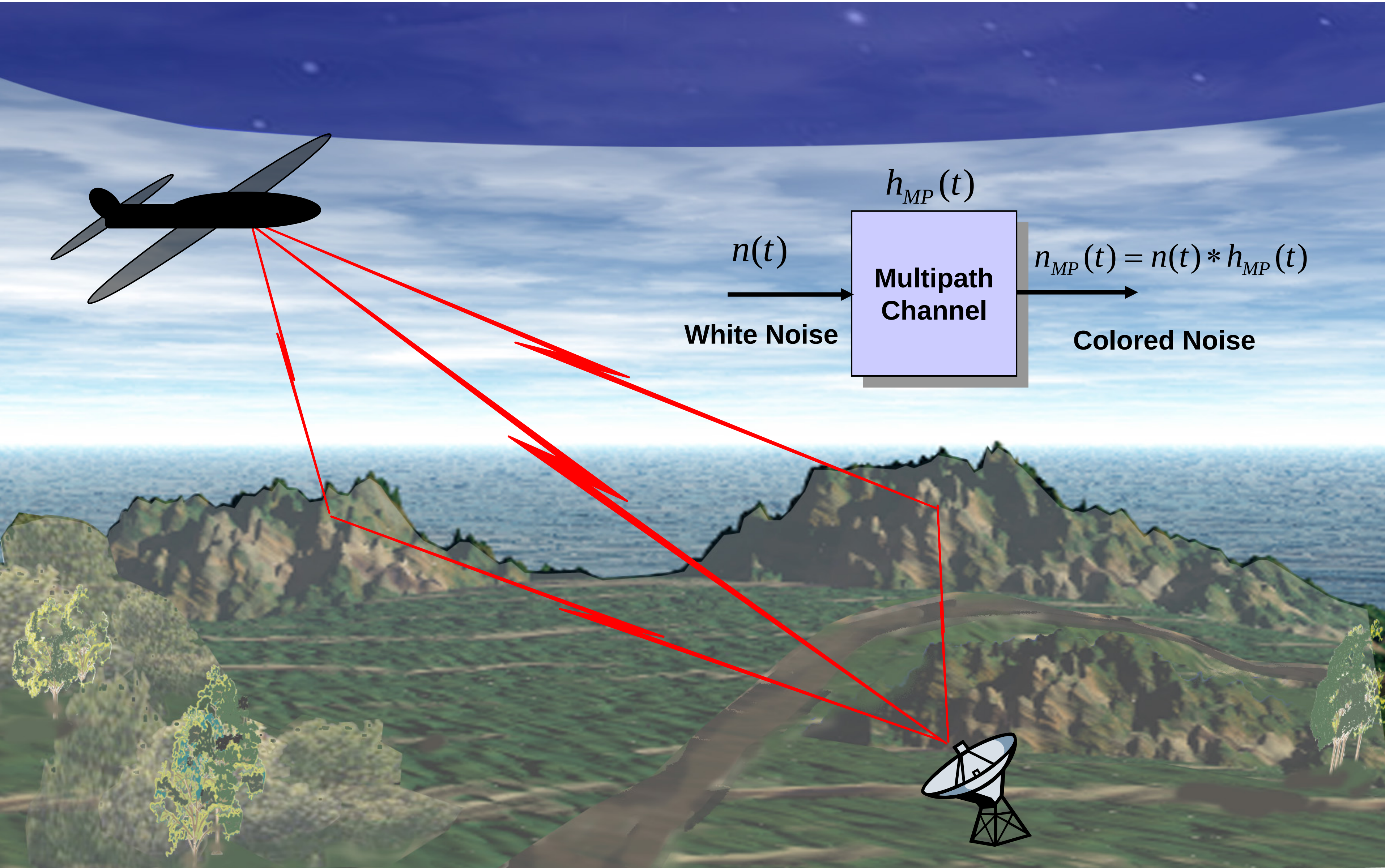
$$\lambda s(\tau_1) = \int_0^T s(\tau_2) K(\tau_1, \tau_2) d\tau_2$$

- Optimal transmit waveform must satisfy a homogeneous Fredholm integral of the 2nd kind with generally Hermitian p.d. kernel (i.e., "eigensystem")

$$\lambda_{\max} s_{\text{opt}}(t) = \int_0^T s_{\text{opt}}(\tau) K(t-\tau) d\tau$$

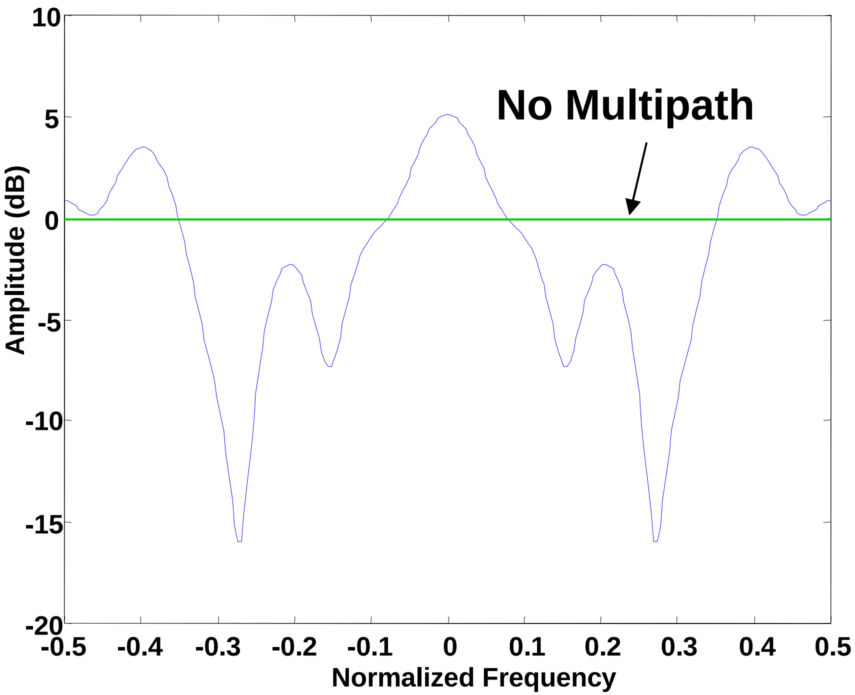
Note: $s_{\text{opt}} \xleftrightarrow{F} S_{\text{opt}}(\omega) \neq H_w^*(\omega)$ (Matched Spectrum Solution)

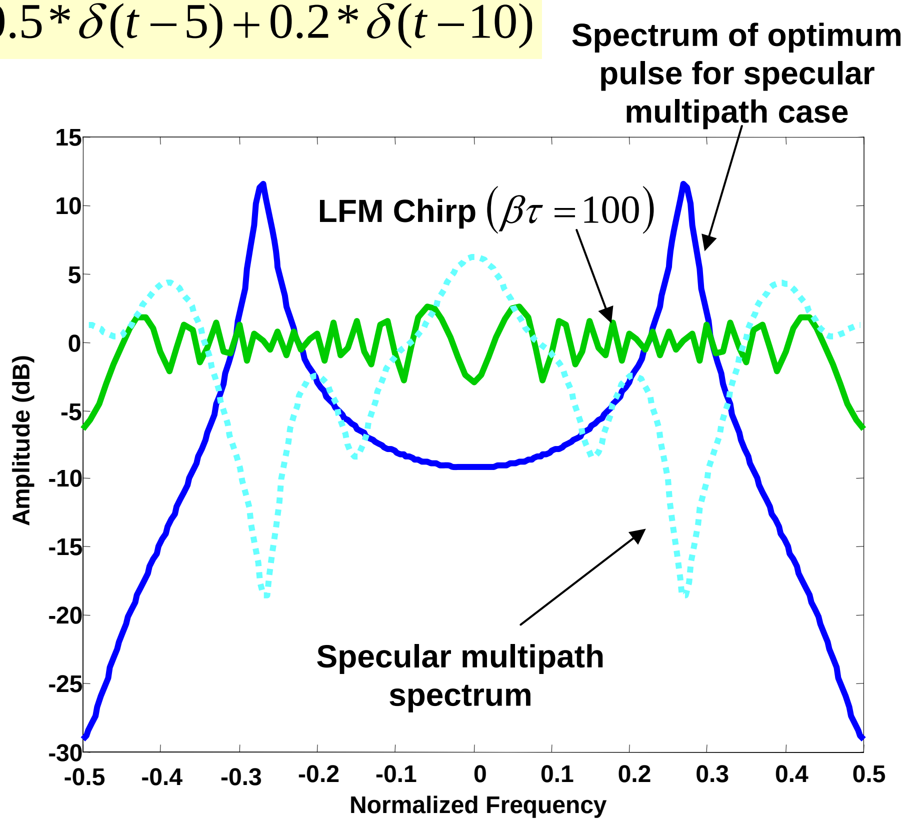
Multipath Interference



Interference Multipath Example

$$h_{MP}(t) = \delta(t) + 0.9\delta(t-2) + 0.5*\delta(t-5) + 0.2*\delta(t-10)$$





- 9.6 dB gain over LFM waveform and MF
  - 8.6 dB matching gain on transmit
  - ~1.0 dB gain on receive (matched filter)

Other Colored Noise Examples

- “Narrowband” EM Interference (EMI)
  - Examples of optimal waveform design
  - Max SINR, Target ID, Colored Noise
- Wideband Spectral Optimization
  - Avoid interfering with other spectral bands yet achieve relatively large bandwidths
- Channel Pre-Equalization
  - Tx/Rx passband filter response matching
    - Example: NLFM
  - Atmospheric effects
    - Particularly in OTH and/or ducting environment

Discrete Case

- Discrete-time equivalent:

$$\mathbf{y} = H\mathbf{s}$$

Composite Channel (target + whitening filter)

Output (echo)

Input (radar pulse)

$$(H'H)\mathbf{s} = \lambda\mathbf{s} \qquad \text{(Optimal Pulse)}$$

where for the causal case

$$H = \begin{bmatrix} h[0] & 0 & \dots & 0 \\ h[1] & h[0] & \dots & 0 \\ h[2] & h[1] & \dots & 0 \\ \vdots & & & \ddots \end{bmatrix}$$

and where for ACN case

$$h[n] = h_T[n] * h_w[n]$$

$$H = H_w H_T$$

$$\mathbf{s} \in C^M, \quad \mathbf{y} \in C^N, \quad H \in C^{N \times M}$$

Discrete MIMO Case

MI: $\mathbf{s} = \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_L \end{bmatrix} \in C^{M_1 M_2 \ldots M_L}$

MO: $\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_K \end{bmatrix} \in C^{N_1 N_2 \ldots N_K}$

$$H = \begin{bmatrix} H_{11} & H_{12} & \ldots & H_{1L} \\ H_{21} & H_{22} & \ldots & H_{2L} \\ \vdots & & \ddots & \vdots \\ H_{K1} & H_{K2} & \ldots & H_{KL} \end{bmatrix}$$

} Very High Dimensionality!

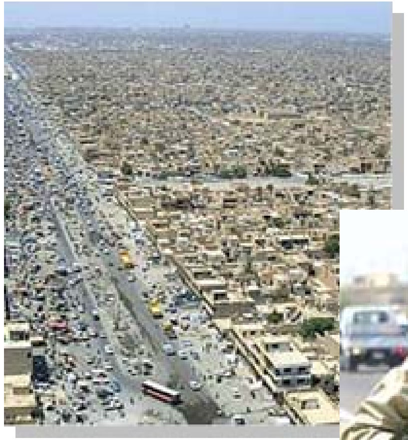
$$\in C^{N \times M} = C^{(N_1 \cdots N_L) \times (M_1 \cdots M_K)}$$

Waveform-Optimized MIMO

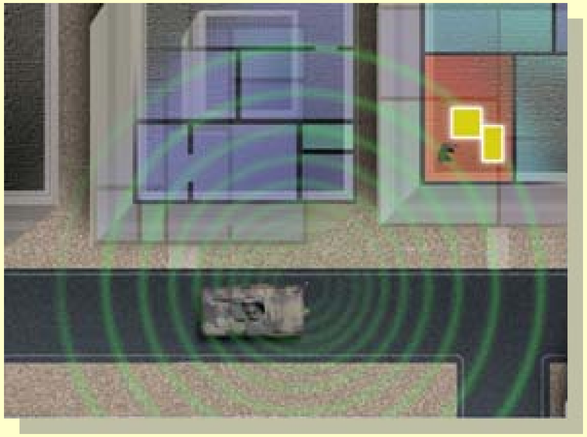
WO-MIMO: $$\boxed{(H'H)\mathbf{s} = \lambda \mathbf{s}}$$

Real-World WO-MIMO: Visibuilding

- Wars increasingly fought in urban domain, but Warfighter has no ISR capability inside of buildings
  - We do not know where to search, or where threatening personnel might be
  - We may not want to search some buildings (e.g., churches, mosques)

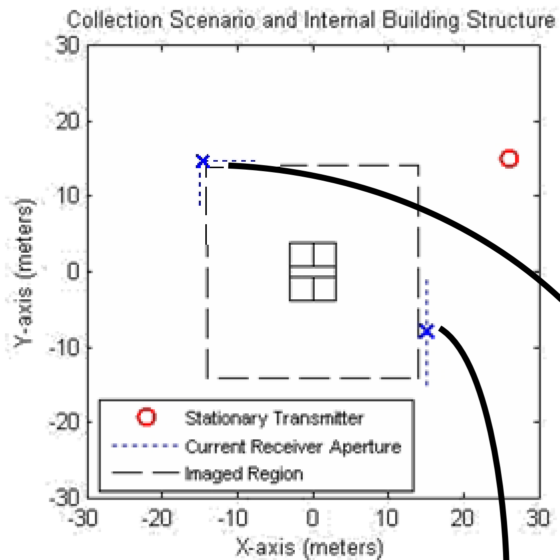


- Need building penetration sensing to own the interior urban space
  - Find personnel inside of buildings
  - Provide building layouts (walls, rooms, stairs, doorways)
  - Identify weapons caches, shielded rooms, etc.

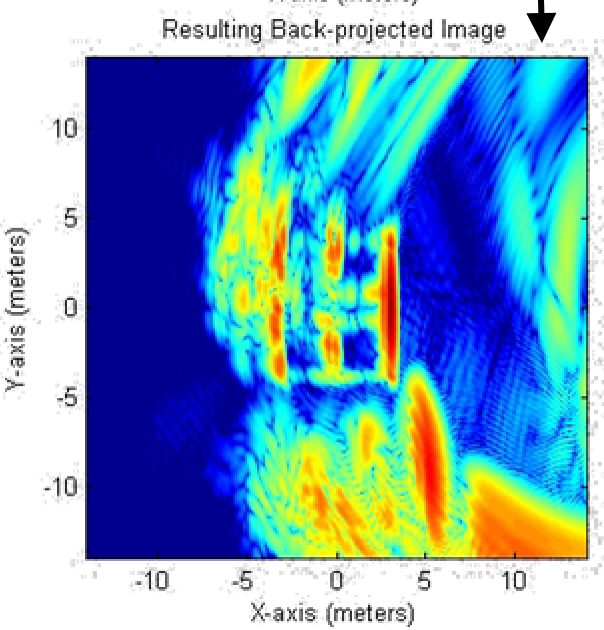


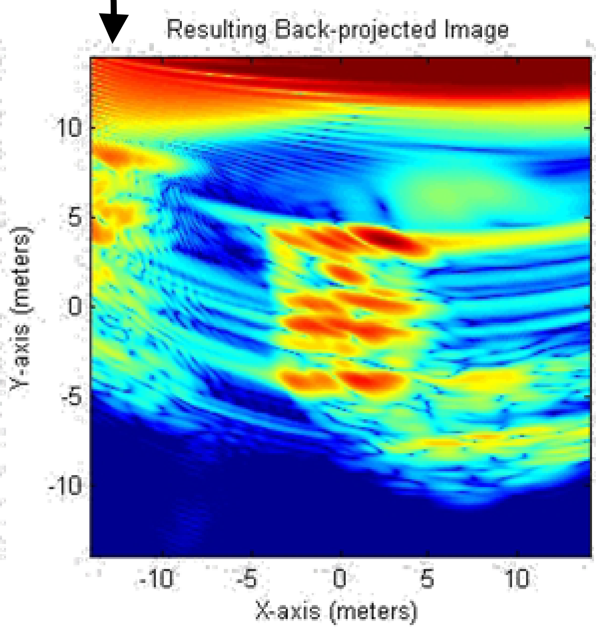
Multi-Sensor CONOPS

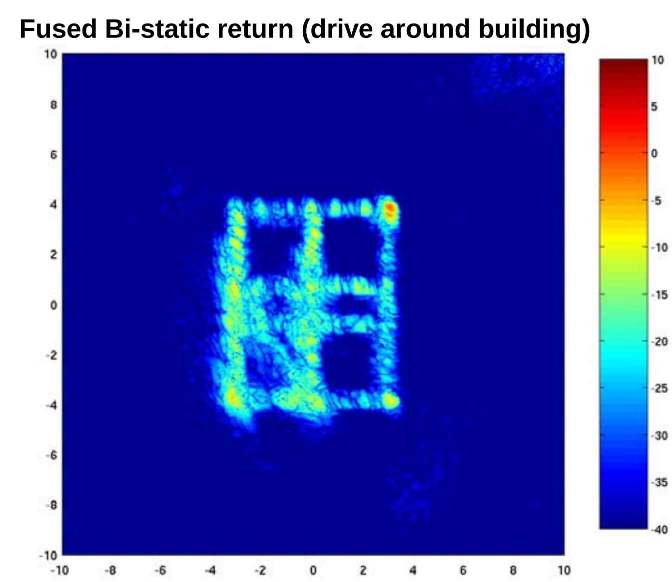
Spatial Diversity Can Increase Information Content



- Angular diversity provides unique reflections from specular surfaces
- Spatial coverage can be accomplished with receiver and/or transmitter diversity
- Various possible CONOPS:
  - Hard: Fixed sensors (< 6 sensors around a building)
  - Harder: Drive-by (vehicle mounted for IPB of broad areas)
  - Hardest: Low-altitude fly-by (UAV-mounted)







The Need for Reduced-Rank WO-MIMO: Adaptive WO-MIMO

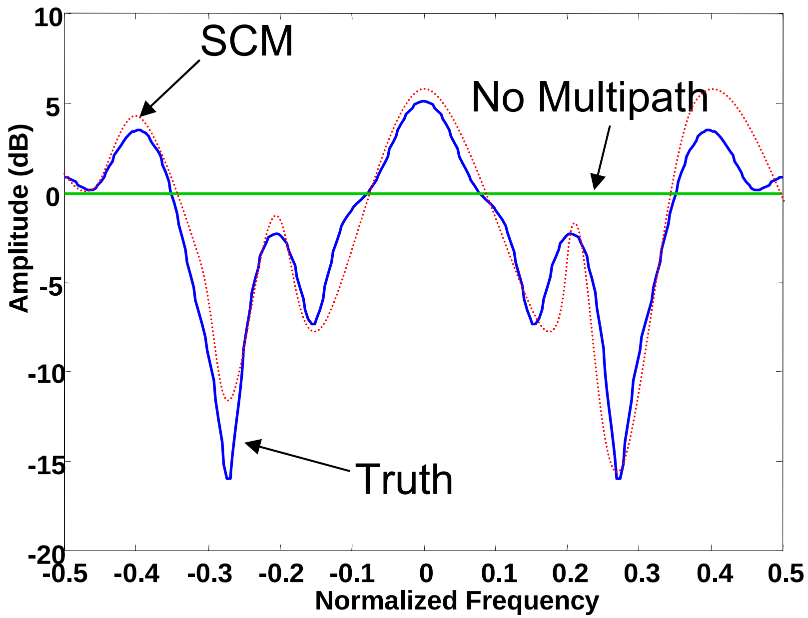
- The real power of waveform diversity is ADAPTIVE waveform diversity!
  - Real-time environmental awareness possible via DARPA KASSPER architecture
- Adaptive waveform optimization can quickly lead to an enormous real-time computing burden
  - Above and beyond adaptive receiver/antenna requirements
  - ALL real-world waveforms are CONSTRAINED
    - Generally highly nonlinear optimization problem
- Some thoughts:
  - KASSPER Architecture
  - Salient Channel Model (SCM)
  - Waveform Projection
  - SWORD (Jian Li et al)

"Channel"

$$\underbrace{(H'H)\mathbf{s} = \lambda \mathbf{s}}_{\text{Waveform Design}}$$

Salient Channel Model (SCM)

- Fact: Large WO-MIMO gains come from significant deviations from all-pass transfer function
- Large deviations are generally easier to predict than small ones
  - Examples: Large discrete scatterers (clutter, target); Resonances; Significant topological features (e.g., mountains); etc.
- SCM solution: *“Don’t sweat the small stuff!”*



Reduced-rank Physics Model

Constrained Optimization

- In general:

$$\max_{\mathbf{s}} \mathbf{s}'(H'H)\mathbf{s}$$

SINR Gain

subject to:

$$\mathbf{f}(\mathbf{s}) = \mathbf{q}$$

Equality

$$\mathbf{g}(\mathbf{s}) \leq \mathbf{r}$$

Inequality

$$\|\mathbf{s}\| < \infty$$

Finite Norm

- Nonlinear multidimensional functional optimization
  - Generally not practical for real-time implementation

Properties of Channel Kernel

- Under fairly general conditions:

$$(H'H)\mathbf{u}_i = \lambda_i \mathbf{u}_i$$

where:

$$\mathbf{u}_i'\mathbf{u}_j = \delta_{ij}$$

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N > 0$$ (assume P.D.)

thus

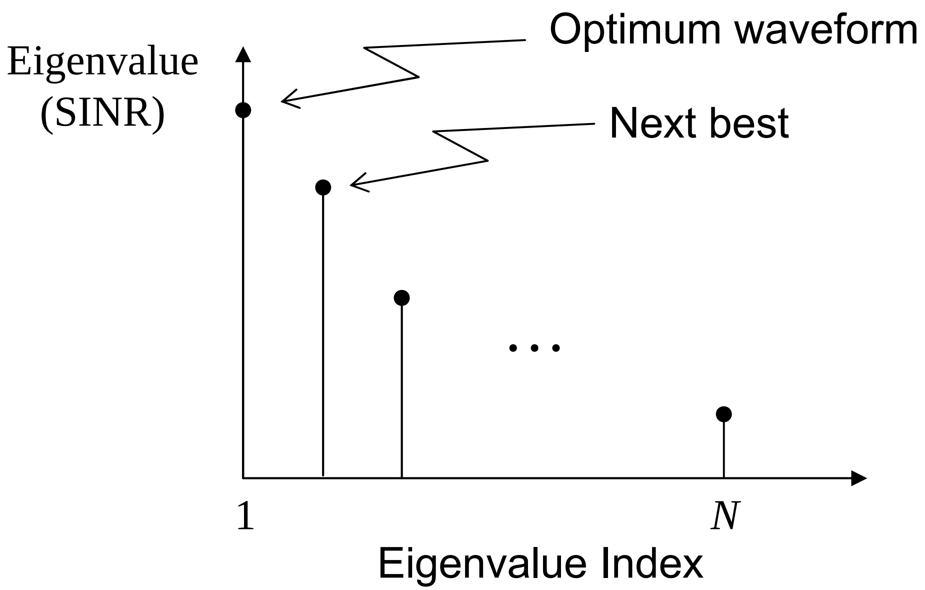
$$\boxed{\mathbf{w} = \sum_{i=1}^{N} (\mathbf{w}'\mathbf{u}_i)\mathbf{u}_i}$$

where

$$\mathbf{w} \in C^N : \|\mathbf{w}\| < \infty$$

Properties of Channel Kernel (cont.)

- General notional eigenvalue distribution:



$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N > 0$$

Observations

- Obs #1: Matched waveform gains only possible if eigenvalue spread is "large"
  - Significant deviation from "all pass" channel required to make waveform optimization worthwhile
  - Potential for real-time gains can be deduced via offline modeling and simulation
    - Determine *a priori* whether matched waveform processing is worth it for a given application

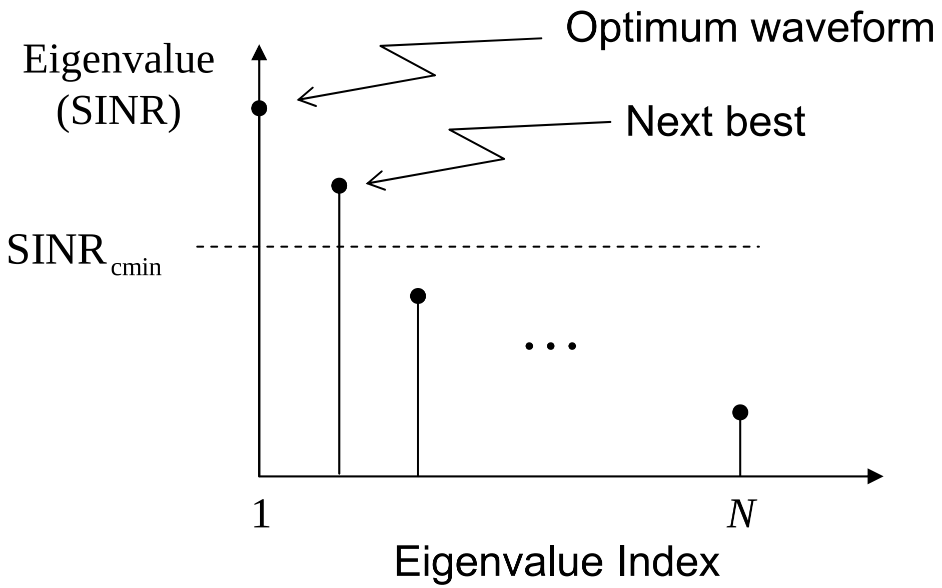
- Obs #2: Let $\mathbf{w}$ be a waveform that meets constraints:
  - Then:

$$SINR_{\mathbf{w}} \leq SINR_{\mathbf{u}_1}$$

  - Thus, one must be willing to lose SINR to achieve waveform constraints

Observations (cont.)

- <u>Obs #3:</u> In practice there is a minimal acceptable constrained matching gain ($SINR_{cmin}$)

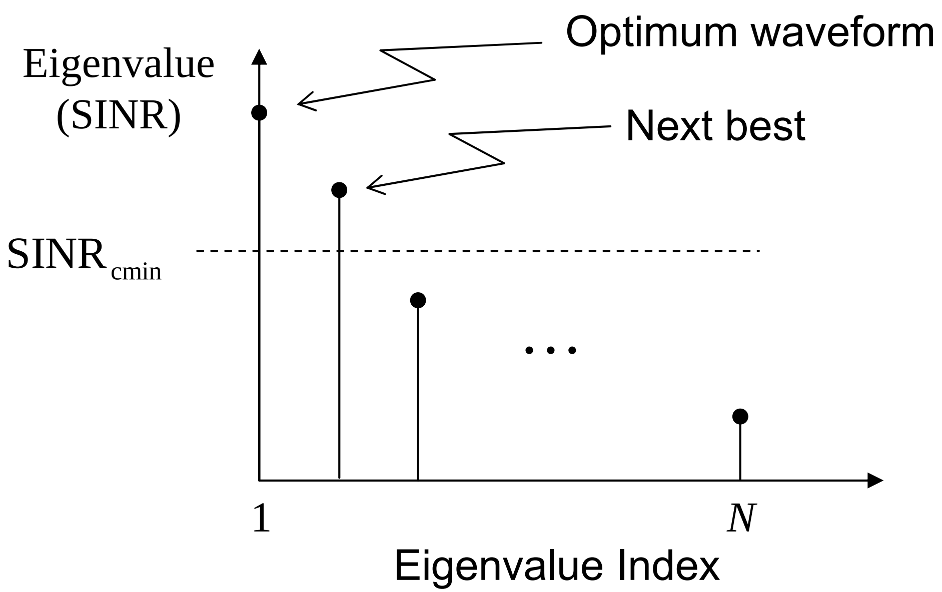


- <u>Obs #4:</u> If the subspace spanned by the constrained set does not have a significant projection onto "matched" subspace", stick with conventional waveform

Observations (cont.)

- Obs #5: "Matched Subspace"

$$\Omega_{\mathrm{MS}} = \left\{ \mathbf{u}_1, \mathbf{u}_2, \ldots \mathbf{u}_k : \lambda_i \geq \mathrm{SINR}_{\mathrm{cmin}}, \forall i \leq k \right\}$$



- Obs #6: If constrained optimization is confined to Ω_{MS}, then solution is guaranteed to meet minimal SINR constraint

Projection onto Matched-Subspace (POMS)

Let $\mathbf{w} \in C^N$ satisfy $\Omega_c = \{\mathbf{f}(\mathbf{s}) = \mathbf{q}, \mathbf{g}(\mathbf{s}) \le \mathbf{r}\}$

$$\min_{\hat{\mathbf{w}}} \quad \|\mathbf{w} - \hat{\mathbf{w}}\|_2$$

subject to: $\hat{\mathbf{w}} \in \Omega_{\text{MS}}$

$\hat{\mathbf{w}}$ is waveform closest to $\mathbf{w}$ in "matched subspace"

$$\hat{\mathbf{w}} = \sum_{i=1}^{k} (\mathbf{w}'\mathbf{u}_i)\mathbf{u}_i$$

Constrained Waveform Design

Special Projects Office

US005146229A

United States Patent [19]

Guerci et al.

[11] **Patent Number: 5,146,229**

[45] **Date of Patent: Sep. 8, 1992**

[54] CONSTRAINED OPTIMUM MATCHED ILLUMINATION-RECEPTION RADAR

[75] Inventors: Joseph R. Guerci, Astoria; Robert W. Schutz, Lindenhurst; John D. Hulsmann, Miller Place, all of N.Y.

[73] Assignee: Grumman Aerospace Corporation, Bethpage, N.Y.

[21] Appl. No.: 720,671

[22] Filed: Jun. 25, 1991

[51] Int. Cl.⁵ G01S 13/28
[52] U.S. Cl. 342/204; 342/132
[58] Field of Search 342/204, 132, 192, 82, 342/83

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Primary Examiner—Gilberto Barrón, Jr.
Attorney, Agent, or Firm—Bacon & Thomas

[57] ABSTRACT

A pulse compression modified OMIR waveform $\bar{s}_N(t)$ is obtained by computing the OMIR eigenfunctions ϕ_i, $i=1, 2, \ldots, \infty$, for an autocorrelation function of the expected target impulse response, specifying a waveform c(t) having a desired pulse compression characteristic, and generating expansion terms

$$\bar{s}_N(t) = \sum_{i=1}^{N} c_i \phi_i(t)$$

for various expansion indices N, until a desired waveform is obtained. The expansion coefficients $c_i(t)$ are given by

$$c_i = \int_0^T c(t)\phi_i^*(t)dt.$$

6 Claims, 4 Drawing Sheets

Eigenspectrum

dB

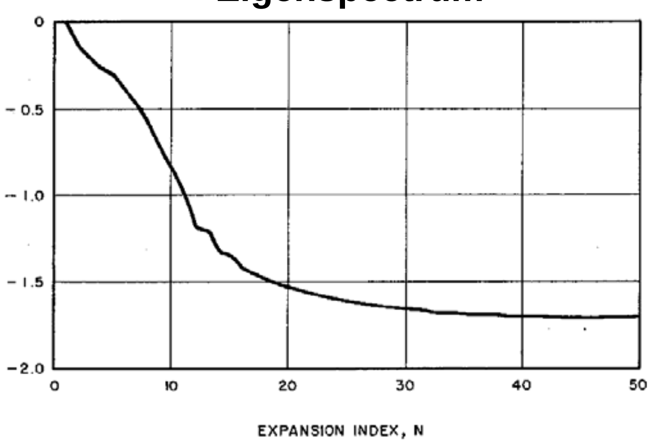
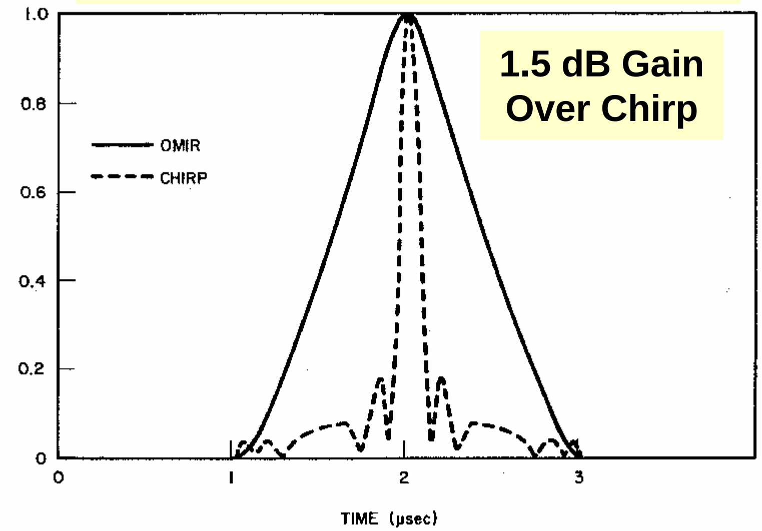


Fig. 3

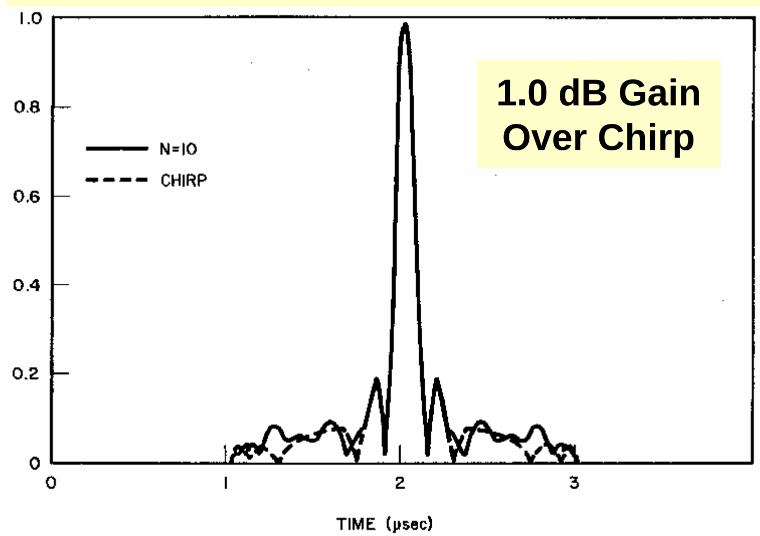
U.S. Patent Sep. 8, 1992 Sheet 3 of 4 5,146,2

Unconstrained Optimum Waveform



1.5 dB Gain Over Chirp

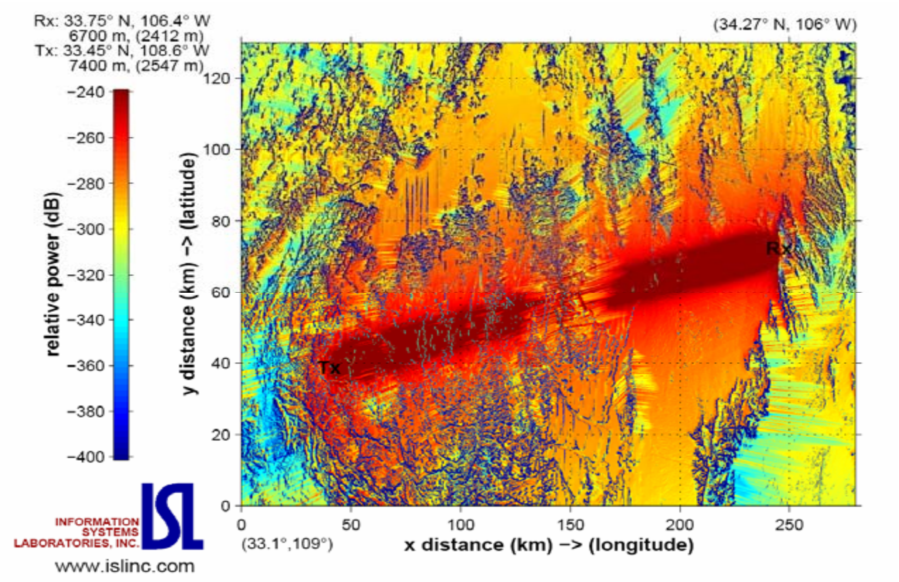
Constrained Waveform w/ 10 Components

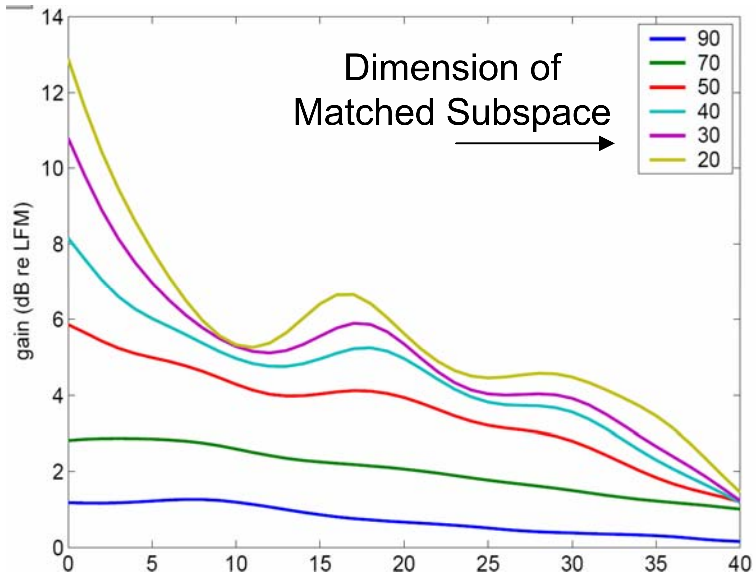


1.0 dB Gain Over Chirp

Terrain Scatter Example*

Interference Source Speed 165 m/s
Receiver Speed 125 m/s





Weight Application Delay (ms)

*J. S. Bergin, P. M. Techau, J. E. Don Carlos, and J. R. Guerci, “Adaptive Waveform Design for Colored Noise”, 2005 IEEE Radar Conf.

Generalized POMS Algorithm (G-POMS)

- Initialization: Create a set of nominal non-matched waveforms that meet the constraints Ω_c

$$\Omega_0 = \{\mathbf{w}_i : \mathbf{w}_i \in \Omega_c, \forall i\}$$

offline

- Step 1: Kernel test for matching gain potential

$$\mu(\lambda_{\max} / \lambda_{\min}) \geq \delta_{\min}$$

- Step 2: $SINR_{cmin}$ defines matched subspace

$$\Omega_{MS} = \{\mathbf{u}_1, \mathbf{u}_2, \ldots \mathbf{u}_k : \lambda_i \geq \lambda_{cmin}, \forall i \leq k\}$$

- Step 3: Project $\mathbf{w}_i$ onto matched subspace and test for gain

$$\mathrm{SINR}_i \overset{?}{\geq} \mathrm{SINR}_{cmin} \rightarrow \{\hat{\mathbf{w}}_i\}$$

Generalized POMS Algorithm (G-POMS)

- Step 4: Of the admissible projection set, calculate constraint deviation and compare to threshold

$$\mathbf{f}(\mathbf{w}_i) = \mathbf{q}_i$$

$$\mathbf{g}(\mathbf{w}_i) = \mathbf{r}_i$$

$$\|\mathbf{q}_i - \mathbf{q}\| \le \delta_q$$

$$\|\mathbf{r}_i - \mathbf{r}\| \le \delta_r \text{ (assuming } \mathbf{r}_i < \mathbf{r}\text{)}$$

- Step 5: Choose best matched waveform from Step 4

Summary & Areas for Future Research

Summary:

- WO-MIMO has huge potential
- Adaptivity *IS* the key to realizing WO-MIMO potential
  - Enormous real-time processing challenge
- Notion of SCM coupled with KASSPER for practical implementation
- POMS, G-POMS & SWORD approaches to achieve practical waveforms

New Areas for Research

- New real-world application areas
- Iterative methods that refine the G-POMS/SWORD solutions to yield global optimality
- Development of a rich and diverse set of “nominal” waveforms meeting non-adaptive constraints (resolution, sidelobes, etc.)
  - Ensures that one or more waveforms will be “close” to the admissible matched subspace
- Other approaches: Waveform Perturbation…?