

Erratum: Optimizing Strategic Safety Stock Placement in Supply Chains

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In Graves Willems (2000) we consider the problem of finding the optimal placement of safety stocks in a supply chain with bounded demand and guaranteed service times. We develop a dynamic programming algorithm for supply chains that can be modeled as spanning trees.

The intent of this note is to correct an error in the algorithm. Ekaterina Lesnaia of MIT and Dr. Salal Humair of Optiant independently discovered the error. We wish to acknowledge and to thank Ms. Lesnaia and Dr. Humair for uncovering the error and for bringing it to our attention; they also have identified how to correct the error.

In order to describe the error and its correction, we will repeat part of the algorithm from Graves Willems (2000). The dynamic program evaluates a functional equation for each node in the spanning tree, in which the nodes have been labeled  $1, 2, \dots, N$  by the labeling algorithm in Graves Willems (2000). We define  $N_k$  for each node  $k$  to be the subset of nodes  $\{1, 2, \dots, k\}$  that are connected to  $k$  on the sub-graph with node set  $\{1, 2, \dots, k\}$ . There are two forms for the functional equation. First, the function  $f_k(S)$  is the minimum holding cost for safety stock in a sub-network with node set  $N_k$ , where we assume that the outbound service time for node  $k$  is  $S$ . Second, the function  $g_k(SI)$  is the minimum holding cost for safety stock in a sub-network with node set  $N_k$ , where we assume that the inbound service time for node  $k$  is  $SI$ .

At node  $k$  for  $1 \leq k \leq N-1$ , the algorithm determines either  $f_k(S)$  or  $g_k(SI)$ , depending upon the location of the node with higher label that is adjacent to  $k$ . If this adjacent node is downstream [upstream] of node  $k$ , then we evaluate  $f_k(S)$  [ $g_k(SI)$ ]. For node  $N$ , we can evaluate either functional equation.

To develop the equations for  $f_k(S)$  and  $g_k(SI)$ , we define the function  $c_k(S, SI)$  to be the minimum inventory holding cost for the sub-network with node set  $N_k$ , where node  $k$  has inbound service time  $SI$  and the outbound service time  $S$ . ***In Graves Willems (2000) the equation for  $c_k(S, SI)$  is incorrect as it is based on faulty assumptions, namely the assumptions that  $f_k(S)$  is non-increasing in  $S$  and that  $g_k(SI)$  is non-decreasing in  $SI$ .***

The correct expression is as follows:

$$c_k(S, SI) = h_k \left\{ D_k (SI + T_k - S) - (SI + T_k - S) \mu_k \right\} + \sum_{\substack{(i,k) \in A \\ i < k}} \min_{0 \leq x \leq SI} [f_i(x)] + \sum_{\substack{(k,j) \in A \\ j < k}} \min_{S \leq y \leq M_j - T_j} [g_j(y)] .$$

The first term is the holding cost for the safety stock at node  $k$  as a function of  $S$  and  $SI$ .

The second term corresponds to the nodes in  $N_k$  that are upstream of  $k$ . For each node  $i$  that supplies node  $k$ , we include the minimum inventory holding costs for the sub-network with node set  $N_i$ , as a function of  $SI$ . As the inbound service time to node  $k$ ,  $SI$ , is an upper bound for the outbound service time for node  $i$ , we need to minimize  $f_i()$  over the range of feasible service times for node  $i$ . In Graves-Willems (2000), we had erroneously assumed that the minimum occurs at  $f_i(SI)$ .

The third term corresponds to the nodes in  $N_k$  that are downstream of  $k$ . For each node  $j$ ,  $j \in N_k$  and  $(k, j) \in A$ , we include the minimum inventory holding costs for the sub-network with node set  $N_j$ , as a function of  $S$ . The outbound service time for node  $k$ ,  $S$ , is a lower bound for the inbound service time for node  $j$ . Thus, we now minimize  $g_j()$  over the range of feasible inbound

service times for  $j$ . In Graves-Willems (2000), we had erroneously assumed that the minimum occurs at  $g_j(S)$ .

The rest of the algorithm remains the same. The computational complexity of the algorithm does not increase with this correction, as one can avoid the additional minimizations within the calculation of  $c_k(S, SI)$  by making this part of the determination of the functions  $f_k(S)$  and  $g_k(SI)$ . Thus, the computational complexity of the algorithm remains of order  $NM^2$  where  $M$

is the maximum service time, which is bounded by the sum of the production lead-times  $\sum_{j=1}^N T_j$ .

## References

Graves, S. C., and S. P. Willems, "Optimizing Strategic Safety Stock Placement in Supply Chains," *Manufacturing & Service Operations Management*, Winter 2000, Vol. 2, No. 1, pp. 68-83.