Applications of Random Matrix Theory to Economics, Finance and Political Science

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Outline

1. Portfolio Selection
2. Factor Models
3. Beyond Covariances

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Econophysics and Portfolio Section

First attempt at applying RMT in Finance - cleaning data (Potters and Bouchaud)

Aim: choose a portfolio of N assets with weights w_i. Variance of portfolio returns is given by $V = \sum_{ij} w_i \sigma_i C_{ij} \sigma_j w_j$.

Minimize risk for a given expected return or hedge assets against each other - choose weights w for assets in portfolio.

Idiosyncratic Volatility

- Improve portfolio choice by removing *idiosyncratic noise* correlation matrix using RMT
- Methods: eigenvalues cut-offs, shrinkage estimators, fixed point covariances (Frahm '05)

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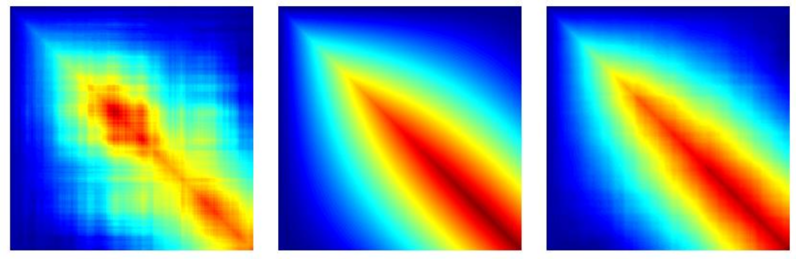
Cleaning correlations by removing idiosyncratic noise



Success?

- Leads to better risk estimates ...
- But focuses exclusively on the noise, no real model of asset pricing
- Sensitive to assumptions on idiosyncratic noise - is there a role for power laws?

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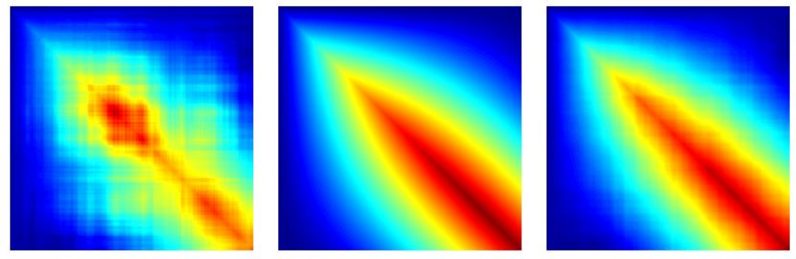

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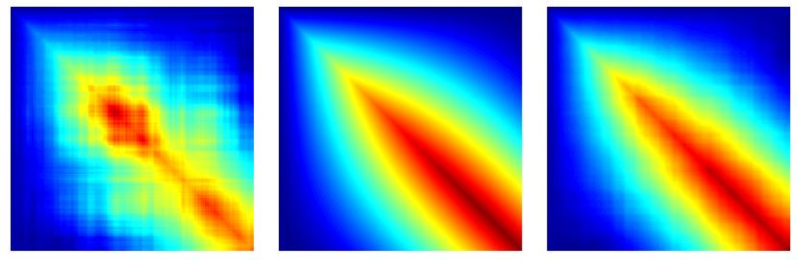

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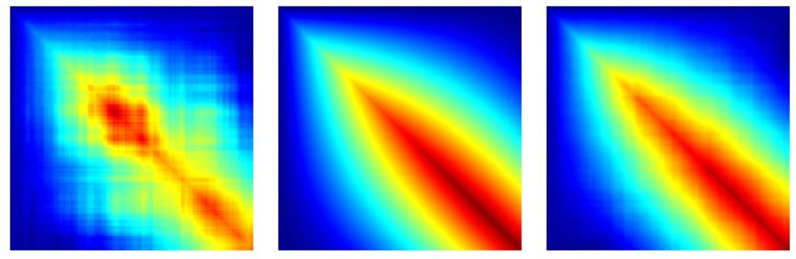

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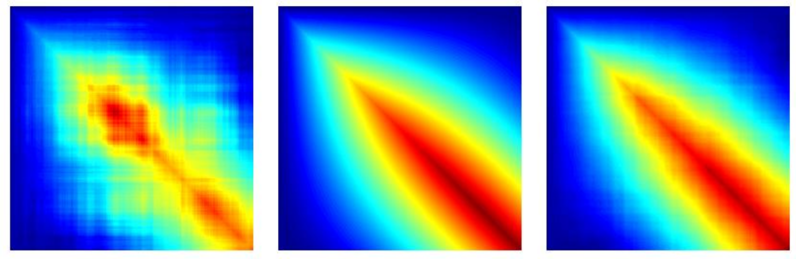

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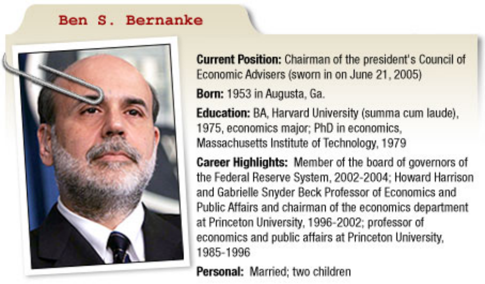

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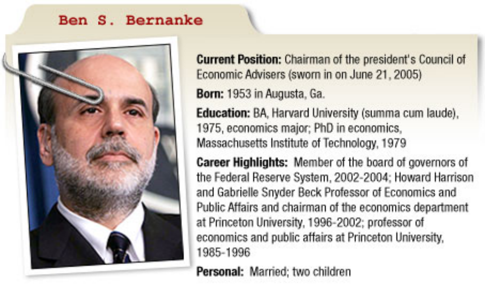

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Factor Models are Back !

$$BY_t + \Gamma Z_t + \Lambda F_t + U_t = 0, \tag{1}$$

where $t = 1..T$.

Assumption 1: The number of endogenous variables increases with the sample size T. Thus, $n \to \infty$, $T \to \infty$ and $n/T \to c \in (0, \infty)$.

Definitions

- Y_t : $n \times 1$dimensional vector of endogenous variables
- Z_t : $k \times 1$vector of exogenous variables
- F_t : $p \times 1$ vector of unobserved factors
- Λ : $n \times p$ matrix of factor loadings

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Special case

$$Y_t = \Lambda F_t + U_t, \tag{2}$$

where $t = 1..T$.

Familiar to signal processing world: k unobserved signals F and noise U.

Follows from **No Arbitrage** restrictions in asset pricing models.

Y usually corresponds to asset returns (stocks, bonds etc.)

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Identification of factors, estimation of number of factors, estimation of loadings, tests on estimated factors ...

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Brown (1989) Puzzle

- An economy with K factors, each of which is priced and contributes equally to the returns and calibrated to actual data from the NYSE.
- Nevertheless, he finds evidence that estimations are biased towards a single factor model.

Answer: Phase transition in spiked model (Paul '05)

b) $\hat{\lambda}_i \stackrel{a.s.}{\rightarrow} \left\{N\sigma_F^2 \ \sigma_\beta^2 + \sigma_\epsilon^2\right\}\left\{1 + \frac{1}{T}\frac{\sigma_\epsilon^2}{\sigma_F^2\sigma_\beta^2}\right\}$, for $i = 2...K$

c) $\hat{\lambda}_i \stackrel{a.s.}{\rightarrow} \sigma_\epsilon^2(1 + \sqrt{N/T})^2$, for $i = 2...K$

depending on $N <> \frac{1}{T}\left(\frac{\sigma_\epsilon^2}{\sigma_F^2\sigma_\beta^2}\right)^2$; (Harding '06 $N > 101,396$)

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Minimum Distance/GMM Estimation of Model Parameters

Assume we observe sample covariance $\hat{\Omega}$ from a model $\Omega(\theta)$. Let $\Pi(\theta) = lim_{N \to \infty} Etr(\Omega^s)$ (Free Probability) and $\widehat{\Pi} = tr(\hat{\Omega}^s)$ (Sample). Then, $\widehat{\theta} = argmin_{\theta}(\Pi(\theta) - \widehat{\Pi})' \, \widehat{V}^{-1} \, (\Pi(\theta) - \widehat{\Pi})$.

Strategy for Identifying the Number of Factors

- Order the sample eigenvalues in decreasing order $\lambda_1, \lambda_1, ..., \lambda_N$.
- Estimate θ using CMD and compute J-test (overid test) Hansen (1982) recursively using the smallest $N - q$ eigenvalues
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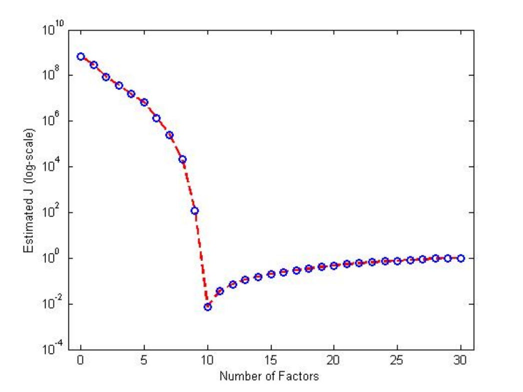

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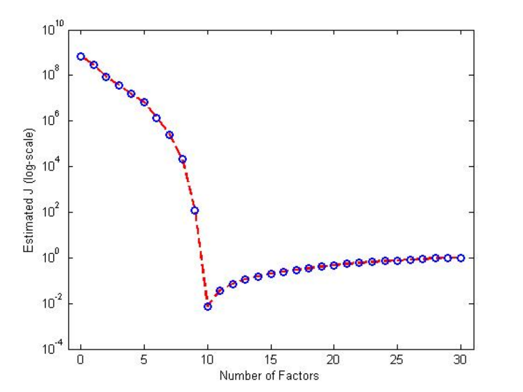

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Extending factor reasoning

Can we extend factor approach beyond covariance matrices?

Testing for Delay correlations

- Application to financial contagion: estimate a factor model based on stock market returns for large number of firms across a region (e.g. South America)
- Estimate numerous factors F_j for $j = 1..p$ (country, industries etc)
- Let τ be a lag
- Is $F_j(0)$ correlated to $F_k(\tau)$?

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Symmetrized delay correlation matrix

$$\Phi = \frac{F'(0)F(\tau) + F'(\tau)F(0)}{2T}$$

- Under Null F_{ij} are iid.
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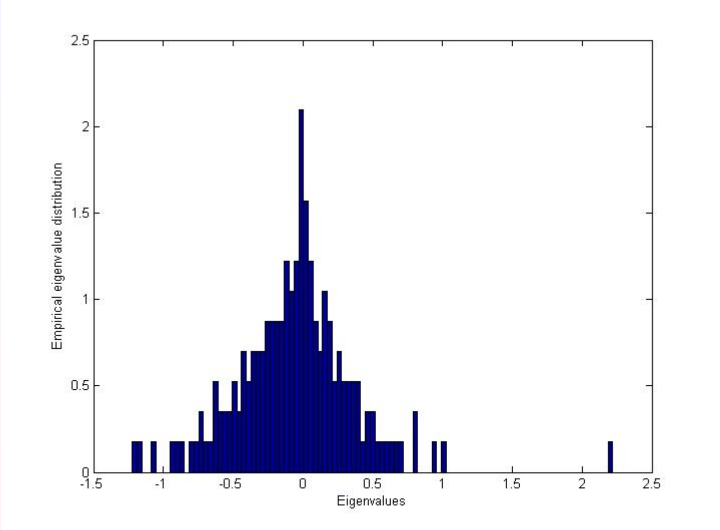
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Empirical eigenvalue distribution
Eigenvalues

Can obtain moments of the eigenvalue distribution from the Cauchy transform and construct J-test ...

Moments of the eigenvalue distribution

- $m_\Phi^1 = 0$
- $m_\Phi^2 = c/2$
- $m_\Phi^3 = 0$
- $m_\Phi^4 = (1/2)c^2 + (3/8)c^3$
- $m_\Phi^5 = 0$
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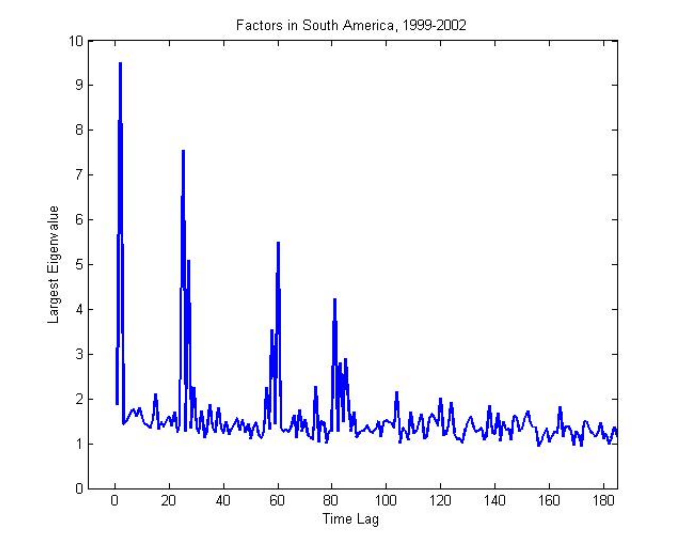
- $m_\Phi^3 = 0$
- $m_\Phi^4 = (1/2)c^2 + (3/8)c^3$
- $m_\Phi^5 = 0$
- $m_\Phi^6 = (5/8)c^3 + (5/16)c^5 + (9/8)c^4$

Or look at the largest eigenvalue ...


Factors in South America, 1999-2002
Largest Eigenvalue
Time Lag
0
1
2
3
4
5
6
7
8
9
10
0
20
40
60
80
100
120
140
160
180

Random Distance Matrices

- Spectral measures for large random Euclidean matrices (Bordenave '06)
- Entries are functions of positions of n random points in a compact set $\mathbb{R}^d$, $A = (D(X_i - X_j))_{i,j=1..n}$.
- Special case $D(X) = D(-X) \in (0,1)$, adjacency matrix for a random graph.
- More general distance in issue space "agreement beyond polarization" study of US congress
- Also risk sharing between companies in developing countries.

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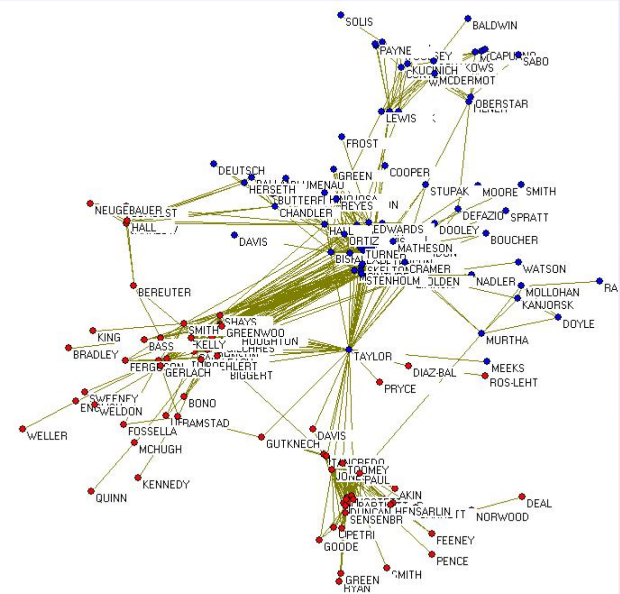
SOLIS
BALDWIN
PAYNE
SABO
KUCINICH
MCDERMOT
OBERSTAR
LEWIS
FROST
DEUTSCH
GREEN
COOPER
HERSETH
STUPAK
MOORE
SMITH
CHANDLER
REYES
NEUGEBAUER
HALL
DEFAZIO
SPRATT
EDWARDS
DOOLEY
DAVIS
ORTIZ
BOUCHER
MATHESON
CRAMER
WATSON
STENHOLM
NADLER
MOLLOHAN
KANJORSK
BEREUTER
DOYLE
SHAYS
SMITH
GREENWOO
KING
MURTHA
BASS
KELLY
BRADLEY
TAYLOR
MEEKS
GERLACH
BIGGERT
DIAZ-BAL
ROS-LEHT
PRYCE
SWEENEY
WELDON
BONO
WELLER
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DEAL
QUINN
SENSENBR
NORWOOD
PETRI
FEENEY
GOODE
PENCE
GREEN