Flow of dilute to semi-dilute polystyrene solutions through a benchmark 8:1 planar abrupt micro-contraction

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\textbf{Abstract}

We have studied the flow of thermodynamically ideal solutions of a high molecular weight (\(M_w = 6.9\) MDa) atactic polystyrene in the \(\theta\) solvent dioctyl phthalate (aPS in DOP) through a micro-fabricated 8:1 planar abrupt contraction geometry. The channel is much deeper than most micro-scale geometries, providing an aspect ratio of 16:1 and a good approximation to 2D flow in the narrow channel. The solutions span a range of concentration \(0.02 \text{wt.}\% < c < 0.6 \text{wt.}\%,\) encompassing the dilute to semi-dilute regimes and providing a range of fluid viscosities and relaxation times such that we achieve a range of Weissenberg numbers \((8.7 < Wi < 1562)\) and Reynolds numbers \((0.07 < Re < 11.2)\), giving elasticity numbers between \(31 < El < 295\). We study the flow using a combination of micro-particle image velocimetry (\(\mu\)-PIV) to characterize the flow field, pressure measurements to evaluate the non-Newtonian viscosity, and birefringence measurements to assess macromolecular strain. Flow field observations reveal three broad flow regimes characterized by Newtonian-like flow, unstable flow and vortex growth in the upstream salient corners. Transitions between the flow regimes scale with Wi, independent of El, indicating the dominance of elastic over inertial effects in all the fluids. Transitions in the flow field are also reflected by transitions in the relative viscosity (determined from the pressure drop) and the macromolecular strain (determined from birefringence measurements). The strain through the 8:1 contraction saturates at a value of \(\approx 4-5\) at high Wi. The results of these experiments broaden the limited set of literature on flow through micro-fluidic planar contractions and should be of significant value for optimizing lab-on-a-chip design and for comparison with modeling studies with elasticity dominated fluids.

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1. Introduction

Flow through a planar abrupt contraction is considered as a benchmark flow problem for research into non-Newtonian fluid flows since it mimics a class of flows which occur widely in industry and nature and due to the relative ease of numerical simulation compared with stagnation flows [1]. Flows of non-Newtonian polymer solutions through contraction geometries are found in diverse applications including ink-jet printing, fibre-spinning, porous media flows, blood circulation and flows of mucin solutions in, e.g., the lungs.

Early studies of this problem with shear thinning polyacrylamide solutions in axisymmetric contraction geometries used streak photography to reveal a fascinating sequence of vortex development and evolution in the salient corners upstream of the contraction entry [2–4]. The behaviour of the fluids was categorized into three broad regimes: vortex growth, diverging flow and unstable flow. Subsequent work, e.g. [5,6], demonstrated a great complexity and variety of vortex development phenomena that depended on fluid rheological properties (i.e. shear-thinning or Boger fluids) and also on the precise geometry of the experiment (i.e. planar or axisymmetric, and the shape of the re-entrant corners). In particular, larger vortices were observed in axisymmetric contractions than in planar contractions, shear-thinning fluids were found to generate larger vortices than Boger fluids in axisymmetric contractions, and in planar contractions no vortex formation was reported with Boger fluids.

Rothstein and McKinley [7,8] measured dimensionless pressure drops and velocity fields via streak photography for the flow of dilute monodisperse atactic polystyrene (aPS) Boger fluids \((0.025 \text{ wt.}\% , M_w = 2.25\) MDa aPS in oligomeric polystyrene) through axisymmetric contractions with various contraction ratios, \(\beta\), and re-entrant corner geometries. For a low contraction ratio of \(\beta = 2:1\) a steady elastic “lip vortex” was observed at the re-entrant corner. For contraction ratios \(4:1 < \beta < 8:1\), no lip vortex was observed but a corner vortex was seen, which was enhanced in size by increasing the...
Weissenberg number, $Wi = \dot{\varepsilon} \tau$, where $\dot{\varepsilon}$ is the strain rate and $\tau$ is the characteristic relaxation time of the fluid). Rounding of the re-entrant corners shifted the behaviour to higher values of $Wi$, but did not qualitatively change the structure of the flow field. Dimensionless pressure drop measurements showed increases corresponding to the reduction in fluid viscosity, suggesting that vortex development is a result of the fluid viscoelasticity and the extension of polymer molecules in the velocity gradient near the contraction throat.

Hunkeler et al.\,[9] and Yu et al.\,[10] have measured flow-induced birefringence in dilute monodisperse aPS in dekalin solutions ($0.01 \text{ wt.}\% < c < 0.04 \text{ wt.}\%$, 2 MDa $< M_w < 10$ MDa, $M_w/M_M = 1 - 1.1$) being sucked from a reservoir into a single-jet apparatus. Values for the birefringence, $\Delta n$, measured near the jet entrance reached up to $\sim$1-10th of the value expected for fully stretched polystyrene molecules, $\Delta n_{0}$. The model of Trelaor\,[11], based on the optical properties of strained networks was used to estimate that molecules stretched to $\sim30\%$ of their contour length at the jet entrance. However, the geometry of this experiment is poorly defined in that the effective contraction ratio is not known (and is probably very large), also the cylindrical symmetry complicates the determination of birefringence from intensity or retardation measurements. In addition, it is unclear that the Trelaor model, which derives the optical properties of strained networks, can be reasonably applied to describe the extension of polymers in dilute solution in transient extensional flows.

Nigen and Walters\,[12] studied the flow of Boger fluids ($M_w = 5$ MDa polyacrylamide in a mixture of glucose syrup and water) in both axisymmetric and planar contractions with various contraction ratios, $4:1 < \beta < 32:1$. The planar contraction geometries approximated 2D flow, having depths up to 5.6 times the width of the wide upstream channel. The excess pressure drop was measured across the contraction and streak images were photographed as the flow rate was varied. In axisymmetric contractions, significant “vortex enhancement” (i.e. vortex growth) was observed with increasing flow rate, which was associated with an increased pressure drop attributed to an increase in polymer extensional viscosity. However, such effects were absent in planar geometries. The lower elastic response in planar compared with axisymmetric contraction flows has been attributed to the lower strain rate and total accumulated fluid strain, $\varepsilon$, through the planar geometry for a given contraction ratio, $\beta$. In axisymmetric flows $\varepsilon = \beta^2$, whereas in planar flows $\varepsilon = \beta^5$\,[8]. However, Nigen and Walters used a variety of contraction ratios to produce higher strain rates and strains in the planar than in the axisymmetric geometry. Based on birefringence measurements in 0.3 wt.% polysisobutylene in polybutene Boger fluids, and comparisons with models, Genieser et al.\,[13] argue that increasing the contraction ratio of a planar contraction flow increases the total accumulated fluid strain through the contraction. However much of this strain occurs in a long tail upstream of the contraction throat at strain rates $\dot{\varepsilon} < \varepsilon / \tau$, where the polymer strain does not increase affinity with the fluid. Hence, increasing the contraction ratio in a planar contraction may provide diminishing returns in terms of increasing the elastic response of a polymer solution. The absence of corner vortex growth in planar contractions is in agreement with numerical simulations with Oldroyd-B fluids, which actually show a reduction in corner vortex size with increasing elasticity\,[14,15]. Additionally, simulations with shear-thinning Phan-Thien–Tanner (PTT) fluids in planar contractions have shown strong corner vortex growth with increasing $Wi$\,[16].

Quinzani et al.\,[17,18] used a 2D planar abrupt 4:1 contraction to study flow structures and flow-induced birefringence in concentrated (5 wt.%) shear-thinning $M_w = 1.2$ MDa polysisobutylene in tetradeacane solutions. The stress-optical rule was used to interpret the birefringence data and calculate the transient extensional viscosity of the polymer solution. The fluid demonstrated slight extension thickening followed by apparent extension thinning with increasing shear rate. The authors attributed the thinning behaviour to the decreased residence time in the region of high velocity gradient near the contraction throat. Reasonable agreement to the experimental data was found using the PTT model.

Within the last decade, experimental researchers have taken advantage of advances in microfabrication techniques in order to study micro-fluidic flows of complex fluids\,[19,20]. Micro-fluidic flows, with characteristic length scales $<100 \mu m$, are particularly relevant to applications such as ink-jet printing and lab-on-a-chip devices\,[21,22]. The reduction in characteristic length scale associated with micro-fluidics allows the study of previously inaccessible flow regimes, due to the combined effects of a reduction in the Reynolds number, $Re$, and a simultaneous increase in the applied strain rate, $\dot{\varepsilon}$ (and hence $Wi$), for a given volume flow rate. A comprehensive review of studies of complex fluid rheometry in micro-scale devices is provided by Pipe and McKinley\,[23]. Even more recent work by Haward et al.\,[24,25] has used a micro-scale cross-slot to study the stagnation point extensional flow of dilute poly(ethylene oxide) (PEO) and dilute closely monodisperse aPS solutions using birefringence and pressure drop measurements in combination with micro–particle image velocimetry (μ-PIV).

Rodd et al.\,[26] utilized a micro-fabricated planar abrupt contraction with $\beta = 16:1$ to study the entry flow of aqueous PEO fluids ($0.05 < c < 0.3 \text{ wt.}\%$, $M_w = 2$ MDa) by measuring the flow field and the excess pressure drop. The small length scale of the contraction throat ($\sim 13 \mu m$), and the variety of fluid rheological properties due to the concentration range, resulted in a range of Reynolds numbers ($0.44 < Re < 64$) and Weissenberg numbers ($0 < Wi < 548$). The authors examined in detail the effect of Re and Wi on the flow dynamics upstream of the contraction throat, finding a number of different flow regimes (Newtonian-like, symmetric vortex growth, bistable vortex growth, asymmetric vortex growth, etc.), which evolved differently depending on the elasticity number ($El = Wi/Re$) of the fluid. Vortex growth was observed for $Wi > 50$.

In a subsequent work, Rodd et al. used a similar planar abrupt contraction, but varied the elasticity number between 2.8 $\leq El < 68$ for a fixed PEO concentration ($c = 0.075 \text{ wt.}\%$) by varying the solvent viscosity with added glycerol\,[27]. Four broad flow regimes were identified in the streak imaging: Newtonian-like flow, steady viscoelastic flow, diverging flow and vortex growth. The transitions between flow regimes were correlated with changes in the measured axial flow velocity and the measured pressure drop. They observed steady Newton-like flow for $Wi < 3$, steady viscoelastic flow for $3 < Wi < 10$, unstable diverging flow for $10 < Wi < 20$ and a vortex growth regime beyond $Wi > 20$. The lower values of onset Wi for the various flow regimes in this, compared with their previous work, were largely attributed to the use of calculated Zimm relaxation times as opposed to using relaxation times measured with a capillary breakup extensional rheometer (CaBER). The Zimm times were shown to be substantially lower than the CaBER times. The varying of the solvent viscosity by using mixtures of aqueous glycerol would also have caused some variation in the solvent quality between good and probably close to theta conditions. These changes in solvent quality will have had a dramatic, and unquantified, effect on the calculated Zimm relaxation time, and therefore also on the apparent Wi and El numbers of the experiments.

A recent study by Gulati et al.\,[28] reports on the flow of a highly elastic semi-dilute monodisperse $\lambda$-phage DNA ($M_w = 31.5$ MDa) in a physiological buffer solution into a planar abrupt micro-contraction with $\beta = 2:1$. The extremely long relaxation time of the $\lambda$-DNA and the small scale of the geometry resulted in high Weissenberg numbers ($Wi$ up to $\sim 629$) for very low Reynolds numbers, achieving elasticity numbers, $El > 10^5$. Gulati et al. found corner vortex growth at much lower Wi than observed by Rodd et al. The differences were attributed to the differing rheological prop-
properties of the DNA and PEO solutions and the comparative stiffness of the DNA molecules. The semi-dilute DNA solutions of Gulati et al. were significantly shear thinning in contrast to the dilute PEO fluids employed by Rodd et al.

The present investigation aims to extend the range of fluids and flow regimes to have been studied in micro-fluidic contraction entry flows, using ideal model solutions of a monodisperse sample of high molecular weight atactic polystyrene in dioctyl phthalate (aPS in DOP) under theta conditions. DOP is a viscous (ηDOP = 46 mPa s) theta solvent for aPS at close to room temperature. The polymer solutions cover a range of concentration spanning from dilute to semi-dilute, in a planar abrupt micro-contraction with an aspect ratio of 2:1, providing a range of elasticity numbers ≤ E ≤ 295. This was significantly deeper than any previously studied microchannels, giving a high aspect ratio of 16:1 and a good approximation to 2D flow in the narrow channel. It should be noted, of course, that this leads to a 2:1 aspect ratio in the wide parts of the channel. Flow field measurements using μ-PIV and pressure drop measurements across the contraction are combined with birefringence measurements in the contraction entry region. Birefringence measurements are compared with models to provide valuable information on the accumulation of macromolecular strain in the velocity gradients close to the contraction throat. This is the first time birefringence observations and measurements have been reported for such model polymer solutions in a planar contraction experiment of such small scale. The results obtained with this well-characterized polymer/solvent system and deep micro-scale planar geometry will be of great value for comparison with numerical modeling work.

2. Experimental

2.1. Micro-channel geometry and flow system

Fig. 1 shows a schematic of the planar 8:1 micro-contraction used in the experiments, showing the upstream channel width, w0 = 1.6 mm, the downstream channel width, wc = 0.2 mm, the contraction length, Le = 2 mm and the uniform channel depth, h = 3.2 mm. The aspect ratio of the narrow downstream channel, α3 = h/wc = 16, provides a good approximation to 2D flow. The length of the wide upstream channel was Lf = 30 mm, providing a ratio of Lu/w0 = 18.75, which should be sufficient to ensure fully developed upstream flow. The channel was cut through 316 stainless steel using wire electrical discharge machining with a 0.1 mm diameter wire. The channel was sandwiched and glued with silicone adhesive between standard glass microscope slides, annealed at 300 °C, to allow observation of flow field and birefringence patterns.

As shown schematically in Fig. 2, the flow was driven using a Chemxy N5000 syringe pump with a 50 ml stainless steel syringe (KD Scientific). The range of volume flow rates thus achieved was 50 ml/h ≤ Q ≤ 4000 ml/h, providing superficial flow velocities of Uc = Q/hw and Uf = Q/hwc in the upstream and downstream channels, respectively. A Druck differential pressure transducer was used to measure the pressure drop, ΔP, across the channel. The ports of the pressure transducer were situated 15 cm apart in the tubing beyond the upstream channels, so will have had a negligible effect on the flow dynamics. On the reasonable assumptions that the pressure drop across the narrow contraction dominates the measured ΔP, and that the velocity profile is parabolic across the channel in the case of the Newtonian solvent, the shear viscosity can be determined using the following formula for Poiseuille flow in a rectangular channel:

\[
\eta = \frac{h w c^2 \Delta P}{12 Q L c}
\]

For polymer solutions, deviations from Newtonian behaviour are quantified by applying Equation (1) and finding the relative viscosity, ηrel, as a function of Q. This is essentially equivalent to the dimensionless pressure drop used in previous related work [26–28], i.e.:

\[
\eta_{rel}(Q) = \frac{\eta_{solution}(Q)}{\eta_{DOP}} = \frac{\Delta P_{solution}(Q)}{\Delta P_{DOP}(Q)}
\]

2.2. Birefringence detection and measurement

Shown in Fig. 2 is a schematic representation of the optical line used for birefringence detection. The optical line consisted...
of a 50 mW temperature stabilized fibre-coupled diode laser from Oz Optics Ltd., which passed a circular, spatially filtered, Gaussian beam through a polarizer and an analyzer (crossed at ±45° to the flow axis on either side of the channel). The signal transmitted due to flow-induced birefringence in the liquid between the polarizer and analyzer was detected by a very low noise, high quantum efficiency, cooled CCD camera from Andor Technology Ltd. In addition, a λ/4 plate was inserted between the polarizer and analyzer to compensate residual birefringence in the optical line and thus improve the extinction. Also, a cylindrical lens was used to focus the laser beam into a narrow sheet aligned along the flow axis, allowing birefringence to be detected along the length of the channel. The birefringence, Δn, was calibrated by rotating a λ/30 plate between the polarizer and analyzer (with no fluid flow) to produce known values of retardation, R, and measuring the resulting intensity detected on the CCD array. Measured values of intensity resulting from flow-induced birefringence could thus be converted to values of retardation and then into values of birefringence using Δn = R/h, where h is the depth of the channel.

The experimental values of Δn are compared with the theoretical maximum value for the birefringence of a solution of fully stretched polystyrene molecules, Δn0 ~ −0.08c, where c is the polymer concentration [29]. Using a model such as the Treloar model for rubber elasticity [11], the value of Δn/Δn0 can be used to estimate the extension ratio (r/L, where L is the contour length and r is the end-to-end length). Values of Δn0 for the various test solutions are provided in Table 1.

The model described by Treloar derives the optical properties of a strained network and uses an approximation to the inverse Langevin function to give the birefringence of the network as:

\[
\frac{\Delta n}{\Delta n_0} = \frac{3}{5} \left( \frac{r}{L} \right)^2 + \frac{1}{5} \left( \frac{r}{L} \right)^4 + \frac{1}{5} \left( \frac{r}{L} \right)^6
\]

(3)

The molecular strain, ε, can then be determined using:

\[
\varepsilon = \frac{r}{L} \epsilon_{\text{max}}
\]

(4)

Where ε_{\text{max}} is the strain to full extension of the polymer molecule. For the M_w = 6.9 MDa aPS molecule used in this study, ε_{\text{max}} has been calculated using the known molecular parameters for dilute theta solutions to be ε_{\text{max}} ~ 84 [30].

The Treloar model assumes that a flexible polymer chain is held by the ends and stretched by increasing the end-to-end length, r. This is valid for a crosslinked rubber, where r represents the distance between crosslinks, and may be reasonable for an entangled network of molecules where (assuming no slip) r would repre-
Table 1

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<th>c, wt.%</th>
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<th>Δn0</th>
<th>η0, mPa s</th>
<th>r, ms</th>
<th>Re</th>
<th>Wi</th>
<th>El</th>
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<td>19.5–1562</td>
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<td>−2.4 × 10⁻⁴</td>
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<td>57</td>
<td>0.22–8.8</td>
<td>24.7–990</td>
<td>113</td>
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<tr>
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<td>30</td>
<td>0.25–9.8</td>
<td>13.0–521</td>
<td>53</td>
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<td>8.7–347</td>
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<td>20</td>
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<td>8.7–347</td>
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</table>

sent the distance between entanglement points. However, it is not clear that this assumption remains reasonable for dilute solutions of molecules being stretched in extensional flows (particularly for dilute solutions in transient extensional flows). Rallison and Hinch [31], Wiest et al. [32], Larson [33], Hinch [34] and Keunings [35] have used computer simulations that predicted kinks and hairpin folds during the chain uncoiling process, which could clearly result in values of Δn close to Δn0 for relatively low molecular strains or end-to-end separations, r. Such conformations have also been observed experimentally by directly imaging fluorescently labeled DNA molecules at the stagnation point of a cross-slot flow [36,37]. Haward et al. [24] have recently made measurements of the transient birefringence of well-defined dilute polymer solutions during the initial stages of stretching in a cross-slot, and found that the birefringence increased with fluid strain much more quickly than predicted by the Treloar model.

Birefringence arises only through the average orientation of chemical bonds to the stretch direction (neglecting a small contribution from form birefringence). Thus hairpins and fully extended chains give essentially the same birefringence, but would yield very different stress levels. In such high deformation regimes, therefore, the birefringence cannot be simply related to the stress through a stress-optical coefficient.

2.3. Micro-particle image velocimetry (µ-PIV)

For µ-PIV experiments, the solutions were seeded with 3 μm diameter fluorescent melamine resin micro-particles (Ex/Em = 560/584 nm) at a concentration of ~0.01 wt.%. An M = 10× objective lens (NA = 0.3) was used to focus on the mid-plane of the channel. The estimated measurement depth is about 30 μm, or ~1% of the channel depth, h [21]. Note that the relative measurement depth (1% of the channel depth) is much smaller than in our previous studies [38,39], hence the relative error in the velocity measurements is also smaller here. The fluorescent particles were excited with a 532 nm Nd:YAG laser and image pairs were captured with a SensicamQE double frame camera. The time interval (Δt) between paired images for generation of the velocity field was optimized to obtain the best results under a given flow condition. Image pairs were captured in sequence at a rate of 4 Hz after the pressure drop had reached steady state. The standard cross-correlation PIV algorithm and Nyquist criterion (TSI Insight3G...
2.4. Solution properties

The polymer sample used in the study was an atactic polystyrene (aPS) with $M_w = 6.9$ MDa and polydispersity index $M_w/M_n = 1.15$, obtained from Polymer Laboratories Ltd. The polymer was dissolved to various concentrations, $c = 0.03$ wt.%, 0.05 wt.%, 0.1 wt.%, 0.3 wt.% and 0.6 wt.% in dioctyl phthalate (DOP), following the method described by Odell and Carrington [40]. DOP is a fairly viscous solvent ($\eta_s = 0.046$ Pa s), which is also a theta solvent for polystyrene at close to room temperature (22 °C) [41]. The overlap concentration, $c^*$, for the 6.9 MDa aPS molecule in DOP is $\sim 0.3$ wt.% based on simple cubic packing of molecules of radius of gyration, $R_g$ [42]. Accordingly, the test solutions span a range of $0.1 \leq c/c^* \leq 2$, i.e. from the dilute to the semi-dilute regime.

The shear viscosities of the polymer solutions were measured using an ARES cone and plate rheometer, and are shown in Fig. 3a. For solutions of $c < 0.1$ wt. % the viscosities are essentially constant over the accessible range of shear rate, and are close to the viscosity of DOP, i.e. the solutions are essentially Boger fluids. Solutions of $c = 0.1$ wt.% and above show mild shear thinning for shear rates larger than 100 s$^{-1}$. The steady shear viscosities, $\eta_\infty$, of the solutions are listed in Table 1. Fig. 3b shows the specific viscosity as a function of the polymer concentration. The gradient of the straight line fit to the data gives an estimate of the intrinsic viscosity, $[\eta] = 235 \pm 23$ cm$^3$ g$^{-1}$. This is in good agreement with the value calculated using the Mark–Houwink coefficients for aPS in DOP: $K = 0.08$ cm$^3$ g$^{-1}$ and $a = 0.5$ [30], giving $[\eta] = KM_n^a = 210$ cm$^3$ g$^{-1}$.

Polymer solution relaxation times, $\tau$, were characterized using a capillary breakup extensional rheometer (CaBER; Thermo Fisher Scientific), the results of which are shown in Table 1. A relaxation time for this molecule in dilute solution in DOP has been measured previously in the extensional flow oscillatory rheometer (EFOR) by looking at the rate of increase in birefringence, $\Delta n$, with strain rate, see Haward et al. [24]. This method gave a value of $\tau \sim 2–4$ ms for the peak of the molecular weight distribution (see Fig. 4), in reasonable agreement with the Zimm value $\tau_{\text{Zimm}} = 1.1$ ms [43], calculated according to:

$$\tau_{\text{Zimm}} = \frac{0.2 \eta_\infty R_g^3}{K_BT}$$

Where $K_B$ is Boltzmann’s constant and $T$ is the temperature. However, the spectrum of relaxation times determined by Haward et al., shown in Fig. 4, extended to values of $\tau > 20$ ms (due to stretching of the high molecular weight tail of the distribution).

This is of a similar order to the values returned by the CaBER for the most dilute test solutions with $c = 0.05$ wt. % and 0.03 wt. %, suggesting the likelihood that in providing a single relaxation time, the CaBER measurement is most sensitive to the high molecular weight part of the polymer sample.

![Fig. 6](image1.png) **Fig. 6.** (a) Normalized transverse velocity profiles of $v_y$ for Newtonian flow of DOP upstream of the contraction plane compared with the analytical solution for fully developed flow, (b) normalized transverse velocity profiles of $v_y$ for Newtonian flow of DOP upstream of the contraction plane. Experiment performed at 23 °C.

![Fig. 7](image2.png) **Fig. 7.** (a) Axial velocity profiles of $v_x$ for Newtonian flow of DOP upstream of the contraction plane, (b) axial extension rates derived from (a). Experiment performed at 23 °C.
Fig. 8. Variation of flow field with Wi for four different solutions of 6.9 MDa aPS in DOP in the 8:1 planar contraction geometry. Experiment performed at 23°C.

Fig. 9. Instantaneous vector field (left) and associated streamline plot for 0.05 wt.% 6.9 MDa aPS in DOP, demonstrating the stable symmetric flow in the “Newtonian-like” flow regime at Wi = 69 (El = 32). Experiment performed at 23°C.
2.5. Flow regime characterization

Following the conventions established in recently published related papers [26–28], there are three dimensionless quantities used to characterize the flow regime in the 8:1 contraction in the present work. These are the Reynolds number, Re, the Weissenberg number, Wi, and the elasticity number, El.

The Reynolds number defines the viscous flow regime and is calculated in terms of the hydraulic diameter of the contraction, \(D_h = \frac{2wch}{wc + h}\), and the superficial flow velocity in the contraction, \(U_c = \frac{Q}{wc}h\).

\[
Re = \frac{\rho U_c D_h}{\nu_0}
\]

(6)

Where \(\rho = 0.985 \text{ g ml}^{-1}\) is the fluid density.

The Weissenberg number is used to define the elastic flow regime and is the product of the characteristic relaxation time of the fluid, \(\tau\), and the average shear rate in the contraction throat, \(\dot{\varepsilon}\):

\[
Wi = \dot{\varepsilon} \tau
\]

(7)

Where \(\tau\) is taken as the value measured with the CaBER, and \(\dot{\varepsilon} = 2U_c/w_c\).

The elasticity number represents the relative importance of elastic and viscous forces and is found to depend only on the fluid properties and the experimental geometry:

\[
El = \frac{Wi}{Re} = \frac{2\tau \eta_0}{\rho c D_h}
\]

(8)

The ranges of Re and Wi and the values of El for the various polymer solutions in the 8:1 contraction are provided in Table 1. At low concentrations the “fluid relaxation time” is actually the Zimm relaxation time of the dilute polymer coils. This becomes essentially concentration invariant at around an average of 10 ms. This would remain true even at close to infinite dilutions, so that the nominal Re, Wi and El become invariant. It is clear, however that such extreme dilute solutions would behave as pure solvent. The simple calculation of an elastic number (as in Table 1) in this way cannot fully characterize a solution. There must be a further concentration effect that allows the elastic properties of the fluid to disappear at low concentrations. Essentially we are calculating an elastic number for the macromolecules, rather than the solution.

3. Results

3.1. \(\mu\)-PIV and flow field observations

Fig. 5 shows the flow field upstream of the contraction plane measured for the flow of the viscous Newtonian solvent, DOP, using \(\mu\)-PIV. The flow field appears smooth and laminar at all three flow rates, i.e. does not change significantly with Re even at the highest Re of \(\sim 14\) (Fig. 5c). These are the typical results for Newtonian fluid in low Re number flow.

Velocity profiles taken transversely across the wide upstream channel at various distances before the contraction plane, for the flow of the Newtonian DOP solvent are shown in Fig. 6. Fig. 6a presents the x-component of the velocity, \(v_x\), and Fig. 6b presents the y-component of the velocity, \(v_y\). The velocity has been normal-
ized by the superficial flow velocity $U_u$ and the position across the channel has been normalized by half the upstream channel width, $w_u$. For distances greater than $\sim$ 1 mm before the contraction plane the $v_y$ tends to zero across the channel (Fig. 6b) and $v_x$ approaches the analytical solution for fully developed Poiseuille flow (Fig. 6a). This indicates that the flow is laminar and unperturbed by inertial effects.

Fig. 7a displays $v_x$ as a function of $x$ along the symmetry axis of the channel ($y=0$). The velocity maintains a constant value far from the contraction plane, but begins to increase rapidly as the contraction plane is approached. When the data of Fig. 7b is differentiated with respect to $x$ the extension rate along the flow axis is obtained (Fig. 7b). The extension rate is zero far from the contraction plane and increases to a maximum at the contraction throat.

Fig. 8 illustrates the effect of Weissenberg number on the flow field with four polymer solutions of concentration from 0.05 wt.% to 0.6 wt.%. The results for the 0.03 wt.% (El = 31) solution were not significantly different from the solvent and so are not shown. At 0.05 wt.% (El = 32) the flow field remains fairly Newtonian-like at low Wi. Streamlines begin to show elastic instabilities at higher flow rates (Wi = 347), however no lip or corner vortices are observed. The 0.1 wt.% solution (El = 53) shows a similar pattern of behaviour, with essentially laminar flow at low Wi (Wi = 104) and increasingly unstable flow for $\sim 200 < Wi < 400$. Increasing Wi further results in the formation of small vortices in the salient corners.

The same pattern is shown for the 0.3 wt.% solution with El = 113. Laminar flow is observed at low Wi (e.g. Wi = 99), followed by a period of unstable flow for $\sim 200 < Wi < 400$. Wi $\sim$ 400 marks the onset of formation of small corner vortices which develop in size (or “enhance”) with further increase in Wi.

In the 0.6 wt.% solution with El = 295 elastic instabilities are already apparent at Wi = 156. Small corner vortices are apparent at Wi = 468, which are enhanced in size with further increases in Wi up to 781.

A broad general trend of flow regimes followed by all of the polymer solutions with Wi can be summarized as follows:

- $Wi < \sim 150$ laminar, Newtonian-like flow,
- $150 < Wi < 400$ increasingly unstable flow,
- $Wi > \sim 400$ onset and enhancement of corner vortex with increasing Wi.

Figs. 9–11 show in greater detail the types of flow fields observed for aPS/DOP solutions within the three identified flow regimes of “Newtonian-like flow”, “unstable flow” and “vortex growth”, respectively.

We saw no clear evidence of lip vortex formation during our experiments, however this may be due to the relatively low resolution of the PIV technique close to boundaries and other regions, such as vortices, where fluid velocities are low. Such vortices can be resolved better by long exposure streak imaging, however this technique relies on the flow being stable, which is generally not the case here. In addition, previous authors have observed lip vortices over only very narrow Wi windows, which we may have missed in the present study [6,12,26,38]. Our examination of the flow field, summarized in Fig. 8, is intended to give an overview, not to pro-
vide a detailed account of the vortex evolution mechanisms, which have been studied previously [26,27,38,39].

In the vortex growth regime for Wi > \sim 400, the vortices grow in an unstable and asymmetric fashion, which was also observed by Rodd et al. [26] for Wi \sim 120 for a range of elasticity numbers.

As also found by Rodd et al. [27] we observe that transitions in the flow field are not strongly dependent on the elasticity number. It is likely that in our experiments, where the lowest elasticity number is 31, elasticity dominates over inertia in all cases.

It should be noted that the most dilute solutions tested here (c = 0.03 wt. % and 0.05 wt. %) can be defined as essentially Boger fluids, whereas the higher concentration semi-dilute solutions (c > 0.1 wt. %) showed a degree of shear thinning in the ARES cone and plate rheometer tests (see Fig. 3). Therefore the fact that vortices were not observed in the dilute solutions, but were observed in the semi-dilute solutions could be taken as consistent with earlier studies (e.g. [6,12]). However, such conclusions should not be drawn as it was not possible to reach Wi > 400 in the dilute solutions, due to the lower relaxation times of the fluids, and the limit on volume flow rate.

The three flow regimes identified here are generally consistent with those identified by Rodd et al. [27], though shifted to rather higher values of Wi. The major reason for this discrepancy is likely to be that Rodd et al. used a calculated Zimm relaxation time for their polymer solutions, which was found to be \sim 10 \times lower than relaxation times measured by the CaBER rheometer. However, our Wi regimes are also rather higher than those in Rodd’s previous paper [26], which also used CaBER measured relaxation times. This is perhaps due to the flow geometry: here we have used an 8:1 planar contraction as opposed to a 16:1 planar contraction as used by Rodd. The present experimental flow geometry also has a substantially larger downstream aspect ratio ($\alpha_d = h/w_c$) than used by Rodd (here $\alpha_d = 16$ compared with $\alpha_d \sim 2$ for Rodd). Omowunmi and Yuan [44] have used the Phan-Thien–Tanner model to show that increasing the aspect ratio of a planar contraction can have a similar effect to altering the fluid properties so as to reduce the elasticity number of the flow. The fact that no lip vortex was observed for any polymer solution under any flow conditions is also consistent with the models of Omowunmi and Yuan for contraction entry flow with high aspect ratio.

To summarize, we report three regimes in the flow field upstream of the contraction plane: Newtonian-like flow, unstable flow and vortex growth. The transitions between the flow regimes depend on Wi, rather than on El. We believe this is due to the dominance of elastic over inertial effects in our test solutions.
3.2. Pressure drop and viscometry

Fig. 12a shows the pressure drop measured across the micro-channel, ΔP, as a function of the Reynolds number, Re, for both the Newtonian solvent, DOP, and the 6.9 MDa aPS/DOP solutions at the various test concentrations. The DOP shows ideal Newtonian behaviour, i.e. a linear relationship between ΔP and Re, the gradient of which provides a value for the viscosity of \( \eta_{\text{DOP}} = 40 \, \text{mPa s} \), close to the expected value at room temperature. This value of \( \eta_{\text{DOP}} \) is used in Equation (2) to determine values of \( \eta_{\text{rel}} \) as a function of Wi for the polymer solutions. This is shown in Fig. 12b. All of the solutions show a region of pseudo-Newtonian constant viscosity (which increases with increasing polymer concentration) at lower Weissenberg numbers, before departing from Newtonian-like behaviour for \( Wi > 150 \), broadly corresponding to the point of departure from the steady Newtonian-like flow field. Beyond \( Wi > 150 \) there is a significant and rapid increase in the relative viscosity, which reaches \( 10–20 \) times the solvent viscosity for the highest concentration solutions. Solutions of concentration \( c > 0.1 \, \text{wt.\%} \) appear to reach plateau values of \( \eta_{\text{rel}} \), which roughly scale with polymer concentration. The plateau value is reached at \( Wi \sim 400 \), which corresponds to soon after the initial point of vortex development observed in the \( \mu\)–PIV measurements.

3.3. Birefringence and molecular strain

Figs. 13–17 show the birefringence observed near the entry region of the 8:1 contraction for progressively increasing concentrations of aPS/DOP solutions, over a range of shear rate and Weissenberg number. The intensity scales in the images are non-linear, meaning that the scale has been individually adjusted in each image to provide the optimum contrast.

The lowest concentration solutions \( c = 0.03–0.05 \, \text{wt.\%} \), Figs. 13 and 14, respectively) are characterized by unusually localized birefringence along the walls of the downstream narrow channel, which originates suddenly from near the re-entrant corners. In both cases, low level birefringence was first observed for \( 100 < Wi < 200 \) and increased in intensity with increasing Wi, though even for the highest accessible Wi (347) the birefringence remained localized close to the channel walls. Within this "wall birefringence" regime, the birefringence remains stable and of reasonably constant intensity for a given strain rate, i.e. fluctuations are low.

Increasing the aPS concentration to 0.1 wt.\%, localized wall birefringence is again observed for \( 100 < Wi < 300 \) (Fig. 15a). However, with increasing Wi, a second definable regime is observed in which additional birefringent strands are formed within the interior of the channel (Fig. 15b–d). In this "stranded birefringence" regime the strands generally increase in intensity with increasing Wi, however both the intensity and position of the individual strands can fluctuate significantly on time scales \( \sim 0.1 \, \text{s} \).

Further increasing the polymer concentration to 0.3 wt.\% wall birefringence is again observed for \( 100 < Wi < 300 \). Higher Wi again result in the stranded birefringent structures (Fig. 16a, b) similar to those observed at 0.1 wt.\% (Fig. 15c, d). However, Fig. 16b (Wi = 590) also marks a point of transition to a third regime of general bright birefringence across the narrow channel (Fig. 16c, d). This regime is also characterized by "upstream birefringence" originating from within the wide upstream channel as opposed to near the contraction throat.
Fig. 15. Birefringence observed in a 0.1 wt.% solution of 6.9 MDa aPS in DOP in the 8:1 contraction under the following flow conditions: (a) $\dot{\varepsilon} = 8700 \text{ s}^{-1}, \text{Wi} = 260$; (b) $\dot{\varepsilon} = 12200 \text{ s}^{-1}, \text{Wi} = 365$; (c) $\dot{\varepsilon} = 14000 \text{ s}^{-1}, \text{Wi} = 417$; (d) $\dot{\varepsilon} = 17400 \text{ s}^{-1}, \text{Wi} = 520$. Experiment performed at 23°C.

Fig. 16. Birefringence observed in a 0.3 wt.% solution of 6.9 MDa aPS in DOP in the 8:1 contraction under the following flow conditions: (a) $\dot{\varepsilon} = 6900 \text{ s}^{-1}, \text{Wi} = 400$; (b) $\dot{\varepsilon} = 10500 \text{ s}^{-1}, \text{Wi} = 590$; (c) $\dot{\varepsilon} = 14000 \text{ s}^{-1}, \text{Wi} = 790$; (d) $\dot{\varepsilon} = 17400 \text{ s}^{-1}, \text{Wi} = 990$. Experiment performed at 23°C.
Fig. 17. Birefringence observed in a 0.6 wt.% solution of 6.9 MDa aPS in DOP in the 8:1 contraction under the following flow conditions: (a) \( \dot{\varepsilon} = 4340 \text{ s}^{-1}, \) \( Wi = 390; \) (b) \( \dot{\varepsilon} = 6900 \text{ s}^{-1}, \) \( Wi = 625; \) (c) \( \dot{\varepsilon} = 10500 \text{ s}^{-1}, \) \( Wi = 940; \) (d) \( \dot{\varepsilon} = 14000 \text{ s}^{-1}, \) \( Wi = 1250. \) Experiment performed at 23\( ^\circ \)C.

Fig. 17 shows more detail of the transition between the stranded and the upstream birefringence regimes in a 0.6 wt.% aPS/DOP solution. Fig. 18 shows how the birefringence increased in the upstream channel approaching the contraction throat. This was measured in a 0.6 wt.% solution at \( Wi = 1090. \) Birefringence could be measured up to almost 3 mm (or \( 15 \times w_c \)) upstream of the contraction plane, and increased monotonically to a maximum at the contraction entry.

The onset \( Wi \) for wall birefringence corresponds closely to the region where the flow field changes from Newtonian-like to unstable and to the point where the relative viscosity starts to increase from its pseudo-Newtonian constant value. It is not surprising that changes in the flow field and viscosity occur when the polymer starts to stretch, however it is unclear why the stretching initiates along the downstream channel walls. It is possible that at these moderate flow rates the highest velocity gradients are experienced along streamlines that pass near the re-entrant corners, which may introduce a significant extensional component into the flow field. The simulations of Coates et al. [45] using a modified upper-convected Maxwell model and a modified Chilcott–Rallison dumbbell model, show high velocity and stress gradients near sharp re-entrant corners in 4:1 and 8:1 axisymmetric contraction geometries. This may explain the observations of previous authors that rounding of the re-entrant corners shifts behaviour to higher values of \( Wi \) [6,8]. Rounding of the corners would be expected to have the effect of reducing the extensional strain rate for a given flow rate. However, calculations using the stress–optical rule \( (\sigma = \Delta n/C, \) where \( \sigma = \text{stress} \) and \( C = \text{stress-optical coefficient} = -5 \times 10^{-9} \text{ Pa}^{-1} \) \([46,47] \)) indicate that the levels of birefringence measured at the walls could be generated by the shear stresses in the narrow downstream channel.

The appearance of upstream birefringence occurs for \( Wi \) slightly greater than that corresponding to the onset of vortex development, near to where the relative viscosity reaches a plateau. The reason for this is fairly clear; the presence of vortices in the salient corners upstream of the contraction has the effect of shifting the contraction plane further back upstream resulting in polymers accumulating strain in the upstream channel. This is certainly due to a complex feedback between polymer orientation and the flow field. The presence of vortices either side of the contraction results in the contraction no longer being planar abrupt, but rather more similar to a hyperbolic contraction geometry. This modification of the flow field by the stretching polymer serves to moderate the applied strain rate, since the velocity increase of \( \sim 8 \) times now occurs over a significantly longer length scale than before. We speculate that there is an energy minimization mechanism involved, whereby the energy cost of vortex development is lower than the energy gain from reducing the velocity gradient.

The birefringence in the contraction throat has been measured and is presented in normalized form as a function of the Weissenberg number in Fig. 19a. Due to the often unstable nature of the flow and the resulting random fluctuations in the position and...
intensity of the birefringent strands in the channel, the birefringence values shown in Fig. 19a are averaged across the channel width, \( w_c \). It is clear how closely changes in the slope of the birefringence, \( \Delta n \), correspond to changes in the flow field, with the initial increase in \( \Delta n \) occurring at \( Wi \sim 150 \), and the plateau occurring at \( Wi \sim 500 \). Maximum values for the birefringence at high \( Wi \) tend to reach \( \approx 0.1-0.2 \) times the value expected for fully stretched molecules, \( \Delta n_{0} \), provided in Table 1. It is possible use \( \Delta n/\Delta n_0 \) in the Treloar model based on an approximation to the inverse Langevin function to estimate values for the polymer molecular strain, \( \varepsilon \) (see Equations (3 and 4)). For \( 0.1 < \Delta n/\Delta n_0 < 0.2 \), this model provides values of \( 33 < \varepsilon < 46 \). Clearly this is unrealistic for flow through an 8:1 contraction, which can provide a maximum fluid strain of \( \sim 8 \). In fact, analytical solutions for the parabolic Newtonian flow profiles across the upstream and downstream channels, show a centerline velocity ratio of only \( \sim 6.2 \).

Haward et al. [24] have measured the birefringence as a function of fluid strain for the transient stretching of a 0.03 wt.% \( M_w = 6.9 \text{MDa} \) aPS in DOP solution during the start-up of a high Weissenberg number (\( Wi \sim 75 \), based on Equation (7)) flow in a micro-scale cross-slot. The results are reproduced in Fig. 19b, alongside the prediction of the Treloar model. The discrepancy between the Treloar prediction and the experiment is most likely explained by the formation of folded or hairpin structures in the real flow, resulting in high values of birefringence for relatively low fluid strains. The experimental data of Haward et al. are well fitted by a Boltzmann sigmoidal curve. Using this to determine the fluid strain from the measured birefringence provides Fig. 19c, showing maximum strain values of \( 4 < \varepsilon < 5 \), much closer to what should be expected through an 8:1 contraction. This is further evidence that the Treloar model does not accurately describe the stretching of polymers in solution in transient extensional flows. Changes in the slopes of the curves on Fig. 19c correspond closely to the transitions between regimes in the flow field and transitions in the relative viscosity.

The salient points are that we see that birefringence also shows three broad regimes of behaviour, similar to the regimes in the flow field to which they are clearly related. The Treloar model results in apparent molecular strains much higher than the maximum possible fluid strain of \( \sim 8 \). Much more reasonable estimates of 4–5 strain. Macromolecular strain fits well with the three identified flow field regimes.

### 4. Summary and conclusions

We have presented the results of flow field observations, pressure drop measurements and observations and measurements of flow-induced birefringence made on dilute to semi-dilute solutions of a well-characterized sample of aPS under theta conditions in DOP in flow through an 8:1 planar abrupt contraction. We have utilized wire electrical discharge machining to fabricate a micro-scale device with a significant depth, providing a large downstream aspect ratio of \( a/d = 16 \) and making birefringence measurements possible for the first time in such a small scale contraction experiment. The variation of the polymer concentration between 0.03 wt.% and 0.6 wt.% results in a variation of the fluid viscosity and relaxation time, which combined with the small scale of the geometry, thus provides a range of elasticity numbers \( 31 < \nu < 295 \). The lower concentration solutions are essentially non-shear thinning (or Boger) fluids, while the higher concentration solutions display mild shear thinning. Due to the long relaxation times of the more concentrated solutions we have been able to access high Weissenberg number regimes up to \( Wi \sim 1500 \).

Using \( \mu-\text{PIV} \) we have observed three distinct regimes in the flow field upstream of the contraction plane: Newtonian-like flow, unstable flow and vortex growth, broadly similar to those reported by Rodd et al. [26,27] with PEO fluids. We find that the transitions between the flow regimes do not significantly depend on \( \nu \), for \( 31 < \nu < 295 \), but rather on the \( Wi \), with the transition between Newtonian-like and unstable flow occurring at \( Wi \sim 150 \) and the transition to the vortex growth regime occurring at \( Wi \sim 400 \). This can be explained by the dominance of elasticity over inertial effects in all our test solutions.

Pressure drop measurements reveal pseudo-Newtonian viscosities for the polymer solutions in the Newtonian-like flow regime. Subsequently, when the flow becomes unstable the pressure begins...
to increase non-linearly with the flow rate, or Wi, resulting in a non-Newtonian increase in the solution viscosity. The viscosity increases with Wi up to a plateau value, which occurs close to Wi ∼400, or the onset of the vortex growth regime.

Observations of flow-induced birefringence made near the contraction throat area also display three broad regimes of behaviour, slightly offset to higher Wi than the regimes in the flow field and pressure drop. Although the transitions in birefringence regimes are offset from the transitions in flow field regimes, the two are clearly related. No significant birefringence is observed until the early stages of the unstable flow regime, where we see localized wall birefringence (most likely shear induced) for ∼100 < Wi < ∼300. As the Wi increases and the upstream flow becomes very unsteady (∼300 < Wi < ∼600) a regime characterized by numerous fluctuating birefringent strands is observed. Finally for Wi > ∼600, following the development of vortices, we observe general bright birefringence across the width of the narrow channel and also observe birefringence originating from significantly upstream of the contraction plane.

Measurements of the birefringence, averaged across the width of the capillary, have been used to estimate the macromolecular strain accumulated on passage through the constriction. The often Treloar model results in apparent molecular strains of between 33 and 46, which is clearly impossible for flow through an 8:1 planar contraction, which can provide a maximum possible fluid strain of ∼8. However, use of previously determined empirical data for the transient build up of birefringence to steady state in a cross-slot experiment, yields values of ∼4–5 for the maximum macromolecular strain through the 8:1 contraction (Fig. 19).

The macromolecular strain as a function of Wi fits well with the three identified flow field regimes. The strain is essentially zero for Wi < ∼150 (i.e. in the Newtonian-like flow regime) and increases rapidly during the unstable flow regime. Finally, the strain reaches a plateau for Wi > ∼400, corresponding to the onset of vortex growth.

Our results indicate that the increasing modifications to the flow field upstream of the contraction plane are due to the non-Newtonian effects of polymer deformation in, and downstream of, the capillary entrance region. This begins with the shear induced orientation of polymer molecules along the capillary walls, leading to unstable flow upstream, and develops with Wi into general stretching of the molecules across the full width of the channel. As the average macromolecular strain across the channel saturates, further increases in Wi, result in the formation and development of upstream vortices. The vortices serve to moderate the effective value of Wi by increasing the length scale over which fluid accelerates into the contraction (effectively modifying the flow geometry into a hyperbolic contraction). The vortices themselves seem to be space-filling regions of recirculating fluid, driven by the shear stresses induced by the fluid flowing along the central region of the upstream channel.

This work with well-defined polymer solutions at theta conditions should be of significant value for comparison with modeling of planar contraction flows dominated by elasticity.

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References

[8] J.P. Rothstein, G.H. McKinley, The axisymmetric contraction-expansion: the role of extensional rheology in the growth of upstream vortices. The vortices serve to moderate the effective value of Wi by increasing the length scale over which fluid accelerates into the contraction (effectively modifying the flow geometry into a hyperbolic contraction). The vortices themselves seem to be space-filling regions of recirculating fluid, driven by the shear stresses induced by the fluid flowing along the central region of the upstream channel.

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