

Pricing Decisions during Inter-generational Product Transition

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Abstract

How should companies price products during an inter-generational transition? High uncertainty in a new product introduction often leads to extreme cases of demand and supply mismatches. Pricing is an effective tool to either prevent or alleviate these problems. We study the optimal pricing decisions in the context of a product transition in which a new generation product replaces an old one. We formulate the dynamic pricing problem and derive the optimal prices for both the old and new products. Our analysis identifies the key dynamics affecting the optimal prices: product replacement, scarcity, inventory, and internal and external competition. In addition, we propose a simple and pragmatic approach to incorporate demand learning into the pricing decisions during a product transition.

Keywords: dynamic pricing; product transition; new product introduction; demand learning

1. Introduction

In high-tech industries, a company periodically replaces the current product with a newer generation product. In many cases, this transition does not occur instantaneously but rather involves a transition period during which the company sells both products. The introduction of a new product creates high uncertainty in both demand and the supply. If many new features are added to the new generation product, it is difficult to predict its acceptance by the customers. Furthermore, there is uncertainty in how smoothly the suppliers handle the technological or production changes for the new product, which often results in delays in the new product release date.

This paper is motivated by collaborative work with a telecommunications equipment manufacturer. In this industry, the replenishment lead time is about 18 weeks: at least 13 weeks for critical components plus 5 weeks for production and testing. A product transition starts with the release of the new product and ends with the old product demand dropping to a negligible level, and usually lasts from a few weeks to a few months. But with an 18-week lead time, any replen-

ishment order placed during the transition will not arrive before the transition ends. Therefore, the managers have little chance to correct their initial inventory decisions once the transition starts, even if a demand-supply mismatch becomes evident. Consequently, the company often runs out of the product that customers want while having excess of the other.

For example, a chip supplier issued an end-of-life notice for the chipset used in one of the company's wireless products, Blofeld, which drove the company to introduce the next generation product Blofeld II. The company had expected the transition to be quick and did not stock up inventory for the old chip. Unfortunately, there were unanticipated design issues around the new chip that delayed the release of Blofeld II. Consequently, the company kept selling Blofeld long after the scheduled release date of Blofeld II, creating a shortage of Blofeld. To counteract such supply risks, operations managers tend to add large buffer inventories for the old product. However, a generous supply cushion often results in excess inventory of the old product at the end of the transition, which is the case for another transition at the same company. This time, they were phasing out a product with high sales volume and had purchased a large amount of buffer inventory for the old product to avoid any supply gap and lost sales. Ironically, when the new generation product was delivered on schedule, it left the managers in another dilemma. If they were to release the new product, Sultan II, as scheduled, they would face large quantities of excess inventory of Sultan, the old product. If they delay the release of Sultan II, they could avoid large excess of Sultan; but this would be a costly option because Sultan II had a better margin than Sultan. Eventually the company decided to delay the new product introduction in selected sales regions and forego some of the margin benefits to alleviate the excess problem.

Because replenishment during the transition is not possible, the managers have very limited options when facing a demand-supply mismatch. We study in Li et al. (2007) the option of prod-

uct substitution: When one product is depleted, a company may offer the other one as a substitute. Pricing is another option: The managers can manipulate the prices of the two products to mitigate the risk of demand and supply mismatch. If sales of the old product are sluggish during the transition, they could put in a promotion and discount it. If the new product does not sell well, managers may increase the price of the old product to make the new appear more attractive.

In this paper, we study the optimal pricing problem for an inter-generational product transition; that is, a product transition that involves a new product and an old product, where the new product is designed with the intention to replace the old product. We do not consider the case of a “completely new” product that has no direct predecessor and that is designed to meet a new set of customer needs. For example, the replacement of Canon’s camera model PowerShot SD700 by SD800 is an inter-generational transition, whereas the first introduction of the “iPhone” is not. While both cases are important for business success, the former is a day-to-day problem facing decision makers in technology companies. In addition, we do not consider cases where the two generations of product belong to different companies.

We consider a finite time horizon from the time a new product is introduced until the transition finishes. We choose the interval length such that by the end of this time horizon, the demand of the old product has dropped to a negligible level and any leftover inventory can only be salvaged at a sizeable discount. At the beginning of the horizon, we are given the initial inventories of the old and new products. There is no option for replenishment as we assume that any replenishment will arrive after the completion of the transition. During the time horizon, the demand of the old product gradually phases out while the new phases in and will continue to be sold beyond the transition. Due to the similarities of the two products, any pricing decision for one product

affects the demand of both products. Therefore, we determine the optimal prices for the two products simultaneously, as a function of time and inventory.

Both uncertainty in demand and uncertainty in the new product release date can lead to demand and supply mismatch. In this paper, we address demand uncertainty only. In Li et al. (2007), we have looked at the timing uncertainty in a new product introduction.

During a product transition, companies adjust price not just to correct an inventory imbalance, but also to adapt to changes in their belief about demand. When a company introduces a new product, often it is not clear what the demand will be. The company might underestimate or overestimate the market's acceptance of the new product. As the transition progresses, the managers learn more about the demand and may change the price to reflect such updated demand belief. A key strength of our model is that it allows a simple demand learning mechanism to be incorporated into the dynamic pricing decision. Specifically, we obtain the demand parameters as a weighted average of the initial estimates and the estimates from a least square regression on the realized demand. We then base the pricing decisions on the updated demand estimation.

The remainder of this paper is organized as follows: In section 2, we review the relevant literature. In section 3, we specify the problem and the demand model. We solve for the optimal prices in Section 4 and identify the key dynamics influencing the optimal prices in Section 5. In Section 6 we describe how to integrate demand learning into the pricing decisions. We conclude with a discussion on the limitations of the model and future research possibilities. The proofs are in a separate document, available on request from the authors.

2. Literature Review

We do not provide a comprehensive review of the existing pricing literature. Rather, we consider papers that are directly related to our work. That is, we focus on dynamic pricing models with limited supply and no replenishment opportunities during the planning horizon.

The majority of these models focuses on a single product with stationary demand functions, i.e., demand is a function of price only. Gallego and van Ryzin (1994) consider a single-product dynamic pricing problem when there is no option for reordering. They study a case when demand's dependence on price and time is multiplicatively separable. Bitran and Mondschein (1997, 1993) study the pricing problem of perishable products in retail. They show that the percentage of price reduction in a retail store should increase over time. Monahan et al. (2004) combine the dynamic pricing problem with the optimal stocking problem. They derive the optimal pricing and stocking solutions for a time-invariant exponential demand function.

Zhao and Zheng (2000) address time-varying demand and derive certain structural properties for the optimal pricing solution that hold in general. For example, they show that the optimal price decreases with inventory and identify a condition under which the optimal price decreases over time for a given inventory level.

With multiple products, an appropriate demand model is necessary. Existing literature considers several ways to model demands. The Multinomial Logit (MNL) model is first proposed by Luce (1959). Under the MNL model, the customer's purchase probability ρ_i depends on his/her

utility from each product u_i through $\rho_i = \frac{e^{u_i}}{\sum_k e^{u_k}}$. The MNL model has been used to predict indi-

vidual choices (McFadden 1986), as well as aggregate market share for new products (Berry 1994, Brownstone and Train 1999, Berry et al. 1995). Another class of choice model incorpo-

rates the customer's budget constraint into the utility maximization problem (Hauser and Urban 1986, Bitran et al. 2004). In the reservation price model (Awad et al. 2000), a customer assigns a maximum amount of money he/she is willing to pay for each product (reservation price). The customer then decides among those products that have a price lower than its corresponding reservation price.

Most dynamic pricing models for multiple products use one of the three choice models, or a variation of them. Awad et al. (2000) study pricing policies for a family of perishable products using the reservation price choice model. Bitran et al. (2004) study a very similar problem. They combine the MNL model with a utility maximization model to describe the demand for substitutable products. Both papers resort to heuristic algorithms to approximate the optimal solution.

Gallego and van Ryzin (1997) study the dynamic pricing problem for multiple products in the context of a network flow problem. They do not model the demand relationships among the products but instead assume a generic set of demand functions. Bitran and Caldentey (2003) also give a generic formulation of the multiple-product problem and provide an optimality condition. In general, these generic formulations can say very little about the optimal policy.

Xu and Hopp (2004a) consider a pricing problem for a single product with one or multiple retailers. Assuming that the demand process follows a geometric Brownian motion, they are able to obtain a closed-form solution for the monopoly case. When there are multiple retailers, they derive the equilibrium pricing policies for the retailers. In contrast, we study the centralized pricing decision, i.e., a company that sells two generations of products and thus has to maximize the total expected profit from the two products.

Kornish (2001) studies the pricing problem for a monopolist with frequent product upgrades. She assumes that the monopolist sells only the latest generation of product in any period. Goet-

tlar and Gordon (2008) study the dynamic pricing and investment decisions for a product that competes in a duopoly. They adopt a Multinomial Logit model for demand. One interesting feature of their model is that the customer's non-purchase option depends on the ownership distribution of the previous generation product. In contrast to these papers, we consider the pricing problem where a company sells two generations of products simultaneously.

Ferguson and Koenigsberg (2007) study the pricing and stocking decisions when a company sells the newly-replenished units in the presence of left-over units from last period. The nature of the problem is closely related to the one we study, although the methods and focus are very different. They use a linear aggregate demand model to derive the optimal prices and quantities for the old and new items. In this paper, we try to capture the time-varying aspects of the demand, in addition to the competition between the old and new products.

Another stream of demand models for multiple products is the diffusion models. Fisher and Pry (1971) propose a technological substitution model in which the ratio of the fractional market share of the new and old technology follows an exponential growth pattern. Norton and Bass (1987) provide a diffusion-theory-based demand model for successive generations of products, extending from the seminal Bass diffusion model (1969). Krishnan et al. (1999) have applied the basic Bass diffusion model to a single-product pricing problem. Bayus (1992) studies a pricing problem in the context of an inter-generational product transition. He uses the original Bass diffusion model for the new generation product demand and generates the old product demand by holding the combined demand of the two products fixed. The difficulty for using the diffusion models is the analytical tractability. It is hard to obtain closed-form or structurally meaningful results, especially with two generations of products.

Most dynamic pricing models with demand learning use Bayesian updating. A typical Bayesian learning model assumes that the demand has a known distribution, but with unknown parameters. When the prior distribution of the parameter under estimation is a conjugate to the demand distribution, there is an easy update equation for the posterior distribution. With demand learning, the optimal dynamic price is difficult to obtain. Researchers often resort to heuristic approaches. Araman and Caldentey (2005) use a linear function to approximate the value function in the dynamic programming formulation. Farias and Van Roy (2007) also use an approximate value function; but they choose the heuristic price based on a “decay balancing” condition which balances the rate of sales and learning with the rate of value discounting. Aviv and Pazgal (2002) propose a “certainty equivalence” heuristic in which they consider demand learning but determine the optimal price by the expected demand, disregarding demand variability. The certainty equivalence heuristic greatly simplifies the problem and is practically appealing. In this paper, we use the same certainty equivalence approximation when incorporating demand learning into the problem. Bertsimas and Perakis (2001) use dynamic programming jointly with least squares estimates to obtain the optimal pricing policies. Similarly, in this paper we combine a regression-based learning model with the dynamic pricing problem.

The learning approach in this paper is inspired by Lenk and Rao (1990) and Smith et al. (1994). They forecast the early sales of a new product from previous sales data of a group of similar products, and use the observed demand to continually update these estimates. The predictive effect from the previous populations diminishes as the amount of sales data for the new product increases. In this paper, we propose a similar demand updating mechanism.

Intel has long recognized the importance of managing product transitions (Hopman 2005). According to Hopman, “*Intel’s business is one of transitions.*” Researchers at Intel have devel-

oped a Product Transition Index (PTI) that consists of eight vectors which the experts believe are the main factors affecting the pace and success of a product transition. They then use the PTI score to predict the rate of a transition and to determine the planning strategy.

3. Problem Description and Demand Model

We present a dynamic programming model that addresses the pricing decisions for a product upgrade during the transition period of length T . We make the following assumptions:

- (i) The transition from the old product to the new one starts at time 0 and is completed within time T ; thus by time T the demand of the old product has become negligible.
- (ii) There is no option for inventory replenishment during the period $[0, T]$.

Given the above, we solve for the optimal prices for the old and new products during the transition period $[0, T]$ as a function of both time and inventory. The justification for (ii) is that the replenishment time is long relative to the transition period.

We adopt the Multinomial Logit (MNL) consumer choice model to describe the demand functions of the two products. Assume that a customer's utility of purchasing product i ($i=1$ refers to the old product and $i=2$ refers to the new product) is $u_i(r_i, t) = a_i(t) - g(r_i) + \varepsilon_i$, where r_i is the selling price of product i , $g(r_i)$ is the disutility of paying r_i and $a_i(t)$ is the time-varying attribute(s) that affects the customer's utility. We also assume that a customer's utility of not purchasing any product from this company is $u_0(t) + \varepsilon_0$. If the disturbances ε_i are iid random variables with Gumbel distribution $F(x) = e^{-e^{-x}}$, the MNL model gives the probability that a customer purchases product i : $\rho_i(\mathbf{r}, t) = \frac{e^{a_i(t) - g(r_i)}}{e^{a_1(t) - g(r_1)} + e^{a_2(t) - g(r_2)} + e^{u_0(t)}}$ $i = 1, 2$, where $\mathbf{r} = (r_1, r_2)$ is

the price vector; then the probability of no-purchase is $\rho_0(\mathbf{r}, t) = 1 - \rho_1(\mathbf{r}, t) - \rho_2(\mathbf{r}, t)$ (McFadden

1973). By using the same $g(\cdot)$ function for both products, we are assuming that a customer's disutility toward price, or equivalently the utility toward money, is the same for both products. The original specification of the MNL model allows the coefficients to vary across individuals, as well as alternatives (Greene 2003). In this model, we assume homogeneous customers and hence focus on the aggregate impact of the customers' choice.

The no-purchase option allows us to explore the pricing problem in a monopolistic situation, as well as in a competitive market. In the monopoly case, we interpret the no-purchase utility as the customer's utility of not obtaining any product; in the case with competition, the no-purchase utility equates to the customer's reservation utility for other market options, such as the utility from buying a competitor's product. We consider both time-invariant and time-increasing $u_0(t)$.

Economists developed the MNL model to describe an individual's choice behavior when facing a set of alternatives (McFadden 1973). It has been widely used to model demand for horizontally differentiated products (e.g., competition among car models by Boyd and Mellman 1980, Cardell and Dunbar 1980) and on different product forms (e.g., different travel modes by McFadden 1974, Hensher 2001). More recently, applications on vertically differentiated products (e.g., high-tech products with multiple generations) have gained more attention. Bresnahan et al. (1997) categorize the PC products into frontier and non-frontier products to reflect the degree of technological innovativeness, which is then incorporated into the choice model along with other attributes to model the demand for PCs. Melnikov (2001) introduces time-related attributes to the choice model to model the inter-temporal demand substitution in computer and printer products. In this paper, we introduce an attribute term $a_i(t)$ to characterize the time-varying customer preference for the old and new products. As customers shift their preference

from the old product (or technology) to the new one, the impact on demand is reflected through an increase in $a_2(t)$ and a decrease in $a_1(t)$.

Assumption 1. We assume that customer arrival is a homogeneous Poisson process with rate λ .

Hence the old and new products have Poisson demand with time-varying rates

$$\lambda_i(\mathbf{r}, t) = \lambda \rho_i(\mathbf{r}, t) = \lambda \frac{e^{a_i(t)-g(r_i)}}{e^{a_i(t)-g(r_i)} + e^{a_j(t)-g(r_j)} + e^{u_0(t)}} \quad i = 1, 2 \quad (1)$$

and the no-purchase rate is

$$\lambda \rho_0(\mathbf{r}, t) = \lambda(1 - \rho_1(\mathbf{r}, t) - \rho_2(\mathbf{r}, t)). \quad (2)$$

This demand model is relatively simple and intuitive. The fact that the time factor and the price factor are separable within each exponent term leads to significant analytical tractability. In addition, it generates a logistic demand pattern that is often observed in practice. For example, when $a_1(t) = 5 - 0.0125t$, $a_2(t) = 0.0125t$, $u_0(t) = 0$, $g(r) = r$, $r_1 = r_2 = 3$ and $\lambda = 0.1$, the above demand model generates a demand pattern shown in Figure 1.

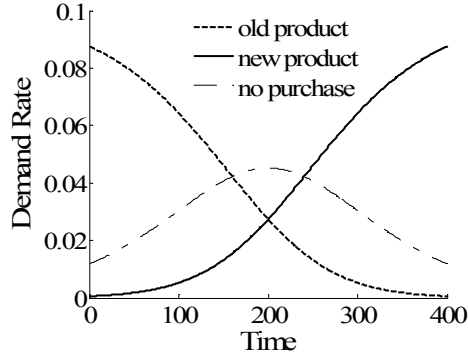


Figure 1: Demand Pattern under Equation (1)

For a time-invariant reservation utility $u_0(t)$, the total demand rate of the two products drops to the lowest level when the two products have equal market shares. This represents a period of time when the customers' preference for the old product is significantly reduced by the introduction of the new product, while the new product itself has not gained full acceptance from the cus-

tomers yet. As a result, customers cannot decide which product to buy and thus are more likely to resort to the no-purchase (or other outside) option.

Assumption 2. When a product stocks out, a customer then chooses between buying the other product and the no-purchase option. Thus, the purchase probabilities when a particular product

$$\text{runs out are: } \bar{\rho}_i(r_i, t) = \frac{e^{a_i(t)-g(r_i)}}{e^{a_i(t)-g(r_i)} + e^{u_0(t)}} \text{ and } \bar{\rho}_0(r_i, t) = \frac{e^{u_0(t)}}{e^{a_i(t)-g(r_i)} + e^{u_0(t)}} \quad (3)$$

Implicitly, we assume that running out of a product is equivalent to setting an infinitely high price for that product (i.e., $\lim_{r \rightarrow \infty} g(r) = \infty$). Consequently, its demand is proportionally split between the other product and the outside option. This also follows directly from the Independence from Irrelevant Alternatives (IIA) property of the MNL model, which states that the ratio of any two choices within a choice set is not affected by the presence of other choices. In this case, IIA

$$\text{implies that } \frac{\bar{\rho}_i(r_i, t)}{\bar{\rho}_0(r_i, t)} = \frac{\rho_i(\mathbf{r}, t)}{\rho_0(\mathbf{r}, t)}.$$

From equation (1), we obtain the following derivatives, which we will use later in deriving the optimal solutions of the pricing problem.

$$\frac{\partial \rho_0(\mathbf{r}, t)}{\partial r_i} = \rho_0 \rho_i g'(r_i), \quad \frac{\partial \rho_i(\mathbf{r}, t)}{\partial r_i} = \rho_i (\rho_i - 1) g'(r_i) \text{ and } \frac{\partial \rho_i(\mathbf{r}, t)}{\partial r_j} = \rho_i \rho_j g'(r_j) \quad (4)$$

4. Optimal Dynamic Prices

In the analysis that follows, we use the “baby Bernoulli process” approximation of the Poisson process (Gallager 1999) to discretize the finite planning horizon, similar to the approach taken in Bitran and Mondschein (1997). We choose the length of each discrete time period such that the probability of more than one demand arrival in each time period is nearly zero. We then assume that there can be at most one demand in each time period. Given this time period, we let T denote

the number of time periods in the planning horizon. Then we rescale the parameter λ so that within each time period t , the probability of no customer arrival is $1-\lambda$, and the probability of exactly one customer arrival is λ . Therefore, for each time unit t , $\lambda\rho_i(\mathbf{r},t)$ is the probability that a demand occurs for the old product ($i=1$) or for the new product ($i=2$); the probability of no purchase after a customer arrival is $\lambda\rho_0(\mathbf{r},t)$.

Let $V_t(x_1, x_2)$ be the value-to-go at the beginning of period t if the company has x_i units of old product ($i=1$) and new product ($i=2$), and makes optimal price decisions at t and thereafter.

We define $V_t(x_1, x_2)$ recursively:

$$V_t(x_1, x_2) = \max_{\mathbf{r}} h_t(\mathbf{r}, x_1, x_2)$$

$$\text{where } h_t(\mathbf{r}, x_1, x_2) \equiv \lambda_1(\mathbf{r}, t)(r_1 + V_{t+1}(x_1 - 1, x_2))$$

$$+ \lambda_2(\mathbf{r}, t)(r_2 + V_{t+1}(x_1, x_2 - 1))$$

$$+ (\lambda\rho_0(\mathbf{r}, t) + 1 - \lambda)V_{t+1}(x_1, x_2) \quad \forall x_1, x_2 > 0 \quad (5)$$

$$h_t(\mathbf{r}, 0, x_2) \equiv \lambda\bar{\rho}_2(r_2, t)(r_2 + V_{t+1}(0, x_2 - 1))$$

$$+ (\lambda\bar{\rho}_0(r_2, t) + 1 - \lambda)V_{t+1}(0, x_2) \quad \forall x_2 > 0 \quad (6)$$

$$h_t(\mathbf{r}, x_1, 0) \equiv \lambda\bar{\rho}_1(r_1, t)(r_1 + V_{t+1}(x_1 - 1, 0))$$

$$+ (\lambda\bar{\rho}_0(r_1, t) + 1 - \lambda)V_{t+1}(x_1, 0) \quad \forall x_1 > 0 \quad (7)$$

The terminal value is the salvage value of products left over after T : $V_{T+1}(x_1, x_2) = s_1x_1 + s_2x_2$ where s_i is the unit salvage value of a product at the end of the transition. The salvage value for the new product reflects the value depreciation of the new product. It does not necessarily imply that the company will salvage any left over units of the new product.

The problem is to find the optimal prices r_1 and r_2 for each (t, x_1, x_2) combination. The value function $h_t(\mathbf{r}, x_1, x_2)$ as defined in equations (5)-(7) is not necessarily concave in prices. However, we establish quasi-concavity of the value function under the following assumption.

Assumption 3. The disutility function $g(\cdot)$ is continuous and twice differentiable and that

$$g'(r_i) + \frac{g''(r_i)}{g'(r_i)} > 0.$$

Assumption 3 is a technical assumption that is satisfied by many increasing utility functions.

Proposition 1. $h_t(\mathbf{r}, x_1, x_2)$ is quasi-concave in r_1 and $r_2 \quad \forall x_1 \geq 0, x_2 \geq 0, t \in [0, T]$.

Under Assumption 3, the Karush, Kuhn and Tucker (KKT) optimality condition for a concave objective function can be extended to a quasi-concave objective function (Avriel et al. 1988). Therefore, we can obtain the optimal solution by solving the first-order condition for equation (5):

$$\rho_i(\mathbf{r}, t) + \frac{\partial \rho_j(\mathbf{r}, t)}{\partial r_i} (r_j - \Delta_j V_{t+1}(x_1, x_2)) + \frac{\partial \rho_i(\mathbf{r}, t)}{\partial r_i} (r_i - \Delta_i V_{t+1}(x_1, x_2)) = 0 \quad i = 1, 2, j \neq i \quad (8)$$

where $\Delta_1 V_{t+1}(x_1, x_2) \equiv V_{t+1}(x_1, x_2) - V_{t+1}(x_1 - 1, x_2)$ and $\Delta_2 V_{t+1}(x_1, x_2) \equiv V_{t+1}(x_1, x_2) - V_{t+1}(x_1, x_2 - 1)$ are the marginal value of inventory. Substituting equation (4) into (8), we can solve equation (8) and rewrite the first order condition as:

$$r_i - \Delta_i V_{t+1}(x_1, x_2) = \frac{1}{\rho_0(\mathbf{r}, t)} [g'(r_i)^{-1} - (g'(r_i)^{-1} - g'(r_j)^{-1}) \rho_i(\mathbf{r}, t)] \quad (9)$$

Economists often assume a quasi-linear utility function to simplify problems and obtain tractable solutions by eliminating the effect of initial wealth (Mas-Colell et al. 1995). In the analysis that follows, we assume that the customers' utilities are linear with respect to money, i.e., the disutility function $g(r_i)$ is a linear function.

Assumption 4. $g(r_i) = \beta_r r_i$ where $\beta_r > 0$.

With Assumption 4 we can reduce condition (9) to

$$r_i - \Delta_i V_{t+1}(x_1, x_2) = \frac{1}{\beta_r \rho_0(\mathbf{r}, t)} \quad (10)$$

The term $r_i - \Delta_i V_{t+1}(x_1, x_2)$ is the marginal gain from selling a unit of product i at time t . Intuitively, the optimal price is set such that the company is indifferent between selling an old and selling a new. Solving the above equation for r_i , we obtain:

$$r_i(t, x_1, x_2) = \Delta_i V_{t+1}(x_1, x_2) + \frac{1}{\beta_r} \{1 + W(e^{a_i(t)-u_0(t)-1-\beta_r \Delta_i V_{t+1}(x_1, x_2)} + e^{a_j(t)-u_0(t)-1-\beta_r \Delta_j V_{t+1}(x_1, x_2)})\}$$

where W is the Lambert's W function, i.e., $W(x)$ solves the equation $we^w = x$ for w as a function of x . We can do a similar analysis for the cases when one of the products runs out. Substituting the optimal prices into equations (5)-(7) yields a recursive formula for computing the value function $V_t(x_1, x_2)$. We summarize these results in Proposition 2.

Proposition 2. *Under Assumption 4, the optimal price of the old and new products at time t for a given inventory level (x_1, x_2) is*

$$r_i^*(t, x_1, x_2) = \Delta_i V_{t+1}(x_1, x_2) + \frac{1}{\beta_r} \{1 + W(Z)\} \quad (11)$$

$$\text{where } Z = \begin{cases} e^{a_i(t)-u_0(t)-1-\beta_r \Delta_i V_{t+1}(x_1, x_2)} + e^{a_j(t)-u_0(t)-1-\beta_r \Delta_j V_{t+1}(x_1, x_2)} & \forall x_1, x_2 > 0 \\ e^{a_2(t)-u_0(t)-1-\beta_r \Delta_2 V_{t+1}(0, x_2)} & \forall x_1 = 0, x_2 > 0 \\ e^{a_1(t)-u_0(t)-1-\beta_r \Delta_1 V_{t+1}(x_1, 0)} & \forall x_1 > 0, x_2 = 0 \end{cases}$$

and we obtain $V_t(x_1, x_2)$ using the following recursive equations:

$$V_{T+1}(x_1, x_2) = s_1 x_1 + s_2 x_2 \quad (12)$$

$$V_t(x_1, x_2) = V_{t+1}(x_1, x_2) + \frac{\lambda}{\beta_r} W(Z) \quad (13)$$

We note from (13) that the marginal value of time is

$$\Delta_t V(x_1, x_2) \equiv V_t(x_1, x_2) - V_{t+1}(x_1, x_2) = \frac{\lambda}{\beta_r} W(Z). \quad \text{This value increases in } a_i(t) - u_0(t), \text{ the}$$

time-varying relative attractiveness of product i , and decreases in the marginal value of inventory

$\Delta_i V_{t+1}(x_1, x_2)$. We can now express the optimal prices as:

$$r_i(t, x_1, x_2) = \Delta_i V_{t+1}(x_1, x_2) + \frac{1}{\lambda} \Delta_t V(x_1, x_2) + \frac{1}{\beta_r} \quad (14)$$

Heretofore we have not made any specific assumptions on $a_i(t)$, the time-varying attribute. In fact, the solution given in Proposition 2 applies to any two substitutable products with time-varying attributes and sold by the same company. To derive structural properties of the optimal dynamic prices in the context of a product transition, we make some additional assumptions regarding $a_i(t)$ and $u_0(t)$. In what follows, we examine in a progressive manner the various factors and dynamics that affect the optimal prices of the two products during the transition.

5. Factors and Dynamics Affecting the Optimal Price

We start with a simple base case. We let both $a_i(t)$ and $u_0(t)$ be constant, and consider infinite supply of inventory, i.e., $a_i(t) = a_i$, $u_0(t) = u_0$, $x_1, x_2 \rightarrow \infty$.

From equations (11), the optimal prices are constants.

$$r_i^* = s_i + \frac{1}{\beta_r} (1 + \mathcal{W}(e^{a_1 - u_0 - 1 - \beta_r s_1} + e^{a_2 - u_0 - 1 - \beta_r s_2}))$$

This is consistent with findings from existing dynamic pricing literature with unconstrained supply and time-invariant attributes. The base case does not necessarily correspond to any realistic situation. However, we can infer the various factors and dynamics affecting the optimal price by comparing the optimal solution under more complex cases with the base case.

5.1 Effect of Product Replacement

The function $a_i(t)$ represents the change over time in customers' attitude toward a product after the new product introduction, independent of the products' prices. We expect $a_i(t)$ to vary over the transition period, and it could take on various functional forms.

Assumption 5. We assume that $a_i(t)$ is given by $a_1(t) = a_0 - kt$ and $a_2(t) = kt$ where $a_0, k > 0$ are known constants.

As justification for this assumption, we consider the behavior of the market shares for the two products. Under the MNL model, this ratio at time t is

$$\frac{\rho_2}{\rho_1} = e^{a_2(t)-a_1(t)-(g(r_2)-g(r_1))} = A(\mathbf{r})e^{a_2(t)-a_1(t)}. \text{ Given that } a_i(t) \text{ is linear, the market share ratio in the}$$

MNL model has the same form as for the Fisher and Pry (1971) model, for which the market share of the old and new technologies (products) is $e^{2\alpha(t-t_0)}$ where t_0 is the time at which the new and the old have equal market shares, and α is a constant that signifies the rate of substitution.

We can also show that Assumption 5 is consistent with the Norton and Bass (1987) model: If we assume that the old and new products have the same customer population, then the market share ratio in the Norton-Bass model differs from the MNL model by a constant. Both the Fisher-Fry model and the Norton-Bass model perform well on empirical data. Thus, we contend that the linear assumption of $a_i(t)$ is reasonable.

As shown in Figure 1, Assumption 5 generates a logistic demand pattern commonly observed during a product transition. The new product will, over time, replace the old product. The magnitude of k represents the rate of the transition, i.e., how quickly the new product replaces the old (excluding the price factor). k may depend on multiple factors including, but not limited to, product capability, timing of the new product introduction, marketing effort and macroeconomic environment. As mentioned in Section 2, Intel generates a PTI score for each transition based on the assessment of these constituent factors. The resulting PTI score is a direct indicator of the transition rate. According to Jay Hopman (Hopman 2005),

“If all vectors are scored down the middle, the product transition should be expected to unfold at a rate on par with the average of past transitions. Hotter scores predict a faster transition, colder scores a slower transition.”

The Intel approach includes price and external competition, along with many other factors in the PTI scores. In our model, prices are decision variables and we model external competition implicitly through $u_0(t)$. We consolidate all other factors into k .

To gain insights into the optimal pricing behavior, we first consider some extreme cases.

Proposition 3. *Under Assumptions 4 and 5, if the no-purchase utility does not vary with time, i.e., $u_0(t) = u_0$, then*

i) *If $x_1 \rightarrow \infty, x_2 = 0$, $r_1^*(t, x_1, x_2)$ decreases in $t \forall t \in [0, T]$*

ii) *If $x_1 = 0, x_2 \rightarrow \infty$, $r_2^*(t, x_1, x_2)$ increases in $t \forall t \in [0, T]$*

iii) *If $x_1, x_2 \rightarrow \infty$, $r_i^*(t, x_1, x_2)$ decreases in t for $t \in [0, \bar{t}]$ and increases in t for $t \in [\bar{t}, T]$,*

where $\bar{t} = \frac{a_0 + \beta_r(s_2 - s_1)}{2k}$.

As inventory increases, the marginal value of inventory $\Delta_t V_{t+1}(x_1, x_2)$ approaches the salvage value s_i . Thus, from equation (14), the optimal price is solely determined by the marginal value of time $\Delta_t V(x_1, x_2)$. For case i), $\Delta_t V(x_1, x_2)$ is an increasing function of $a_1(t)$, which decreases linearly in time by assumption. Therefore, the optimal price of the old product decreases over time. Similarly, in case ii), $\Delta_t V(x_1, x_2)$ is an increasing function of $a_2(t)$, which increases in t . Therefore the optimal price of the new product increases over time. When both products are available, the optimal price behavior is more complex. In case iii), $\Delta_t V(x_1, x_2)$ is an increasing function of $Ae^{a_1(t)-u_0} + Be^{a_2(t)-u_0}$ where A and B are constants. Thus the price trend depends on which term dominates.

We illustrate the behavior of case iii) graphically in Figure 2. In this example, we assume the inventory level to be (40,40) throughout the transition. This inventory level represents, for all practical purposes, an infinite amount of inventory since the maximum total expected demand for both products for the planning horizon is $\lambda T = 40$. We observe from the optimal demand curve that the new product gradually replaces the old as the dominating product. The no-purchase option reaches its highest level midway through the transition when neither product dominates.

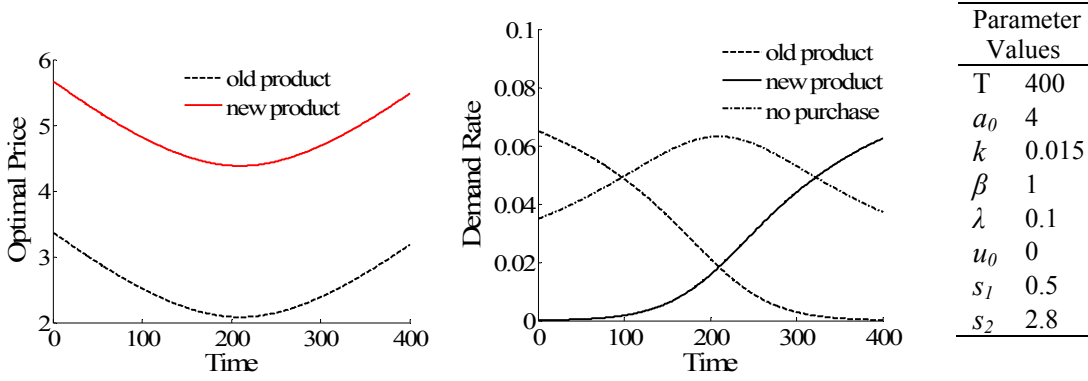


Figure 2: Optimal Prices and Demand for Given Inventory Level (40,40)

Comparing this with the base case, we see that the price trend is due to changes in $a_i(t)$. Initially, the new product is at its infancy and the old product offers the highest utility to customers (i.e., the term $Ae^{a_1(t)-u_0}$ dominates over $Be^{a_2(t)-u_0}$); the competition is essentially between the old product and the non-purchase option. As t increases, customer preference for the old product decreases whereas that for the new increases. The overall impact is that the no-purchase option becomes relatively more attractive over time before the new product gains strong hold. Therefore, the optimal pricing strategy during the first half of the transition is to decrease price in order to compete with the no-purchase option. Later, the new product replaces the old product to become the main product that competes with the no-purchase option, i.e., the term $Be^{a_2(t)-u_0}$ dominates over $Ae^{a_1(t)-u_0}$. Therefore, as the new product's attractiveness increases with time, the company ought to increase price to maximize revenue.

In Figure 2, the optimal prices of the two products differ by a constant amount. From equation (14), we know that $r_2^*(t, x_1, x_2) - r_1^*(t, x_1, x_2) = \Delta_2 V_{t+1}(x_1, x_2) - \Delta_1 V_{t+1}(x_1, x_2)$. With infinite supply, $\Delta_2 V_{t+1}(x_1, x_2) - \Delta_1 V_{t+1}(x_1, x_2) \rightarrow s_2 - s_1$, thus the gap between the two prices equals $s_2 - s_1$.

5.2 Effect of External Competition

In the demand model from Section 3, the no-purchase option may represent different customer behaviors. In the case of a monopoly, a customer's no-purchase utility, u_0 , represents his/her preference for not getting any product, old or new. Such preference may change over time. For example, in the high-tech industry, customers may expect a product's price to drop over time even in the absence of competition. Alternatively, technology development may make it easier (or harder) over time for a customer to get by without buying either product.

In a competitive market, a customer's no-purchase utility represents the best outside option. He/she may buy from a competitor that offers similar products. If the competitor continuously improves its product attractiveness, whether through enhanced features or lower prices, the value of u_0 will increase over time. Furthermore, in a mature industry, customers have a preconceived expectation on the price trend and evaluate the product against this preconception. For example, from 2000 to 2006, the microprocessor price drops at an annual rate of 48.9%, PC at 25.6%, and software at 0.8% (Cooper 2008). Therefore, a time-varying u_0 allows us to model the impact of these various factors on the optimal pricing strategy during a product transition.

When the no-purchase utility u_0 increases over time, the optimal prices of both the old and new products experience downward pressure. In Figure 3 we show the optimal prices and demand patterns when u_0 increases linearly with time, i.e., $u_0(t) = u_c + k_c t$, $u_c = 0$, $k_c = 0.0075$.

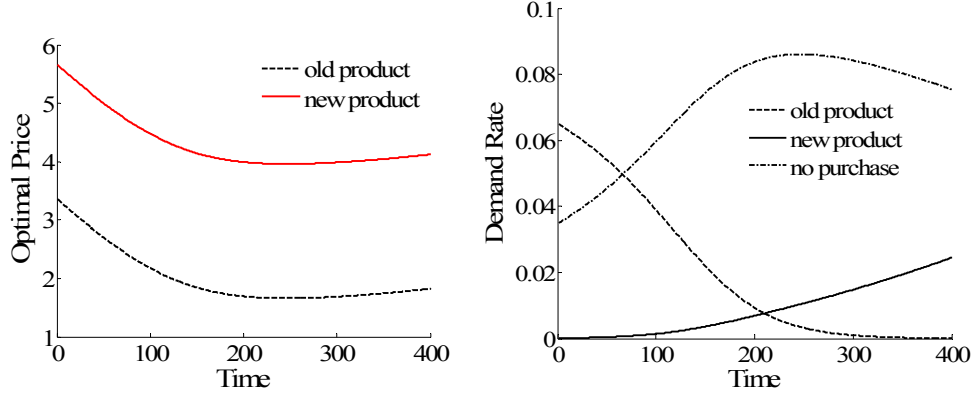


Figure 3: Optimal Prices and Demand with Time-Increasing u_0

All other parameters are the same as in Figure 2. The inventory level is again $(40,40)$, representing the case of abundant supply; hence, the marginal value of inventory $\Delta_i V_{t+1}(x_1, x_2)$ is constant. Thus u_0 affects the optimal price through the marginal value of time $\Delta_t V(x_1, x_2)$, more specifically, through the terms $e^{a_1(t)-u_0(t)}$ and $e^{a_2(t)-u_0(t)}$. As $u_0(t)$ increases in time, the optimal prices for both products will experience downward pressure. We observe from the resulting demand pattern that the time-increasing u_0 reduces the sales of the new product and limits the company's ability to increase price. Indeed, price increase is rare in practice for technology products. We have grounds to believe that a time-increasing u_0 is the cause. As design and production technology advance, a customer's outside option comes in the form of a cheaper or better product, prohibiting any price increase.

5.3 Effect of Scarcity

When supplies are limited, the optimal price behavior, as well as the resulting demand, has quite different characteristics. With limited supplies, the marginal value of inventory is not a constant but varies with time. Figure 4 demonstrates the optimal price over time using the same parameters as in Figure 2 but with the inventory level fixed at $(1,1)$. The no-purchase utility is held constant at zero to remove the impact of external competition.

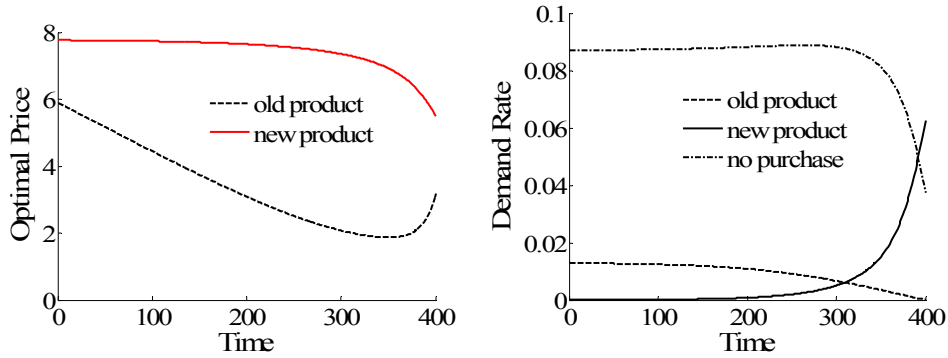


Figure 4: Optimal Prices and Demand at Inventory Level (1,1)

The optimal price tends to be higher when there is a positive probability of running out of a product. That is, scarcity itself increases the optimal price relative to ample inventory for a given time t . But for a “fixed” level” of scarcity, the optimal price has a stronger tendency (compared to the case of ample supply) to decline over time as the selling window becomes shorter. This constitutes a third dynamic factor, in addition to replacement and external competition that affects the pricing behavior over time. In the pricing literature, price decline over time due to the scarcity effect has been widely studied (e.g., Bitran and Mondschein 1997, Zhao and Zheng 2000).

In Figure 4, we still observe the replacement factor at the end of the transition. The price trend changes as the new product replaces the old. The replacement factor, and therefore the price uptick, depends on the rate of transition, namely the value of k . For a small k or constant $a_i(t)$, this factor disappears and the optimal prices would strictly decrease over time as the scarcity effect is the only remaining factor affecting price change over time.

5.4 Effect of Internal Competition

A fourth factor affecting price arises from competition between the old and new products. From equation (1), a price decrease of one product will not only increase its own demand, but also decrease the demand of the other product. The optimality condition requires the company to be in-

different between selling the two products, i.e., $r_1 - \Delta_1 V_{t+1}(x_1, x_2) = r_2 - \Delta_2 V_{t+1}(x_1, x_2)$; thus the price of the other product should decrease as well. Therefore, the competitive nature of the two products explains why the optimal prices tend to move together. We observe this clearly in Figures 2 and 3. In Figure 4, this characteristic is less pronounced as the impacts of product replacement and scarcity are also in effect. These four dynamic factors (replacement, scarcity, internal and external competition) are critical for understanding the optimal pricing strategies over time. One or more of these dynamics can become the dominant force that affects the shape of the price path under specific conditions. We summarize the impact of each factor in Table 1.

Dynamic Factors	Impact on Optimal Prices
Product Replacement	the price of the old decreases in time the price of the new increases in time
Scarcity	the prices tend to decrease over time for both products for a given amount of scarcity
Internal Competition	the prices of the old and new products move together
External Competition	the prices of the old and new products move in the opposite direction to changes in $u_0(t)$; typically external competition increases with time, so this factor results in downward pressure on both prices

Table 1: Impact of the Key Dynamics on Optimal Prices

5.5 Effect of Inventory

As products are sold, the inventory level for each product changes, which affects the optimal prices. When either $x_1 = 0$ or $x_2 = 0$, the relationship between r_i^* and x_i is as expected.

Proposition 4.

If $x_j = 0$, $r_i^(t, x_i)$ is non-increasing in x_i .*

The higher the inventory level, the lower the optimal price. However, in the presence of the other product, it is not clear how the inventory level affects the optimal price. Numerically, we find that a product's price still decreases with its own inventory level (Figure 5 (a)). The impact of inventory on the optimal price of the other product is more intriguing. Figure 5(b) shows how the optimal price of the old product changes with the inventory of the new product.

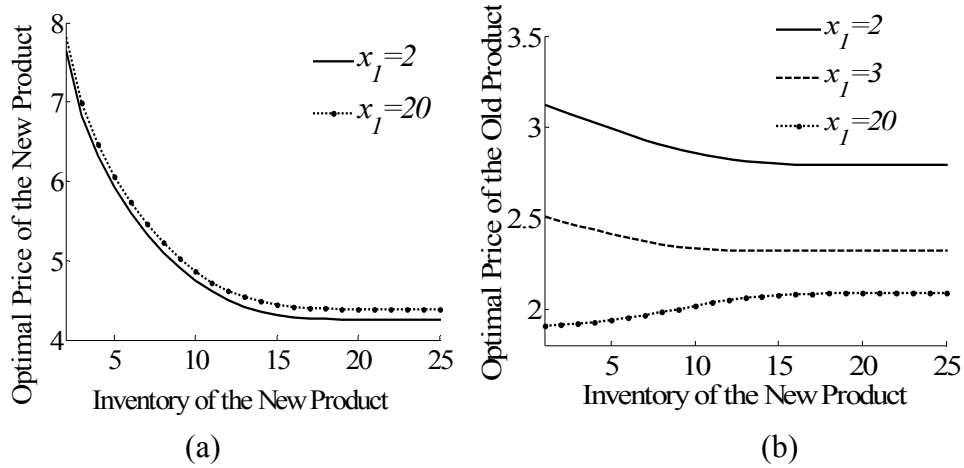


Figure 5: Optimal Price vs. Inventory

The vertical axis is the optimal prices of the new (Figure 5a) and old (Figure 5b) product at time $t = 200$. Other parameters are the same as in Figure 2 except for the inventory levels. In Figure 5(b), when the old product inventory is low, its optimal price decreases as the new product inventory increases; when the old product inventory is high, its optimal price increases in the new product inventory. This pattern arises from multiple dynamics that are at play. The two products compete with the outside option; thus there is pressure to decrease price when the total inventory of the two products goes up. In the mean time, there is competition between these two products. Increased inventory of the new product increases the risk of excess for the new product and calls for a price increase of the old to make the new appear more attractive. With more old product (e.g. $x_1 = 20$), the competitive nature of the two products are more pronounced as it becomes more likely that the company has to make a choice of which product to sell in the transition period; therefore the dominating impact is a price increase for the old product when the inventory of the new product increases.

6. Optimal Pricing with Periodic Demand Update

In a product transition, one major source of uncertainty comes from the demand. Demand, in turn, is largely determined by the customers' perception of the value of both products, which is

very difficult to predict. Experienced managers come up with an initial estimate. As new demand information becomes available, they use that information to update their beliefs. This suggests the possibility for Bayesian updates for the demand parameters, provided we have a tractable conjugate pair for the demand and parameter distributions. The MNL model assumes that a customer's utility contains a noise term that has type I extreme value (Gumbel) distribution, which does not have a simple form of conjugate distribution. Therefore, the Bayesian approach cannot be easily implemented for the MNL model. However, with our prior assumptions, the MNL demand model does lend itself to regression-based parameter estimates.

Under the MNL demand assumption, the log ratio of the demand for the old and new products is

$$\ln \frac{\lambda_1(r, t)}{\lambda_2(r, t)} = a_1(t) - g(r_1) - (a_2(t) - g(r_2)) = (a_1(t) - a_2(t)) - (g(r_1) - g(r_2)) \quad (15)$$

We assume further that $a_1(t) - a_2(t)$ and $g(r_1) - g(r_2)$ are linear (Assumptions 4 and 5). Thus

$$\ln \frac{\lambda_1(r, t)}{\lambda_2(r, t)} = a_0 - 2kt + \beta_r (r_2 - r_1)$$

Given the linear equation above, we can use ordinary least squares (OLS) regression to estimate the coefficients. We let $-2t$ and $(r_2 - r_1)$ be the independent variables and $\ln \frac{\lambda_1(r, t)}{\lambda_2(r, t)}$ be the dependent variable. Then we have the least squares specification $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\omega}$ where \mathbf{y} is the vector of observations of the dependent variable, \mathbf{X} is the matrix of observations on the independent variables and $\boldsymbol{\omega}$ is the noise vector.

Proposition 5.

Under the additional conditions that \mathbf{X} is full rank and $\boldsymbol{\omega}$ is uncorrelated with \mathbf{X} , the OLS regression yields unbiased, consistent and asymptotically efficient estimators of the coefficients a_0, k , and β_r .

From Proposition 5, we propose a more intuitive and pragmatic method to incorporate demand learning into the pricing model. As a product transition involves a new product, the company has to rely on expert knowledge, which is typically based on historical demand for similar product transitions, as well as estimates of the impact of product specific attributes.

The initial pricing decisions should be based on the initial parameter estimates. As we observe the demand realizations, we can update the initial parameter estimates. To do that, we perform regression on the actual demand data to get new parameter estimates. However, the accuracy and validity of these estimates depend on the amount of data that becomes available. As we observe more and more demand, we will have more confidence in the parameter estimates from the regression than from our initial estimates.

We use a weighted average of the initial estimates and the regression estimates as the new estimates for demand parameters, with the weight on the regression estimates increasing over time: Let $\hat{\boldsymbol{\beta}}(0) = (a_0(0), k(0), \beta_r(0))$ be the initial parameter estimates. At time t , we run a linear regression on equation (15) to obtain the parameter estimates $\hat{\boldsymbol{\beta}}(t)$ from the actual demand data. We then update the estimate of $\boldsymbol{\beta}$ using the following equation:

$$\boldsymbol{\beta}(t) = (1 - \alpha(t))\hat{\boldsymbol{\beta}}(0) + \alpha(t)\hat{\boldsymbol{\beta}}(t) \quad (16)$$

where $\alpha(t)$ is an increasing function of t , and is the weight given to the regression estimates at time t . For instance, we might set $\alpha(t) = t/T$. In this case, when t reaches T , the estimate of $\boldsymbol{\beta}$ should be based entirely on the actual demand.

The proposed regression method can help determine coefficient on prices, time, and potentially other attributes as well. Therefore, it provides room to assess the effect of various product attributes and/or marketing initiatives on the optimal prices and the expected profits. For example, if the managers believe that a particular attribute q (other than price or time) is important in a customer's choice, and that its impact on a customer's utility can be approximated by a linear equation, then we can use the equation $\ln \frac{\lambda_1(r, t)}{\lambda_2(r, t)} = a_0 - 2kt + \beta_r(r_2 - r_1) + \beta_q(q_1 - q_2)$ to perform the regression and incorporate attribute q into the demand learning model and ultimately the dynamic pricing decisions.

7. Discussions and Future Research

The main contribution of this research is to address the pricing problem in a special albeit ubiquitous industry context – inter-generational product transition. We solve for the optimal prices of the two generations of products for any given inventory level at any given time during the transition. From the optimal solution, we systematically identify the various factors and dynamics affecting the optimal prices. Lastly, we propose a simple technique to incorporate demand learning into the pricing decisions for a product transition.

We make several simplifying assumptions so that we can develop meaningful results and insights from the model for the complicated real problem under study.

The MNL choice model is the foundation of the demand model in this paper. Therefore, any limitations of the MNL model also apply to our model. For example, the IIA property may not be valid (Train 2003). Consider a case where customers are choosing laptop computers among Dell Latitude 620 (the old product), Dell Latitude 630 (the new product), and Lenovo Thinkpad T60 (the best external option). When the Latitude 630 runs out, the demand may not split proportion-

ally between Latitude 620 and Thinkpad T60, but rather favors the Latitude 620 if the customers are brand loyal. When the IIA property does not hold, we might consider an alternative formulation such as the nested logit model. In the nested logit model, customers first choose between the brands (Dell Latitude vs. Lenovo Thinkpad), and then choose within a brand (Latitude 620 vs. Latitude 630). In addition, if there are other factors that affect demand such as complementary product offerings, the MNL model, as presented in this paper, may not be appropriate.

The linear relationships in Assumptions 4 and 5 may invite further scrutiny. Clearly, if the quasi-linear utility assumption is dropped, the problem is less tractable. We might also relax the assumption that the two products have the same coefficient β . However, doing so would imply that the customer's attitude toward money depends on the product, which seems dubious.

We justify the linear assumption on $a_i(t)$ by deriving mathematical equivalency with the Fisher-Fry and Norton-Bass models. This assumption is also crucial for the demand learning approach we propose. However, it is possible to adapt the demand learning approach to a polynomial functional form of $a_i(t)$, as well as to allow interaction terms.

We assume a stationary customer arrival process. We can adapt our model to permit a time-varying λ to address seasonality or other demand cycles. In fact, the solution in Proposition 2 still holds with a time-varying λ .

Lastly, we address a dynamic pricing problem with only two generations of products. Often there are multiple generations of products selling during the same time period. In that case, the MNL model allows an easy extension - we simply add more choices to the MNL model. The diffusion-theory-based approach can also be extended to more than two generations of products (Norton and Bass 1987); but the Norton-Bass model becomes even more complex and intractable.

ble. Therefore, from this perspective, the MNL model has an advantage when considering multiple generations of products.

The demand learning approach we propose is relatively simple. For instance, it weighs each data point (more precisely, each time period) equally, which may be suboptimal. Nevertheless, it seems a reasonable place to start and a company might experiment to find, empirically, the optimal balance between the estimates from past sales of similar products and that from new demand observations of the products undergoing a transition.

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Pricing Decisions during Inter-generational Product Transition

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Appendix

Proof of Proposition 1.

From equation (1) and (4), we have

$$\frac{\partial^2 \rho_0(\mathbf{r}, t)}{\partial r_i^2} = \rho_0 \rho_i [(2\rho_i - 1)g'(r_i)^2 + g''(r_i)],$$

$$\frac{\partial^2 \rho_i(\mathbf{r}, t)}{\partial r_i^2} = \rho_i (\rho_i - 1) [(2\rho_i - 1)g'(r_i)^2 + g''(r_i)],$$

$$\frac{\partial^2 \rho_i(\mathbf{r}, t)}{\partial r_j^2} = \rho_i \rho_j [\rho_j g'(r_i) g'(r_j) + (\rho_j - 1)g'(r_j)^2 + g''(r_j)],$$

Therefore,

$$\begin{aligned} \frac{\partial^2 h_t(\mathbf{r}, x_1, x_2)}{\partial r_1^2} &= 2g'(r_1)\rho_1(\rho_1 - 1) \\ &\quad + [g''(r_1) + g'(r_1)^2(2\rho_1 - 1)][\rho_1(\rho_1 - 1)(r_1 - \Delta_1 V_{t+1}(x_1, x_2)) + \rho_1 \rho_2 (r_2 - \Delta_2 V_{t+1}(x_1, x_2))] \end{aligned}$$

Substituting equation (8) into the second order derivative yields

$$\begin{aligned} \frac{\partial^2 h_t(\mathbf{r}, x_1, x_2)}{\partial r_1^2} &= 2g'(r_1)\rho_1(\rho_1 - 1) + [g''(r_1) + g'(r_1)^2(2\rho_1 - 1)] \left[-\frac{\rho_1}{g'(r_1)} \right] \\ &= -\rho_1 \left[g'(r_1) + \frac{g''(r_1)}{g'(r_1)} \right] \end{aligned}$$

From Assumption 3, $\frac{\partial^2 h_t(\mathbf{r}, x_1, x_2)}{\partial r_1^2} \leq 0$ at the points with zero slope. Similarly, we can derive

that $\frac{\partial^2 h_t(\mathbf{r}, x_1, 0)}{\partial r_1^2} = -\bar{\rho}_1 \left[g'(r_1) + \frac{g''(r_1)}{g'(r_1)} \right] \leq 0$ at the points with zero slope. Thus, $h_t(\mathbf{r}, x_1, x_2)$ is

quasi-concave in r_1 .

Similarly, we show that $h_t(\mathbf{r}, x_1, x_2)$ is quasi-concave in r_2 . \square

Proof of Proposition 2.

From equation (5), we have

$$V_t(x_1, x_2) = V_{t+1}(x_1, x_2) + \lambda \rho_1(\mathbf{r}^*, t)(r_1^* - \Delta_1 V_{t+1}(x_1, x_2)) + \lambda \rho_2(\mathbf{r}^*, t)(r_2^* - \Delta_2 V_{t+1}(x_1, x_2))$$

Substituting condition (10) into the above yields

$$V_t(x_1, x_2) = V_{t+1}(x_1, x_2) + \frac{\lambda}{\beta_r} \frac{\rho_1(\mathbf{r}^*, t) + \rho_2(\mathbf{r}^*, t)}{\rho_0(\mathbf{r}^*, t)}$$

From equation (11),

$$\begin{aligned} \frac{\rho_i(\mathbf{r}^*, t)}{\rho_0(\mathbf{r}^*, t)} &= \exp(a_i(t) - u_0(t) - \beta_r r_i^*) \\ &= \exp(a_i(t) - u_0(t) - 1 - \beta_r \Delta_i V_{t+1}(x_1, x_2) - W(e^{a_1(t) - u_0(t) - 1 - \beta_r \Delta_1 V_{t+1}(x_1, x_2)} + e^{a_2(t) - u_0(t) - 1 - \beta_r \Delta_2 V_{t+1}(x_1, x_2)})) \end{aligned}$$

Therefore,

$$\frac{\rho_1(\mathbf{r}^*, t) + \rho_2(\mathbf{r}^*, t)}{\rho_0(\mathbf{r}^*, t)} = Z e^{-W(Z)} = W(Z)$$

$$\text{where } Z \equiv e^{a_1(t) - u_0(t) - 1 - \beta_r \Delta_1 V_{t+1}(x_1, x_2)} + e^{a_2(t) - u_0(t) - 1 - \beta_r \Delta_2 V_{t+1}(x_1, x_2)}$$

$$\text{Hence, } V_t(x_1, x_2) = V_{t+1}(x_1, x_2) + \frac{\lambda}{\beta} W(Z).$$

Similarly, we can show that this holds when one product runs out. \square

Proof of Proposition 3.

From equations (11),

$$r_1^*(t, \infty, 0) = s_1 + \frac{1}{\beta_r} [1 + W(e^{a_1(t) - u_0 - 1 - \beta_r s_1})] \quad \text{and} \quad r_2^*(t, 0, \infty) = s_2 + \frac{1}{\beta_r} [1 + W(e^{a_2(t) - u_0 - 1 - \beta_r s_2})]$$

As $a_1(t)$ decreases in t and $a_2(t)$ increases in t , we have $r_1^*(t, \infty, 0)$ decreases in t and $r_2^*(t, 0, \infty)$ increases in t .

$$\text{From equation (11), } r_i^*(t, \infty, \infty) = s_i + \frac{1}{\beta_r} [1 + W(Z)] \quad \text{where } Z = e^{a_i(t) - u_0(t) - 1 - \beta_r s_i} + e^{a_j(t) - u_0(t) - 1 - \beta_r s_j}.$$

$$\text{From Assumption 5, } \frac{da_1(t)}{dt} = -k \quad \text{and} \quad \frac{da_2(t)}{dt} = k.$$

$$\text{We then have } \frac{dr_i^*(t, \infty, \infty)}{dt} = \frac{1}{\beta_r} \frac{kW(Z)}{Z(1+W(Z))} (-e^{a_1(t) - u_0 - 1 - \beta_r s_1} + e^{a_2(t) - u_0 - 1 - \beta_r s_2}).$$

Therefore, $\frac{dr_i^*(t, \infty, \infty)}{dt} \leq 0$ iff $a_1(t) - \beta_r s_1 \geq a_2(t) - \beta_r s_2$, or equivalently $t \leq \frac{a_0 + \beta_r (s_2 - s_1)}{2k}$.

That is, $r_i^*(t, x_1, x_2)$ decreases in t for $t \in [0, \bar{t}]$ and increases in t for $t \in [\bar{t}, T]$, where

$$\bar{t} = \frac{a_0 + \beta_r (s_2 - s_1)}{2k}. \quad \square$$

Proof of Proposition 4.

From equation (11), $r_1^*(t, x_1, 0) = \Delta_1 V_{t+1}(x_1, 0) + \frac{1}{\beta_r} [1 + W(Z_1)]$ where $Z_1 \equiv e^{a_1(t) - u_0(t) - 1 - \beta_r \Delta_1 V_{t+1}(x_1, 0)}$

For the Lambert's W function, $W(Z_1) = \ln Z_1 - \ln W(Z_1)$. Thus we have

$$r_1^*(t, x_1, 0) = \Delta_1 V_{t+1}(x_1, 0) + \frac{1}{\beta_r} [1 + \ln Z_1 - \ln W(Z_1)] = \frac{1}{\beta_r} [a_1(t) - u_0(t) - \ln W(e^{a_1(t) - u_0(t) - 1 - \beta_r \Delta_1 V_{t+1}(x_1, 0)})]$$

Thus $r_1^*(t, x_1, 0)$ is increasing in $\Delta_1 V_{t+1}(x_1, 0)$.

Next we show by induction that $\Delta_1 V_{t+1}(x_1, 0)$ is non-increasing in x_1 .

When $t = T$, $\Delta_1 V_{T+1}(x_1, 0) = s_1$, thus the base case is true.

Assume for induction that $\Delta_1 V_{t+1}(x_1, 0)$ is non-increasing in x_1 , we show that $\Delta_1 V_t(x_1, 0)$ is non-increasing in x_1 .

From equation (13),

$$\begin{aligned} V_t(x_1, 0) &= V_{t+1}(x_1, 0) + \frac{\lambda}{\beta_r} W(Z_1) = V_{t+1}(x_1, 0) + \frac{\lambda}{\beta_r} [\ln Z_1 - \ln W(Z_1)] \\ &= (1 - \lambda)V_{t+1}(x_1, 0) + \lambda V_{t+1}(x_1 - 1, 0) + \frac{\lambda}{\beta_r} [a_1(t) - u_0(t) - 1 - \ln W(e^{a_1(t) - u_0(t) - 1 - \beta_r \Delta_1 V_{t+1}(x_1, 0)})] \end{aligned}$$

We then have

$$\begin{aligned} \Delta_1 V_t(x_1 + 1, 0) &= (1 - \lambda)\Delta_1 V_{t+1}(x_1 + 1, 0) + \lambda\Delta_1 V_{t+1}(x_1, 0) \\ &\quad - \frac{\lambda}{\beta_r} \ln W(e^{a_1(t) - u_0(t) - 1 - \beta_r \Delta_1 V_{t+1}(x_1 + 1, 0)}) + \frac{\lambda}{\beta_r} \ln W(e^{a_1(t) - u_0(t) - 1 - \beta_r \Delta_1 V_{t+1}(x_1, 0)}) \\ \Delta_1 V_t(x_1, 0) &= \Delta_1 V_{t+1}(x_1, 0) + \frac{\lambda}{\beta_r} W(e^{a_1(t) - u_0(t) - 1 - \beta_r \Delta_1 V_{t+1}(x_1, 0)}) - \frac{\lambda}{\beta_r} W(e^{a_1(t) - u_0(t) - 1 - \beta_r \Delta_1 V_{t+1}(x_1 - 1, 0)}) \end{aligned}$$

Thus

$$\begin{aligned} \Delta_1 V_t(x_1+1,0) - \Delta_1 V_t(x_1,0) &= (1-\lambda)[\Delta_1 V_{t+1}(x_1+1,0) - \Delta_1 V_{t+1}(x_1,0)] \\ &\quad - \frac{\lambda}{\beta_r} \ln W(e^{a_1(t)-u_0(t)-1-\beta_r \Delta_1 V_{t+1}(x_1+1,0)}) + \frac{\lambda}{\beta_r} \ln W(e^{a_1(t)-u_0(t)-1-\beta_r \Delta_1 V_{t+1}(x_1,0)}) \\ &\quad - \frac{\lambda}{\beta_r} W(e^{a_1(t)-u_0(t)-1-\beta_r \Delta_1 V_{t+1}(x_1,0)}) + \frac{\lambda}{\beta_r} W(e^{a_1(t)-u_0(t)-1-\beta_r \Delta_1 V_{t+1}(x_1-1,0)}) \end{aligned}$$

By induction assumption, $\Delta_1 V_{t+1}(x_1+1,0) < \Delta_1 V_{t+1}(x_1,0) < \Delta_1 V_{t+1}(x_1-1,0)$. As $W()$ is an increasing function, $\ln W(e^{a_1(t)-u_0(t)-1-\beta_r \Delta_1 V_{t+1}(x_1+1,0)}) > \ln W(e^{a_1(t)-u_0(t)-1-\beta_r \Delta_1 V_{t+1}(x_1,0)})$ and $W(e^{a_1(t)-u_0(t)-1-\beta_r \Delta_1 V_{t+1}(x_1,0)}) > W(e^{a_1(t)-u_0(t)-1-\beta_r \Delta_1 V_{t+1}(x_1-1,0)})$.

Therefore, $\Delta_1 V_t(x_1+1,0) - \Delta_1 V_t(x_1,0) < 0$, i.e., $\Delta_1 V_{t+1}(x_1,0)$ is non-increasing in x_1 . We already show that $r_1^*(t, x_1, 0)$ is increasing in $\Delta_1 V_{t+1}(x_1, 0)$; thus $r_1^*(t, x_1, 0)$ is non-increasing in x_1 .

Similarly, we can show that $r_2^*(t, 0, x_2)$ is non-increasing in x_2 . \square

Proof of Proposition 5.

In the specification $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\omega}$ where $\boldsymbol{\beta} = (a_0, k, \beta_r)$, the disturbance is $\boldsymbol{\omega} = \varepsilon_1 - \varepsilon_2$ where ε_i are iid random variables with Gumbel distribution $F(x) = e^{-e^{-x}}$

Thus, $E(\varepsilon_i) = \gamma$ where γ is the Euler-Mascheroni constant 0.57721... and $\text{var}(\varepsilon_i) = \frac{\pi^2}{6}$.

Therefore, $E(\boldsymbol{\omega}) = \mathbf{0}$ and $\text{var}(\boldsymbol{\omega}) = \frac{\pi^2}{3} \mathbf{I}$

The OLS yields $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}(\mathbf{X}'\mathbf{y})$.

We have

$$E(\hat{\boldsymbol{\beta}}) = E[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}] = E[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'(\mathbf{X}\boldsymbol{\beta} + \boldsymbol{\omega})] = E(\boldsymbol{\beta}) = \boldsymbol{\beta}$$

$$\text{var}(\hat{\boldsymbol{\beta}}) = \text{var}[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}] = \text{var}[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'(\mathbf{X}\boldsymbol{\beta} + \boldsymbol{\omega})] = \text{var}(\boldsymbol{\omega}) = \frac{\pi^2}{3} \mathbf{I}.$$

Thus, $\hat{\boldsymbol{\beta}}$ is unbiased.

Given that \mathbf{X} has full column rank, and $\boldsymbol{\omega}$ is uncorrelated with \mathbf{X} , then by Greene (2003),

$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}(\mathbf{X}'\mathbf{y})$ is also consistent and asymptotically efficient. \square