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LOGISTICS NETWORK DESIGN WITH SUPPLIER CONSOLIDATION HUBS AND MULTIPLE SHIPMENT OPTIONS

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Abstract. An important service provided by third-party logistics (3PL) firms is to manage the inbound logistics of raw materials and components from multiple suppliers to several manufacturing plants. A key challenge for these 3PL firms is to determine how to coordinate and consolidate the transportation flow, so as to get the best overall logistics performance. One tactic is to establish consolidation hubs that collect shipments from several suppliers, consolidate these shipments, and direct the consolidated shipments to the appropriate manufacturing plant. We consider the network design problem to implement this tactic, namely deciding the number, location and operation of consolidation hubs so as to minimize the total logistics costs for the network. To solve this network design problem, we define candidate shipping options for each potential hub, for which we can pre-compute the shipping quantities required from each supplier, and the incurred shipping costs and inventory holding costs. We formulate the problem as an integer linear optimization model and illustrate how to solve large instances using Lagrangian relaxation and a subgradient optimization algorithm. Our results indicate that the bounds obtained are fairly tight and are superior to the bounds obtained from the solution of the LP relaxation.

1. Introduction.

YCH Group, established in 1955, is a large Singapore-based 3PL company. It provides services to various multi-national companies (MNCs) operating in the electronics, chemicals and consumer products industries. It currently has operations throughout Asia Pacific, such as in Singapore, Malaysia, Thailand, Indonesia, China, Taiwan, Hong Kong, Philippines, and Australia (please see www.ych.com). Motorola chose YCH to be its logistics partner to manage its incoming inventories from suppliers in Asia. To serve Motorola, YCH has opened three sourcing hubs in Asia, namely Tianjin, Hangzhou and Singapore. YCH has implemented their V-Hub^{TM} solutions for Motorola, where the suppliers no longer

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need to send their materials separately to each of Motorola's plants, which are located not just in Asia but throughout the world. They now send their materials to the nearest hubs, and these hubs will then consolidate the materials before sending them to Motorola's plants. YCH Group's V-Hub^{TM} has helped Motorola to slash inventory holdings by more than US$70 million.

(Source: MIS ASIA September 2004, 'MIS Innovation Awards 2004 - Winner Profiles', Pg 109, and http://www.ych.com/press/y200405.htm)

Designing a logistics network that maximizes the utilization of the transport and warehouse capacity in the network with minimal inventory, supports the material flows in the supply chains of multiple clients, and delivers superior performance for each client, is a key imperative for the success of a third party logistics (3PL) firms. The 3PL firm must balance the need to provide customized solutions to its clients with the economic benefits of maximizing consolidation in terms of freight and warehouse capacity, using a single network. An important service provided by 3PL firms is to manage the inbound logistics from multiple suppliers to several manufacturing locations. A crucial challenge that the 3PL firm faces is to determine how to coordinate and consolidate the transportation flows, so as to get the best overall logistics performance. One tactic is to establish consolidation hubs that collect shipments from multiple suppliers and then send consolidated shipments directly to each manufacturing site. We consider the network design problem to implement this tactic, namely deciding the number, location and operation of consolidation hubs so as to minimize the total logistics costs for the network.

Consider a 3PL firm that is responsible for managing the material flows for the suppliers of two manufacturers. Typically, the 3PL firm would customize the network design for each manufacturer (see Figure 1). The incoming components and raw materials from the suppliers are sent directly to dedicated warehouses which serve as supply hubs and replenish the manufacturing plants on a regular basis. Prior research on such short interval, regular replenishment strategies or "supply hub research", has focused on Vendor Managed Inventory (VMI) arrangements rather than on network design issues. The supply hub concept is commonly used by computer manufacturing firms such as Hewlett-Packard, Apple and Dell, to reduce cost and/or improve responsiveness in the face of increasingly short product life-cycles and high demand variability.

In the scenario described above, suppliers for each manufacturing plant ship components directly to the dedicated warehouse, regardless of their location. The network performance suffers as suppliers are unable to capture economies of scale in long haul transportation by consolidating less–than-container-load (LCL) shipments. Moreover, inventory visibility from the supplier to the warehouse is often poor.

An alternative approach to network design is to locate an appropriate number of consolidation hubs near the suppliers and service the manufacturers from these hubs (Figure 2). The suppliers ship components to the nearest hub, and the hubs consolidate all the components bound for the same manufacturer before shipping them.

Such a "consolidated design" can yield economic benefits for all parties. Each supplier benefits from this arrangement because the consolidation hub is closer than the manufacturer warehouse. The 3PL firm benefits because it can integrate the network of the two clients and improve consolidation in shipping. With the consolidation hubs, the 3PL firm can have better control on the inventories coming from

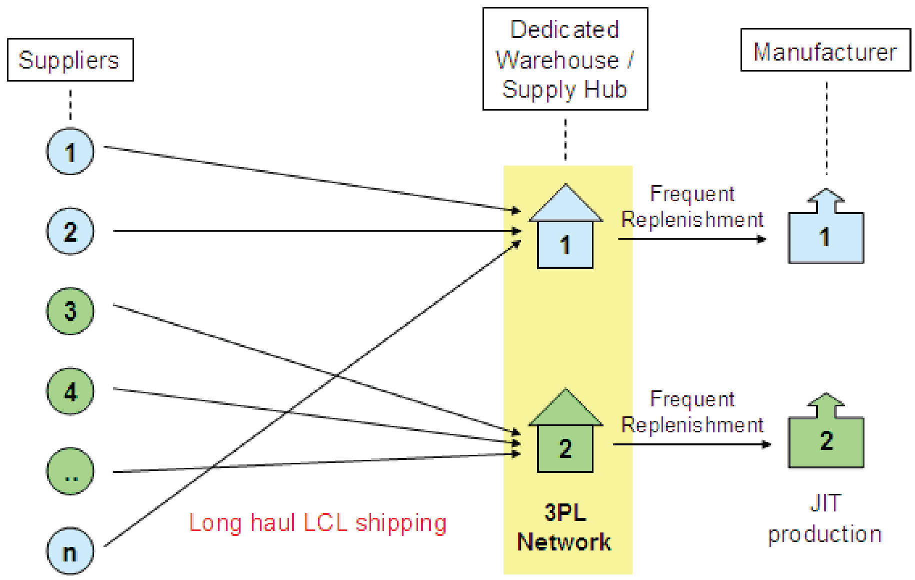

FIGURE 1. Traditional Management of Supply Hubs for Two Manufacturers by 3PL Firm

different suppliers, since any incorrect deliveries or inferior quality components can be returned to the suppliers quickly. This advantage gives manufacturers greater confidence in allowing the 3PL firm to manage their logistics network. Manufacturers also gain because they have better visibility of supply chain inventory and this allows them to manage uncertainty in their assembly plans more effectively.

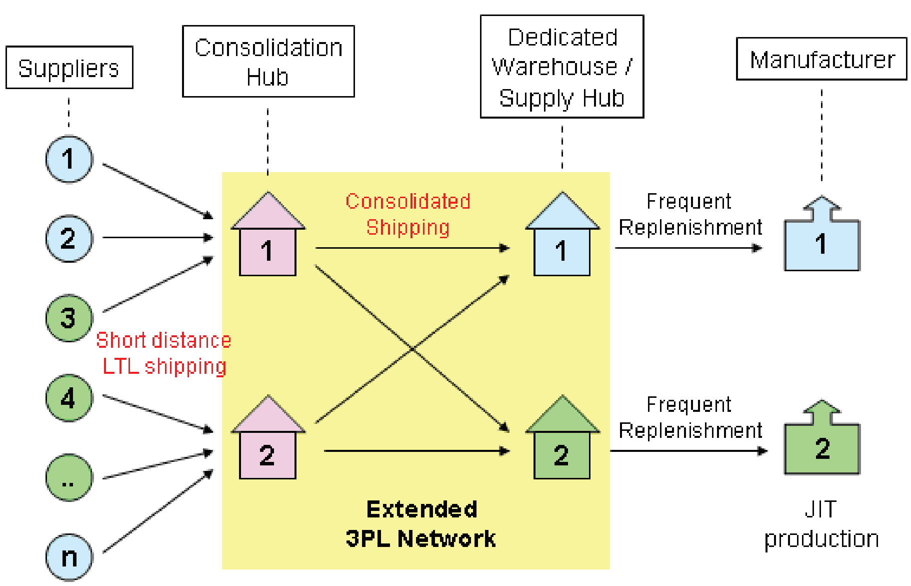

FIGURE 2. Extended 3PL Network with Consolidation Hubs to Serve Two Manufacturers

Designing such consolidated networks to serve multiple manufacturers is a challenging task. It requires the coordination of several interlinked aspects of the distribution system. For example, the right amount of components must be sourced from suppliers, proper inventories must be maintained at the warehouses, and appropriate replenishment policies must be established between the suppliers and warehouses, and between the warehouses and the manufacturers. Additional complexity comes from the fact that the choice of hub location impacts the shipping options that might be used. Our objective here is to select and locate a suitable number of consolidation hubs between the suppliers and the manufacturers' warehouses to minimize the overall network inventory holding and transportation costs, as well as the costs for operating the hubs.

The rest of this paper is organized as follows. In Section 2, we discuss the relevant literature which covers some of the aspects similar to our work. The model, model assumptions and parameters are presented in Section 3. In section 4 we use a Lagrangian Relaxation method, with subgradient optimization, to obtain both lower and upper bounds on the problem. We demonstrate the effectiveness of this solution method on a set of large test problems. Section 5 presents the conclusions and highlights the possible future work.

2. **Literature Review.** Hubs are transshipment facilities that allow the construction of a network where large numbers of direct connections between nodes (including suppliers, warehouses and customer locations) can be replaced with fewer, indirect connections. In solving hub location problems, two distinct aspects need to be resolved: finding the best location for the hubs, and identifying the best route for the material flows from the origin nodes to the destination nodes via the hubs.

One of the earliest works in hub location is by O'Kelly (1986) who demonstrated that the one hub location problem is equivalent to the Weber least cost location model. The author also discussed the two hub location problem in a plane. For the location of two interacting hubs, the flows between the hubs are an endogenous function of their relative location, and a gravity model is linked into the objective to allow for complete interdependence between the interaction and hub location.

O'Kelly (1987) presented a quadratic integer programming formulation for a discrete hub location problem, and discussed two enumeration-based heuristics. The first heuristic allows each demand point to be allocated to the nearest hub, while the second heuristic allows each demand point to be allocated to either the first or second nearest hub. For n demand points, the second heuristic will need to consider 2^n allocation combinations for every hub combination, and thus can only be used for small n.

Klincewicz (1991) proposed a node substitution heuristic that uses a multi-criteria distance and flow-based allocation procedure rather than assigning the nodes to hubs based on distance alone. The exchange heuristic first determines the hub locations and then assigns the nodes to the hubs, with improvements to the initial solution made by exchanging hubs with nodes. The clustering heuristic, on the other hand, first clusters the nodes into p groups and then assigns a hub to each group.

Metaheuristic approaches were explored by Klincewicz (1992) and Skorin-Kapov and Skorin-Kapov (1994). They considered the hub location problem as two sub-problems, hub location and routing, and used tabu-search to find good solutions to each of the sub-problems.

In 1994, two review papers were published by Campbell, and by O'Kelly and Miller. Campbell (1994) reviewed over 70 papers on hub network optimization, while O'Kelly and Miller (1994) reviewed hub locations problems, identifying eight prototype models. Each model is characterized by three decision types: whether each node is assigned to one hub or multiple hubs; whether node-to-node links, bypassing the hub are allowed; and whether the hub-to-hub links are full or partial.

Campbell (1996) discussed the hub location and the p-hub median problem. He defined the p-hub median problem, analogous to p-median problem, as a linear integer program. He formulated economies of scale in transportation by using a discount factor for flows between an O-D pair via the hub, and for flows between hubs. The solution to the multiple-allocation p-hub median problem is obtained by a greedy-interchange heuristic and used as a starting point for developing the solution to single-allocation p-hub median problem. Two new heuristics are used to evaluate the single-allocation p-hub median problem. Computational results for problems with 10 to 40 origins/destinations and up to 8 hubs are presented.

O'Kelly et al. (1995) developed a lower bound for the hub location problem where distances satisfy the triangular inequality. The lower bound is improved by linearization and incorporating a known heuristic solution. Pirkul and Schilling (1998) formulated a new procedure that develops solutions with tight upper and lower bounds, in polynomial time. The solution procedure begins with a previously proposed tight LP formulation and uses subgradient optimization on a Lagrangian relaxation of the model. To improve the performance, the authors added a constraint that provides a cut for one of the sub-problems and solved the new model using subgradient optimization.

O'Kelly et al. (1996) presented exact solutions to hub location models and discussed the sensitivity of these solutions to the inter-hub discount factor used for economies of scale in transportation. Two different hub network designs are considered, single- and multiple-allocation to hubs. This solution approach reduces the problem size but is still able to find integer solutions to the relaxed LP problem in most instances. The compact formulation is used to perform a large computational study, from which sensitivity results for several key parameters of the model are obtained.

The previous research discussed above did not model the economies of scale in transportation cost, but rather assumes linear transportation costs. O'Kelly and Bryan (1998) incorporated a piecewise linear cost function into the multiple-assignment hub location model, and showed that it is a more reliable representation of the hub and spoke networks. They applied the model on a small example of airline passenger travel, and compared the results on network cost with a traditional model without economies of scale in transportation cost. Klincewicz (2002) applied the model of O'Kelly and Bryan (1998) on a problem with a fixed set of hubs, and showed that the model could be solved using the classic uncapacitated facility location problem. This observation motivated an optimal enumeration procedure, and some search heuristics based on tabu search and greedy algorithm random adaptive search procedures. By using these search procedures, the author showed that it is possible to solve large-sized problems.

Horner and O'Kelly (2001) explored embedding non-linear cost functions into hub network design. In their work, they did not assume *a priori* a hub and spoke network structure; rather their objective was to make both link discounts and hub locations endogenous. They applied their model to air passenger flights for 100

US cities and concluded that large discounts are necessary to encourage significant bundling of flows.

We note that the hub location models reported in previous research do not account for the tradeoff between transportation costs and inventory costs. For a portion of the prior research, this is not a relevant tradeoff as the research addresses the design of air passenger networks. However, in logistics networks, there is usually a critical tradeoff between the transportation costs and the resulting inventory. Efforts to achieve greater transportation economies of scale through consolidation inevitably lead to less frequent shipments, which result in more inventory. This point was also reinforced in our discussions with the managers at YCH, as this is the challenge they face in designing their logistics networks for customers such as Motorola.

Shen, Coullard and Daskin (2003) described a joint location-inventory problem where they consider the location of distribution centers to serve a set of retailers, and explicitly account for the inventory implications of the location decisions. They formulate the problem as a non-linear integer optimization and show how to transform this into a set-covering model. They solve the set-covering problem with a column-generation method. The pricing problem for finding new columns is a non-linear optimization that can be solved efficiently. The authors report computational results for problems with up to 150 retailers, and find that the LP relaxation to the set-covering problem always yields the optimal solution.

Our paper is distinguished from the work of Shen, Coullard and Daskin (2003) in that we model the trade-off between inventory holding costs and transportation cost at a greater level of detail. We explicitly account for the inventory holding costs for both the cycle stock and safety stock associated with different inventory replenishment policies from the suppliers, via the hubs, to the warehouse. Our model captures the transportation economies of scale for each link from each supplier to each hub; we assume a piecewise linear concave cost function to model these economies of scale. We also differ from the previous research in the solution method used in dealing with the concave cost function by using candidate shipping options and pre-calculating costs for each and every possible link. This enables us to transform the non linear cost function into a linear, mixed integer program.

3. **Model, Assumptions and Parameters.** We consider a layered network with I suppliers, K warehouses and K manufacturers. The suppliers ship components to the warehouses, which in turn replenish the manufacturers on a regular basis to support a make to order (MTO) production/assembly process. Each warehouse is dedicated to serve one manufacturing plant. Given the daily final assembly schedule of each manufacturer, each warehouse needs to place replenishment orders on the suppliers.

Due to the long haul transportation from the suppliers to the warehouse, our objective is to locate consolidation hub(s) between the suppliers and the warehouses to improve the network performance. We select an appropriate number of hubs among potential hub locations, and assign suppliers to the hubs. The hubs will receive the components from the suppliers, and then consolidate all of the components bound for the same warehouse, before shipping these to the warehouse.

We make the following assumptions:

- Each manufacturer assembles one unique product with a known bill of material (BOM), and is supplied by a dedicated warehouse. Each manufacturer follows a make-to-order policy; each day it observes demand and pulls the necessary components from its warehouse to assemble the orders to meet demand. This assumption is in line with the practice between YCH and Motorola. Each day Motorola's manufacturing plants observe the random demand and then procure the components from the warehouse according to the BOM of a certain cell phone.

- Each warehouse holds an inventory of components sufficient to provide a high level of service to the manufacturer. YCH's warehouses receive components from the consolidation hubs in Singapore, Tianjin and Hangzhou, and maintain inventories to supply to Motorola's plants.
- Each supplier supplies a unique component or component kit, which is required for the final assembly of one or more products. [If a supplier supplies different components to different manufacturers, we can create an additional dummy supplier for each additional component type. However, this formulation would overlook consolidation opportunities when shipping from the supplier to the hub.]
- Each warehouse operates with a periodic review, order-up-to policy; without loss of generality, we assume the period is a day. Thus, each day a warehouse places a replenishment order for each of its components equal to the observed daily demand.
- Each supplier ships directly to a single hub. This assumption reduces the shipping cost involved since it is more economical to ship a larger quantity to a single hub, than to ship smaller quantities to more than one hub. For instance, Motorola's suppliers in Malaysia, Indonesia, Thailand and Vietnam ship only to YCH's Singapore hub, which is the nearest hub.
- Each hub can serve more than one warehouse and each warehouse can be served from more than one hub. This allows us to ignore hub-to-hub links. YCH's Singapore hub makes regular shipments to the warehouses in Singapore, Tianjin, Mexico, Brazil and Germany
- Each hub operates as a cross-dock facility and does not hold inventory. Associated with each hub is a shipping frequency, which denotes the frequency with which it makes shipments to the manufacturing warehouses. For instance, a hub with a shipping frequency of three days makes a shipment once every three days to each warehouse that it serves. Since the hub operates as a cross-dock facility, each supplier synchronizes its shipments to the hub to match the shipping frequency of the hub; if the hub operates with a three-day shipping frequency, then each supplier makes a shipment to the hub once every three days, where the shipment quantity equals the cumulative orders for the three most recent days.

Given a set of candidate hub locations, we want to determine which hubs to open. For each hub that we select, we need to decide its shipping frequency. Finally, we assign each supplier to a hub. Our objective is to minimize the total logistics costs that include transportation costs from the suppliers to the hubs, transportation costs from the hubs to the warehouses, inventory holding costs at the warehouses, and the fixed and variable costs for operating the hubs.

The number of possible shipping frequencies for each hub is unbounded, in theory; however, in practice, it is quite feasible to identify a limited set of realistic possibilities. In particular, the time between shipments needs to be sufficiently long so that the size of a typical shipment achieves reasonable transportation economies, e.g., shipping one container to each warehouse. But once these shipment economies are achieved, there is little benefit from making less frequent, larger shipments; for instance, there is little cost benefit to shipping two containers every six days in comparison with one container every three days.

Based on this observation, we pre-specify for each hub j a set of shipping options. The shipping option m for hub j is defined by the following parameters:

- Shipping frequency, t_{jm} (days between shipment)
- Nominal or target shipping capacity available from hub j to warehouse k, G_{jkm} (m^3)
- Shipping lead time from hub j to warehouse k, S_{jkm}(days)
- Annual fixed cost for shipping from hub j to warehouse k, CS_{jkm}($)

We observe that in theory, the shipping cost and the inventory cost are non-linear expressions which are dependent on t_{jm} and S_{jkm}. By defining these parameters for each shipping option m, we can pre-compute the shipping cost (C_{ijm}) and inventory cost (CI_{ijkm}) for each supplier i, shipping to warehouse k via hub j, using shipping option m. We can overcome the complexities in such non-linear expressions without compromising much of the accuracy in the costs.

We now formulate an optimization problem for the design of the logistics network.

Input Parameters

- i = index for suppliers
- j = index for hubs
- k = index for warehouses
- m = index for the available shipping options at each hub j
- μ_k = mean of daily demand at warehouse k (number of units of final product k required to be assembled per day)
- σ_k = standard deviation of daily demand at warehouse k
- Φ_{ik} = number of units required from supplier i for each unit required at warehouse k (number of units of component i per unit of product k)
- U_i = shipping capacity required per unit of component i (m^3)
- h_{ik} = inventory holding cost of component i at warehouse k ($ per unit of component i per year)
- V_{ijk} = variable cost of handling a unit of component i from supplier i to warehouse k via hub j ($ per unit of component i expressed in terms of shipping capacity)
- S_{ij} = transportation time from supplier i to hub j (days)
- t_{jm} = shipping frequency for hub j with option m (days between shipment)
- G_{jkm} = nominal or target shipping capacity available from hub j to warehouse k with option m for each shipment (m^3).
- S_{jkm} = shipping lead time from hub j to warehouse k with option m (days)
- F_{jm} = annual fixed cost of opening hub j and operating with shipping option m ($)
- μ_{ijkm} = average shipping lot size per shipment from supplier i to warehouse k, via hub j using shipping option m, expressed in terms of shipping capacity $= \Phi_{ik}\mu_k t_{jm} U_i(\mathrm{m}^3)$.
- CS_{jkm} = annual fixed cost for shipping from hub j to warehouse k using shipping option j ($)
- C_{ijm} = annual logistics cost for supplier i to ship to hub j using shipping option m, including any variable costs for shipping from hub j to warehouse

k (\$). We pre-calculate this cost parameter with the following formula:

$$C_{ijm} = \left(F_{ijm} + R_{ijm}\left(Q_{ijm}\right) \times Q_{ijm} + \sum_k V_{ijk} \times \mu_{ijkm} \right) \times N_{ijm},$$

where,

- F_{ijm} is the fixed logistics cost per shipment for supplier i to ship to j with shipping option m;
- $R_{ijm}(q)$ is the variable shipping rate for shipping from supplier i to hub j with option m, assuming quantity q. As we are modeling concave shipping costs, different shipping rates will apply to different shipping quantities.
- $Q_{ijm} = \sum_k \mu_{ijkm}$ is the average amount of component i required in each shipment from supplier i to hub j under option m, measured in units of shipping capacity; We assume an average shipment size for determining the variable shipping cost per shipment. This is an approximation, as the actual shipping quantities will depend on the demand outcomes, and thus will vary around the average. However, we expect this will be a reasonable approximation given that the purpose of our model is to support strategic planning decisions, namely what hubs to open.
- V_{ijk} is the variable cost of handling a unit of component i from supplier i to warehouse k via hub j (\$ per unit of component i expressed in terms of shipping capacity)
- N_{ijm} is the number of shipments per year from supplier i to hub j under option m.
- CI_{ijkm} = Annual inventory holding cost at warehouse k, attributable to components from supplier i, who uses hub j and shipping option m (\$)

  To calculate this parameter, we assume a periodic-review inventory replenishment policy at warehouse k, with a review period equal to the selected shipping frequency (t_{jm}) of hub j serving warehouse k. We approximate the expected inventory of component i at warehouse k as the sum of the cycle stock and safety stock (z is the safety factor), given by a standard approximation:

$$E[I_{ijkm}] = \frac{t_{jm}\phi_{ik}\mu_k}{2} + z\sigma_k\phi_{ik}\sqrt{LT_{ijk} + t_{jm}}$$

  The effective lead-time for the safety stock computation (LT_{ijk}) is equal to the maximum of the lead times for all suppliers i who supply manufacturer k (that is $\Phi_{ik} > 0$), and are assigned to hub j ($i \to j$),

$$LT_{ijk} = \underset{i \in \{\ i\,:\,\phi_{ik} > 0 \quad and \quad i \to j\ \}}{Max} \left(S_{ij} + S_{jkm}\right)$$

  So, the annual inventory holding cost at warehouse k, attributable to components from supplier i, who uses hub j and shipping option m, is $CI_{ijkm} = E[I_{ijkm}]h_{ik}$.

Decision Variables

- X_{ijm} = binary variable to denote if supplier i is assigned to hub j, using shipping option m
- Y_{jm} = binary variable to denote if hub j is open and uses shipping option m
- Z_{jkm}= binary variable to denote if hub j ships to warehouse k using shipping option m

The model minimizes the network logistics cost given by,

Problem (**P**):

$$\text{Min} \sum_{j,m} F_{jm}Y_{jm} + \sum_{i,j,m} C_{ijm}X_{ijm} + \sum_{i,j,m}\left(\sum_{k} CI_{ijkm}\right)X_{ijm} + \sum_{j,k,m} CS_{jkm}Z_{jkm}$$

Subject to:

$$\sum_{j,m} X_{ijm} = 1 \qquad \forall i \qquad (1)$$

$$X_{ijm} \leq Y_{jm} \qquad \forall i,j,m \qquad (2)$$

$$\sum_{m} Y_{jm} \leq 1 \qquad \forall j \qquad (3)$$

$$\sum_{i} \mu_{ijkm}X_{ijm} \leq G_{jkm}Z_{jkm} \qquad \forall j,k,m \qquad (4)$$

$$Z_{jkm} \leq Y_{jm} \qquad \forall j,k,m \qquad (5)$$

$$X_{ijm}, Y_{jm}, Z_{jkm} \in \{0,1\} \qquad \forall i,j,k,m$$

Constraint (1) ensures that each supplier is assigned to exactly one hub and one shipping option. Constraint (2) ensures that supplier i is assigned to a hub j and shipping option m, only if the hub is open with the specific shipping option m. Constraint (3) requires that at most one shipping option is chosen for each hub j. Constraint (4) limits the average shipment along each link from hub j to warehouse k to the nominal or target capacity. Constraint (5) ensures that if a link from hub j to warehouse k is selected with option m, hub j must be open with option m.

Constraint (4) merits some comment. In actual operation, the shipments from hub j to warehouse k will vary, depending on the random demand. We desire to set this constraint so as to assure that there will be sufficient capacity to cover each shipment. We propose to do this by setting the right-hand side parameter G_{jkm} to reflect the target capacity utilization on the shipping link between hub j and warehouse k. For instance, based on experience, we might expect that the utilization target cannot be any more than 80% in order to handle the variations in the shipping quantity; in this case, we would set G_{jkm} to equal 80% of the actual capacity on each shipment, e.g., 80% of a shipping container. Alternatively, we might base the parameter G_{jkm} on an estimate of the coefficient of variation for the shipment quantity. For instance, if we let cv denote the coefficient of variation, then we could set

$$G_{jkm} = \frac{Max_{jkm}}{1 + z \times cv}$$

where Max_{jkm} denotes the maximum capacity per shipment from j to k for option m, and z is a safety factor; for instance, if we assume the shipping quantities were normally distributed, we might set $z = 1.64$ to have a 0.95 probability that there would be sufficient capacity. In either case, some care is needed in setting up constraint (4), so as to get a reasonable approximation of the shipping limits on each link, but to do so without adding undue complications to the model.

We have based our mathematical model on YCH's business model with Motorola. We note that, in fact, YCH also practices the same business model with several other clients. One example is Dell computers where YCH manages the incoming components from the Dell suppliers using the YCH hubs and replenishes Dell's assembly plants on a regular basis. We can foresee that such a business model will become very common, especially as global sourcing and manufacturing becomes increasingly pervasive in the electronics and computer industry.

In summary, we can achieve potential network cost savings by designing a network to serve more than one client, maximizing the utilization of the facilities and shipping links, and leveraging on the economies of scale and scope. We develop our model based on the concept of establishing consolidation hubs that consolidate shipments from multiple suppliers to multiple manufacturers. Logistics managers can use this result to help them understand the complexities involved in such analyses, the economics of consolidation versus non-consolidation, as well as the factors affecting consolidation.

4. **Solution Procedure and Computational Test.** Cheong (2005) reports on an illustrative example of applying our model to the personal computer (PC) industry in South-east Asia. This case illustrates how the logistics network for two PC makers located in Philippines and Indonesia, can be consolidated by using consolidation hubs in Shanghai and Singapore. This problem has 6 suppliers, 4 potential hubs and 2 manufacturers, and is readily solved by branch-and-bound using the Lingo solver. Cheong (2005) also reports on solving a small set of larger problems using the Lingo solver, with the largest problem having 19 suppliers, 4 potential hubs, 2 warehouses and 3 shipping options per hub. For this problem there are 264 binary variables. However, we expect the computational difficulty for our model to increase rapidly as the number of binary decision variables increases. Hence, we need to explore more specialized solution approaches so as to have the ability to solve even larger problems.

In this section, we introduce a Lagrangian relaxation to problem **P**. We apply subgradient optimization to the Lagrangian relaxation to find a good lower bound on **P**. We also develop a heuristic procedure to find feasible solutions from the solutions to the Lagrangian; these feasible solutions provide a way to obtain a good incumbent solution and a corresponding upper bound to **P**. We demonstrate the effectiveness of this procedure on a set of test problems.

The Lagrangian relaxation approach is widely used in solving location problems. Fisher (1981) reviews how Lagrangian relaxation can be used to solve problems which are complicated by a small set of constraints, and discusses the details of using Lagrangian relaxation and the usefulness of the approach. One relevant example is Klincewicz and Luss (1986), who develop a Lagrangian relaxation heuristic to solve the capacitated location problem with a single-source constraint. We present our Lagrangian relaxation next and then describe how we use it to find a good solution to **P**.

Lagrangian Relaxation (LR) Solution Procedure

We relax constraint (1) in Problem **P** to obtain the problem given below, with Lagrangian multipliers (λ_i).

Problem (**LR**):

$$\text{Min} \sum_{j,m} F_{jm}Y_{jm} + \sum_{i,j,m} C_{ijm}X_{ijm} + \sum_{i,j,m}\left(\sum_k CI_{ijkm}\right)X_{ijm}$$

$$+\sum_{j,k,m} CS_{jkm}Z_{jkm} + \sum_i \lambda_i\left(\sum_{j,m} X_{ijm} - 1\right)$$

Subject to:

$$\begin{aligned} X_{ijm} &\le Y_{jm} && \forall i,j,m \\ \sum_m Y_{jm} &\le 1 && \forall j \\ \sum_i \mu_{ijkm}X_{ijm} &\le G_{jkm}Z_{jkm} && \forall j,k,m \\ Z_{jkm} &\le Y_{jm} && \forall j,k,m \\ X_{ijm}, Y_{jm}, Z_{jkm} &\in \{0,1\} && \forall i,j,k,m \end{aligned}$$

By relaxing constraint (1), we see that **P** separates into a set of single-hub problems, one for each possible hub location $j \in J$. For each sub-problem j, we decide whether or not to open the hub and with what shipping frequency m, and then assign suppliers to the hub. Although each of these sub-problems is still an integer linear program, we can solve these problems very quickly. Furthermore, the relaxation **LR** does not satisfy the *Integrality Property* given by Geoffrion (1974). In particular, the optimal value of **LR** does depend on the integrality requirements on the variables X_{ijm}, Y_{jm}, Z_{jkm}. Thus, from the *Integrality Property,* we expect a relatively tight lower bound from the solution of the dual problem implied by **LR,** in comparison to the lower bound from the LP relaxation of **P**. Indeed, we will show this to be the case on a set of test examples.

We now state our solution procedure.

1. Use heuristic H1 to find an initial feasible solution to the original problem **P** to obtain an initial upper bound (UB_{LR}) and incumbent solution.
2. Solve the Lagrangian Relaxation **LR** using an IP code (e.g., Lingo) to obtain a lower bound (LB_{LR}). In the first iteration we initialize the Lagrangian multipliers (λ_i) to zero.
3. Use heuristic H2 to perturb or fix the solution found in Step 2 to obtain a feasible solution to the original problem **P**, and a new upper bound (New-UB_{LR}).
4. If New-$\text{UB}_{LR} < \text{UB}_{LR}$, then UB_{LR} = New-UB_{LR} and replace the incumbent with the new feasible solution.
5. Compute the subgradient from the solution found in step 2 and use it to update the Lagrangian multipliers (λ_i) using the step size:

stepsize $= \frac{\Theta * (UB_{LR} - LB_{LR})}{\sum_i \left(\sum_{j,m} X_{ijm} - 1 \right)^2}$ where Θ is a scalar defined between 0 and 2. Θ is set to an initial value of 2, and is halved whenever there is no improvement in LB_{LR} obtained in Step 2.

6. Repeat Step 2 to Step 5 using the new Lagrangian multipliers (λ_i) values obtained in Step 5, until a convergence criterion is satisfied or the maximum number of pre-defined iterations is reached. The convergence criteria are when the percentage gap between UB_{LR} and LB_{LR} falls to below 0.001%, or when the change in each Lagrangian multiplier is less than a delta value. In our experimental runs, we use a delta value of 1 (since our multipliers have values in the magnitude of 10^5 and 10^6) and we set the maximum number of iterations to be 40.

LP Relaxation (LP) Solution Procedure

To assess the effectiveness of the Lagrangian relaxation solution procedure, we will compare it to a solution procedure that is based on solving the LP relaxation to **P**. We specify the LP relaxation solution procedure as follows:

1. Solve the LP relaxation of problem **P** to obtain a lower bound (LB_{LP}). The LP relaxed problem can be solved quickly using LP code.
2. Use heuristic H3 to perturb or fix the solution found in Step 1 to obtain a feasible solution to the original problem **P** and an upper bound (UB_{LP}).

We provide the details for each heuristic in the Appendix.

In designing a set of test problems, we are guided by our interactions with several 3PL firms that have operations in Singapore. There are several aspects that limit the size of real life problems. In most contexts, the network designer is unlikely to be faced with a choice involving large numbers of potential hubs (j), warehouses (k) and available shipping options (m). In contrast, we would expect instances with a large number of suppliers. For a 3PL firm attempting to position consolidation hubs, usually the potential hubs are existing facility locations, and this number is expected to be small. As our model attempts to design a single network to serve multiple clients using dedicated warehouses (supply hubs), we expect there to be a small number of clients that share a supply base and are suitable to having their networks consolidated. Finally, the number of available shipping options is also small; when a consolidation hub serves more than one warehouse, it can only consider options with a common shipping frequency, in order to function as a cross-dock facility.

Our experimental runs for the larger problems are based on the illustrative example given in Cheong (2005). We used random functions using normal distribution and uniform distribution, to randomly generate the parameters ($\Phi_{ik}, U_i, h_{ik}, V_{ijk}, S_{ij}, t_{jm}, G_{jkm}, S_{jkm}$ and F_{jm}) to simulate different suppliers, different hubs and different manufacturers. Test problems can be downloaded from www.mysmu.edu/faculty/michcheong/. To assure feasibility, we need to make sure that the capacity constraint is not violated, by setting G_{jkm} to be larger than $\sum_i \mu_{ijkm} X_{ijm}$ for every j, k, m.

In Table 1 we report the bounds as the number of suppliers increases for the case with 4 potential hubs, 2 manufacturers and 3 shipping options for each hub. We observe that Lagrangian relaxation performs very well, finding the optimal solution in six out of seven cases; for the seventh case, the gap between the upper and lower bound is 0.04%. The average number of iterations for the subgradient optimization is 16 for these seven problems. In comparison, the solution procedure based on the LP relaxation does not do as well, with an average gap of about 4%. We observe that the main difference the two procedures is that the Lagrangian consistently finds a much tighter lower bound.

# suppliers	**Lagr. relaxation**		**LP relaxation**	
	Best LB	Best UB	LB	UB
6	3,524,051	3,524,051	3,250,503	3,524,051
	gap % =	0.00%	gap % =	8.42%
12	6,814,972	6,814,972	6,547,598	6,814,972
	gap % =	0.00%	gap % =	4.08%
18	10,274,725	10,278,480	9,969,190	10,392,868
	gap % =	0.04%	gap % =	4.25%
24	13,540,901	13,540,901	13,224,020	13,655,289
	gap % =	0.00%	gap % =	3.26%
30	18,442,226	18,442,226	18,112,810	18,517,596
	gap % =	0.00%	gap % =	2.23%
36	21,352,007	21,352,007	20,954,680	21,352,007
	gap % =	0.00%	gap % =	1.90%
42	24,548,786	24,548,786	24,157,520	24,548,786
	gap % =	0.00%	gap % =	1.62%

Table 1. Comparison of Bounds Between LR and LP Relaxation with Increased Number of Suppliers

In Table 2 we report the performance of the solution procedures for a set of test problems with 42 suppliers, but with varying numbers of hubs and manufacturers. The number of options for each hub is 3 for each test problem. Again, the Lagrangian relaxation solution procedure dominates the LP relaxation solution procedure. The largest gap for the Lagrangian relaxation is 1.68%, with all other problems having gaps of less than 0.55%. The average number of iterations for the twelve test problems is 36. In contrast, for the LP relaxation solution procedure, the smallest gap is 1.62%, and the average gap over the twelve problems is around 5%; furthermore, the gap between the upper and lower bounds increases when the number of hubs and number of manufacturers increase.

5. **Conclusions and Future Work.** The main contribution of our work is in terms of formulating a representative model to encompass the different aspects encountered by 3PL firms in designing a consolidated network to serve multiple clients. The critical aspects include concave shipping costs, inventory holding costs, hub locations, flow assignments, and finally the inventory replenishment policies to support a make-to-order production system, prominent in the supply hub concept. We are not aware of any previous research which has addressed these issues concurrently in a single model.

We introduce the concept of candidate shipping options to overcome the complexities inherent in modeling non-linear costs, both for inventory holding and shipping. We pre-compute these costs for the candidate options, as input parameters for a linear binary integer program. When solved, the binary decision variables in our

# hubs	**# manuf.**	**Lagr. relaxation**		**LP relaxation**	
		Best LB	Best UB	LB	UB
4	2	24,548,786	24,548,786	24,157,520	24,548,786
		gap % =	0.00%	gap % =	1.62%
6	2	23,815,172	23,848,272	23,075,340	23,848,272
		gap % =	0.14%	gap % =	3.35%
8	2	23,465,208	23,471,574	22,591,010	23,540,036
		gap % =	0.03%	gap % =	4.20%
10	2	22,454,724	22,577,848	21,359,600	22,614,720
		gap % =	0.55%	gap % =	5.88%
4	3	25,783,039	25,783,039	25,171,220	25,802,817
		gap % =	0.00%	gap % =	2.51%
6	3	25,013,488	25,064,716	23,936,690	25,109,525
		gap % =	0.20%	gap % =	4.90%
8	3	24,705,584	24,706,138	23,501,730	24,917,087
		gap % =	0.00%	gap % =	6.02%
10	3	23,792,890	23,879,534	22,225,320	24,013,011
		gap % =	0.36%	gap % =	8.04%
4	4	26,782,282	26,782,438	25,879,700	26,786,995
		gap % =	0.00%	gap % =	3.51%
6	4	26,091,288	26,112,344	24,590,930	26,195,785
		gap % =	0.08%	gap % =	6.53%
8	4	25,792,506	25,793,848	24,220,660	26,145,908
		gap % =	0.01%	gap % =	7.95%
10	4	24,769,180	25,192,804	22,902,550	25,338,632
		gap % =	1.68%	gap % =	10.64%

TABLE 2. Comparison of Bounds Between LR and LP Relaxation with Increased Number of Hubs and Manufacturers

model tell us which hubs to open, which shipping option to select, which hubs to assign to each warehouse, and which suppliers to assign to each hub. In addition, the selected shipping option also sets the inventory replenishment cycle for each component at each warehouse. We have made possible the integration of network design with inventory replenishment policy, by allowing the model to make the decisions for both concurrently. Finally, we have developed and tested a solution procedure, based on a Lagrangian relaxation of the original model. On a series of test problems we demonstrate that this procedure is very effective at solving large problems.

In framing this research, we have tried to be quite faithful to our observations of real practice in the design of 3PL logistics networks that entail consolidation hubs. Nevertheless, we have employed a number of simplifying assumptions. We can relax some of these assumptions as follows:

1. We assume that each supplier assigned to a hub, ships with the same shipping frequency. We can relax this assumption and allow suppliers to ship in multiples of the selected shipping frequency. For instance, the shipping frequency of a hub might be 3 days, then we could consider letting the suppliers assigned to the hub to ship every 3 days, or every 6 days, or every 9 days, and so on.

Less frequent shipments by a supplier would result in reduced transportation costs from the supplier to the hub, but increased inventory holding costs at either the warehouses or the hubs, depending on whether or not the hub must hold inventory to smooth the shipments to the warehouses.

2. We assume each supplier must ship to a hub. We can relax this assumption to permit some suppliers to ship directly to the warehouse, bypassing the hubs. This is especially important when the potential consolidation benefit for some suppliers is low, or when there is already full-container-load shipping from these suppliers.

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Appendix.

Heuristic (H1) – To obtain initial upper bound for problem P

This heuristic is modified from the Add heuristic from "A Lagrangian Relaxation Heuristics for Capacitated Facility Location with Single-Source Constraint" by J.G. Klincewicz and H. Luss, *The Journal of the Operational Research Society*, Vol.37, No.5, pp 495-500.

1. Initialization

   a). Set Q = set of all facilities to consider (facility q here means hub j with particular option m, and each hub q has m_j options, so the initial total number of facilities in $Q = \sum_j m_j$)

   b). Set S = set of open facilities (open facility s here means hub j is open with option m, so the initial number of open facilities in S = null)

   c). $Z^U = \infty$

2. For each facility $(q = jm) \in Q$, compute R_{jm} (= total savings that will result if facility $q = jm$ is open) to determine which facility to open.

   a). For each supplier i, compute,

$$W_{ijm} = \max\{\min_{s \in S} (CT_{is} - CT_{ijm}), 0\}$$

where,

$$CT_{ijm} = C_{ijm} + \sum_k CI_{ijkm}$$
$$CT_{is} = \left(C_{ijm} + \sum_k CI_{ijkm}\right)_{jm=s}$$

Note: Here, W_{ijm} will take the value of either 0 or the smallest positive difference between CT_{is} and CT_{ijm}. If $W_{ijm} > 0$, then it means that for that supplier i, CT_{ijm} will be lower than all CT_{is}, thus, a lower cost will be incurred when facility $q = jm$ is open.

   b). Compute Ω_{jm}= total savings in CT if facility $q = jm$ is open,

   c).

$$\Omega_{jm} = \sum_i W_{ijm}$$

$R_{jm} = \Omega_{jm} \times \text{Min}\left\{\left(\frac{G_{jkm}}{\sum\limits_{\{i|W_{ijm}>0\}} \mu_{ijkm}}\right)_{\forall k}, 1\right\} - \left(F_{jm} + \sum\limits_{k} CS_{jkm}\right)$. This saving is computed by deducting the fixed cost incurred from the total savings in CT if facility $q = jm$ is open. However, the total savings in CT is discounted if the capacity constraint is violated. The greater the violation, the larger will be the discount factor. The discount factor is the ratio between the available capacity and the capacity required for each warehouse k.

d). Open facility q (= hub j with option m) which has the maximum R_{jm}, and remove all facilities in Q which has hub index $= j$.

Note: For the initial iteration where set S is null, all W_{ijm} will be equal to 0 and thus, Ω_{jm} will also be 0. In this case, R_{jm} will be the negative of the fixed cost. The facility to open will be the maximum R_{jm}, corresponding to the facility with the lowest fixed cost.

3. For each path k provided by hub j with option m, determine if the capacity constraint is violated using a binary parameter B_{jkm}. If $\sum\limits_{jm\in S} G_{jkm} < \sum\limits_{jm\in S} \sum\limits_{i} \mu_{ijkm} \Rightarrow \quad set \quad B_{jkm} = 1 \quad else \quad B_{jkm} = 0$

   If any capacity constraint is violated, that is, if any $\text{B}_{jkm} = 1$, go to step (2) to add more facilities to set S, else continue to step (4).

4. For each supplier i, compute,

$$\Delta CT_{is} = CT_{is}^2 - CT_{is}^1 \qquad \forall s \in S$$

   where, $CT_{is} = \left(C_{ijm} + \sum\limits_{k} CI_{ijkm}\right)_{jm=s}$ and superscript 1 and 2 signifies the best and second best assignment of supplier to the open facilities in set S.

5. Rank ΔCT_{is}in descending order.

6. For each supplier i in the order given in step (5), assign supplier i to the hub j with option m where $jm \in S$, with the smallest possible CT_{ijm}, and with sufficient capacity G_{jkm} for each k. If no assignment is possible, this implies that insufficient capacity is available using only the facilities in set S, then we go back to step (2) to add more facilities to set S.

7. When all suppliers are assigned, any facility $s \in S$ with no supplier assigned to it should be removed from set S.

8. Compute the total cost of the solution to get the upper bound.

Heuristic (H2) – To obtain feasible solution to problem P and new upper bound (New-UB$_{LR}$)

1. For each supplier i, compute,
   $CT_{ijm} = C_{ijm} + \sum\limits_{k} CI_{ijkm}$ for each jm
2. Consider each supplier for which the LR solution assigns to more than one hub, or more than one option for the same hub. Assign each such supplier to the cheapest hub j with option m, given by the lowest CT_{ijm}, for the same i.

3. Consider each supplier for which the LR solution does not assign to any hub at all. Assign each such supplier to the cheapest hub j with option m, given by the lowest CT_{ijm}, for the same i, provided this does not violate the capacity constraint (4). Otherwise, select the next lowest CT_{ijm} and repeat the step.
4. Check that each hub j does not have more than one option selected
   (a) If an open hub has more than one option selected. For each option m, compute the cost of opening hub j with option m and assigning all suppliers (which are assigned to hub j regardless of options) to it.
   (b) Select the lowest cost option with sufficient capacity and assign all suppliers (which are assigned to hub j regardless of options) to it. We assume that the problem has been specified so that for each hub there is always one option with adequate capacity.
5. Determine Z_{jkm} according to constraint (3) and constraint (5).
6. Compute the new objective function value according to the decision variable X_{ijm}, Y_{jm} and Z_{jkm}, to obtain the new upper bound New-UB$_{LR}$

Heuristic (H3) – Perturb LP Relaxed solution to obtain feasible solution and upper bound (UB$_{LP}$)

1. Obtain all values of X_{ijm}
2. For each supplier i, find the max(X_{ijm}) with corresponding best-j and best m
3. For each i,
   (a) Compute the capacity required to ship components from supplier i to each manufacturer k, for the best-j and best-m, that is,

$$\mu_{ijkm}|_{j=best-j,m=best-m} \qquad \forall i,k$$

   (b) Check if capacity constraint is violated using,

$$\sum_i \mu_{ijkm}|_{j=best-j,m=best-m} \leq G_{jkm}|_{j=best-j,m=best-m} \qquad \forall k$$

   where the summation is over all suppliers that have been assigned.
   i) if capacity is not violated, set $X_{ijm} = 1$ for the best-j and best-m, and set all other $X_{ijm} = 0$ for the remaining j and m, else
   ii) if capacity is violated, set best-j and best-m to the next best max(X_{ijm}) and repeat steps 3a) to 3b)
4. For each hub j with option m, compute $\sum_i X_{ijm}$. If $\sum_i X_{ijm} > 0$, set $Y_{jm} = 1$. (This may cause more than one option to be selected for the same j)
5. For each hub j,
   (a) Find out all the suppliers which are assigned to hub j for all its available options m, that is, set $X(i) = 1$ if there exists an m for which $X_{ijm} = 1$.
   (b) For each k, set $Z(k) = 1$ if there exists an m for which $\sum_i \mu_{ijkm} X_{ijm} > 0$; otherwise $Z(k) = 0$.
   (c) Compute the cost of assigning all these suppliers to j for each option m

$$Y_{jm} + \sum_i C_{ijm} X(i) + \sum_i \left(\sum_k CI_{ijkm} \right) X(i) + \sum_k CS_{jkm} Z(k)$$

(d) Find the option m with the least cost given in step 5c), and with sufficient capacity. We assume that the problem has been specified so that for each hub there is always one option with adequate capacity.
(e) Set $Y_{jm} = 1$ for the least cost option m, and set $Y_{jm} = 0$ for all other options at hub j.
(f) Set $X_{ijm} = 1$ for the least cost option m for suppliers with $X(i) = 1$; set all other $X_{ijm} = 0$.

6. For each j, k, m, if $Y_{jm} = 1$ and $\sum_i \mu_{ijkm} X_{ijm} > 0$, set $Z_{jkm} = 1$.

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